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"DEVELOPING AN AUTOMATIC REAL MAP"

By

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A Thesis
Presented in partial fulfilment for the requirements of
The of Doctor of philosophy in
Surveying Engineering

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DEVELOPING AN AUTOMATIC REAL MAP

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STATEMENTS

This thesis is submitted to Shoubra Faculty of Engineering, Benha University for the degree of Doctor of philosophy in Surveying Engineering.

The work included in this thesis was carried out by the author in the Department of Surveying, Benha University, from 2/ 2010 to 5/ 2019.

No part of this thesis has been submitted for a degree or qualification to any other University or institution.

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Abstract

The earth as a planet is geometrically represented as an ellipsoid or a sphere where geodetic computations should be followed. In small areas and as a special case, the considered area can be treated as a plan and plan metric computations are followed. The surveying elements to be introduced to the user could be distances, bearings, azimuths, and areas. These elements can be obtained by computing them from either map (projected) coordinates or from geodetic coordinates.

In the past, not everybody could deal with the geodetic coordinates, so map projection has been introduced to facilitate dealing with the map using metric units. Nowadays computers and computer programming enable us to deal easily with geodetic computations and geodetic maps. In this research, the proposed computerized real geodetic map is introduced. The computations which have been done to clear the idea of the proposed map and their results are tabulated and illustrated.

The distortion is the less if all the earth was projected on aspects of a cube than the projection on single plane only; the distortion would be less than the projection in the polyhedral. This means that the distortion disappears when the earth's surface was treated without projection; then it is possible to produce a map directly from the representation of latitude and longitude. So, the values could be gotten directly from geodetic calculations (geodetic azimuths, geodetic distances and ellipsoidal area) without projection or distortions. By this way the dilemma of distortion and scale factor will be finished.

In this thesis trial have been made to obtain automatic real map; It is digital map which is represented by latitude and longitude directly; this map depends on digital form in point tables by latitude and longitude coordinates. Calculation of distances, azimuths, and areas will be done by using the appropriate geodetic equations by hot keys ad-joint to the digital map; these points are defined in geodetic datum like WGS84 and EGD (Egyptian Geodetic Datum). The map can be plotted when a hard copy is needed.

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Abbreviations

- EGD30 Egyptian Geodetic Datum
- GPS Global Position System
- TS Total Station
- GTS Geodetic Total Station
- WGS84 World Geodetic System1984 ellipsoid
- Φ Astronomical latitude
- Λ Astronomical longitude
- ϕ Geodetic latitude
- λ Geodetic longitude
- ξ Meridional component of the deflection of the vertical
- η Prime vertical component of the deflection of the vertical
- N Geoid height (geoidal undulation)
- h Ellipsoid height
- H Orthometric height
- a Semi-major axis of reference ellipsoid
- f Flattening of reference ellipsoid
- e Eccentricity of reference ellipsoid
- α Geodetic Azimuth
- UTM Universal Transverse Mercator
- UPS Universal Polar Stereographic
- M and N The meridian and prime vertical radii of curvature of the ellipsoid
- (X,Y,Z) Geodetic Cartesian Coordinates.
- ϵ The spherical excess
- F area Area of spherical triangle
- P area Area of plan triangle computed by geodetic distances
- (u, v, w) The local horizon system coordinates
- GNSS Global Navigation Satellite System
- GMIS Global Map Index System

1. INTRODUCTION

Mapping is important for the progress in any country; Map Production depends on the science of map projection; itis representation of the earth or a portion of the earth on a plane or developer surface which is expressed mathematically and converts geographic coordinates (latitude and longitude) to Projected coordinates (Easting and Northing). The projection surface is sometimes plane or cone or cylinder.

The projection surface may take a normal position, oblique or transverse with respect to the earth surface. The two surfaces are sometimes touch or intersect surfaces. Also, the projection can be from central point and it may be expressed mathematically through equations only. Equal-area projection can be used to preserve the area without distortion. Conformal projection can be used to preserve the directions and shape without distortion. Equidistant projection is used to keep the distances without distortion. All of these projections are aimed to decrease the distortions.

What Is a Map? Surprisingly, this is a question for which there is no easy answer. One knows what map is, but that definition can vary from person to another and from culture to another. A general definition from 40 years ago was, “A graphic representation of the globe or a part of the globe drawn to scale upon a plane”. However, questions arose. What about the moon and other extra-terrestrial features? If it looks like a map but lacks an indication of its scale, is it a map? Can an annotated satellite image (one with names of features printed on it) be considered a map? Is a globe a map? What about 3-Drepresentations? Purists would say that a “map” with no scale is a diagram and that3-D representations are models. The moon and planets could be handled by inserting “or other celestial body” into the basic definition [Tyner J. A., 2010].

In cartography and surveying, the earth’s surface is substitute by a sphere or ellipsoid that approximates the natural ground surface of the globe in overall shape. In the calculation of map projections, the earth sphere or ellipsoid is called the datum. Transformations between the 3-dimensional earth curvature and a2-dimensional map are called map projections. The small change in dimensions distorts at least one of the following properties: distance, area, direction, and shape. Different map projection systems have developed according to the purpose and scale of the map. There is no ideal map projection for all applications of mapping.

When a survey has to be done for the construction of engineering works, such as a line of communication, a port or a harbour, that map or plan must be drawn so that the engineer may

apply a scale to any part of it and read of actual distance. A plan of this kind is constructed without reference to the earth's curvature, or reduction to sea level. When, however, the area covered by the survey is large, some method of representation must be adopted which will allow for the spheroidal form of the earth, and for the reduction of the whole survey to a common datum plane – usually that of the mean sea level. With the exception of the geographical globe, all maps are drawn on a plane surfaces – i.e., it cannot be unrolled into a flat sheet – all systems of map projection produce distortion in a great or less degree, the amount of produced distortion depends on the method of projection adopted and the extent of the area represented.

It is useful to express the 3D positions of points near the earth’s surface in terms of latitude, longitude and height above an ellipsoid of defined shape. This system gives a precise and straightforward definition of any point’s location, in terms of parameters (east, north and up) which are convenient to use everywhere around the world. It is often necessary to record the positions of points, boundaries and natural features such as coastlines, on a map—and this too is usually done by showing the (east, north) positions directly on a flat 2D projection, and (where necessary) the heights by means of contour lines, etc. The complication which arises is that the surface of any ellipsoid, including a sphere, is doubly curved, and therefore cannot be developed (i.e. unwrapped) to form a planar projection.

One solution is to project the surface features of the ellipsoid directly onto a plane; alternatively, they can be projected onto a developable surface such as a cone or cylinder, a developable surface is then developed to give a planar projection. In each case, the resulting projection can subsequently be scaled down to give a map of a useful size.It is not possible to project a doubly curved surface onto a planar or developable surface in such a way that the scale of the projection is unity (or any other constant value) at all places. Thus, all such projections involve some degree of distortion on the resulting map, except at certain points or along particular lines. By varying the exact method of projection, it is possible to manipulate the changes in scale so as to avoid some aspects of distortion on a map, but usually at the expense of increased distortion in other respects, [Johnson, A., 2004].

Although a projection must be expressible as a pair of formulas for converting latitudes and longitudes into a plane coordinate system, it is usual to describe a projection by explaining the geometrical structure of the network of graticule of lines on the map that will represent the network of meridians and parallels on the sphere or spheroid. Few projections have geometrical sense, and some of them cannot be drawn by ruler and compasses methods.

Drawing must usually be done by plotting calculated coordinates of a series of a point and joining by smooth curves. However, the computer – controlled plotting is now available, not only for ruler and compasses jobs, but for plotting a complete map from stored positional information and programmed projection formulas, [Jackson, J. E., 1980].

The surface of datum, representing a portion of the globe, may be represented on a plane, i.e. $E = f_E(\varphi, \lambda, a, f)$ and $N = f_N(\varphi, \lambda, a, f)$. Where E is easting and N is northing on the classic map of the represent latitude and longitude on the geodetic datum. Representation of a globe surface on a plane must result in some distortions, therefore the properties required of the map projection must be considered, [Schofield, W. and Breach, M., 2007].

The map projections have been discussed in multi thousands of books and papers dating at least from the Claudius Ptolemy time (about AD 150), and projections are known from three centuries earlier. Most of the widely used map projections date from the sixteenth to nineteenth centuries, but many of variations have been modified during thetwentieth century.

For almost five hundred years, it has been conclusively proven that the globe is essentially a sphere shape, although a number of intellectuals and scientists nearly two thousand years earlier were believed of this. Even to the scientists who considered the Earth plane surface; the skies appeared part of sphere, however. It was created at an early date that attempts to draw a plane map of a surface curving leads to distortion of form or another form. A projection of map is a systematic representation of the globe or part of it. This usually contents graticules describing parallels and meridians, as required by some basics of a map projection.

A map projection is required in any condition. Since this can’t be done without map distortions in areas, distances or azimuths, the draft man of cartography must choose the map property which must appear accurately at the expense of anothers, or a compromise of several properties. If the map covers the Earth or a continent, distortion will be visually obvious. If the region is limited area as small town, map distortion may be hardly measurable using several projections, but it can still be problem with other projections. There is an infinite number of map projections that can be used and designed, and many hundred have been publishedover the world, most of which are rarely used innovation.

Most map projections may be infinitely differed by choosing many points on the Earth as the center or as a base map point. It can’t be said that there is type best map projection. It is even

risky to say that type has found the best projection .A constructed globe map is not the best map for most applications because its map scale is by necessity very small. The earth shape is awkward to use in general, and a ruler can't be satisfactorily used to measure distance, [Snyder, J. P., 2003].

1.1 DISTORTION IN MAP PROJECTION

Some distortions of conformality area, conformality, direction, and distance always result from projection of maps. Some projections minimize distortions in some properties at the expense of maximizing errors in other properties. Some map projections are attempts to only reasonably distort all of these properties.

- **Conformality**

When the scale of any map is the same in any direction at any point, the projection is conformal. Meridians and parallels (lines) intersect at right angles, shape is kept on conformal maps, [Dana, P. H., 2000]

A conformal projection manages the scaling effects such that, at any point on the map, the scale in all directions is the same value. Such maps are also orthomorphic, which means that small shapes on the ground (buildings, etc.) are shown as the same shape on the map. The result of this is that the angles at which two lines cross on the earth's surface are preserved exactly on the map. However, the shortest distance over the ellipsoid between two distant points (a geodesic) does not in general plot as a straight line on the map.

- **Distance**

A map is equidistant when it measures distances from the center of the projection to any other location on the map. In an *equidistant projection,* the scale of the projection is maintained at unity along a particular set of geodesics. Equidistant projections are normally azimuthal projections, the scale is preserved along all geodesics radiating out from the central point of the projection. These geodesics also plot as straight lines, so that distances from the central point to any point on the map can be measured directly. However, all other geodesics will plot as curved lines on the map.

- **Direction**

A map preserves direction when azimuths (for any line connected between two points) are measured correctly in all directions.

- **Area**

When a map measure area over the entire map so that all mapped area has the same relative relationship to the area on the globe, the map is an equal area map, [Wolfe, J., 2000].

An equal area projection manages scaling such that if the scale at a point is increased in one direction, then it is reduced in another. Thus, the area of any feature on an equal area map is preserved exactly, subject to the quoted scale of the map. However, the shape of the area will not be preserved exactly; the scale in one direction (e.g. east to west) will generally differ from the scale in any other direction (e.g. north to south), at any point on the map. Small circles drawn on the surface of the earth would plot as ellipses of the same area on the projection, but with greater eccentricity in places where the distortion of the projection is higher.

1.2 MAP PROJECTION AND COMPUTERIZATION

No projection can have more than one of these properties over an extended area, and others have none of them, preferring instead to strike a compromise between them. Conformal projections are generally preferred for surveying purposes, because angles measured in the field can be plotted directly onto the map, [Johnson, A., 2004].

Chosen projection may be corresponded to some region but it is not suitable for another region. Specific projection is suitable at the poles but it cannot be suitable for representing the equatorial zone. The goal of this is to obtain specific map characteristics (equal area, equidistance or conformal).

With developed computerization, it is very important to know that projected coordinates (map coordinates) for all these projections may be mathematically computed with mathematical model which would have appeared too difficulted in the past, but nowadays they may be programmed easily, especially if helped by numerical examples, [Snyder, J.P., 2003].

It is believed that scientists have taken care in map projection to avoid long difficult geodesic equations to deal at this time; where there are not calculating machines or computers. The user takes the projected value and forgets the sphere and ellipsoid, he enforced calculation of observations by using map equations, where it is treated as plan surveying (contrary to the fact) to escape from complex geodetic computations. The calculation problem still existing in converting geodetic coordinates to map coordinates, as John P Snyder mentioned in the previous paragraph. Nowadays the computerized routine solution can apply in map projection

as well as in geodetic computations (geodetic azimuth, ellipsoid distance and ellipsoidal area and etc…).

There is another reason for inventing the map projection, where it starts from holism to represent the globe in a single map. The globe shape whether a sphere or an ellipsoid is impossible to be developed as a plane surface, so scientists refuge to the map projection.

A layman might wonder, why map projection is a big problem at all? A map of a limited area, such as a village or city, is almost distortion free. If the projection of the globe in one map is disregarded, **an area of earth bounded by two latitudes and two longitudes could be represented with its geodetic coordinates keeping the values (areas, bearings and distances) without using classic projection.** This could be true for all surveying mapping scales because the Earth's curvature does not appear in the limited areas. When this operation is repeated at neighboring areas, then the total area is collected in series of maps.

The question now is why classic map projection is still used if a digital real geodetic map can be developed. This map will be presented using geodetic coordinates directly. The required surveying elements (distances, directions, areas) will be obtained, by push button keys, computed from the geodetic coordinates using computer programs ad joint to the map. The new map will get rid of the noisy distortion. The map can be plotted whenever it is needed. Maps all over the world will be in one mapping system matching the requirements of the non-proceeded satellite gravity missions, satellite positioning (GNSS)and satellite remote sensing imagery.

The aim of this thesis is introducing a proposal of that digital geodetic real map after studying and investigating the related topics and the required computations.

1.3 THE OBJECTIVE OF THE THESIS

The basic target of this research is producing digital maps presented by geodetic coordinates (latitude and longitude) directly; these maps depend on digital form in basic entities point, line and arc in tables form by latitude and longitude coordinates. Calculation of areas, azimuths, and distances will be done using the appropriate geodetic mathematical model.

Producing maps by geodetic coordinates (latitude and longitude) directly could be better than the traditional maps presented by projected coordinates (E, N) because:

- There is no distortion in the proposed map because already the required values (distance, direction, area) will be computed from geodetic latitude and longitude.
- The proposed map is proper for all mapping scales(topographic and cadastral).
- The actual shape in cadastral and topographic maps will be preserved.
- No need to divide the globe in zones like UTM zones.
- Any point on the globe will have unique geodetic value of coordinates latitude and longitude (ø, λ), but different points have the same values of coordinates (E, N) (represent 60 points in 60 zones in UTM as example), also the same point in border zones has two pairs of coordinates for each zone; (E, N) in east zone and different (E, N) in west zone.
- Very easy to calculate the distances between any two points on the earth by latitude and longitude but it is difficult if the points are in different zones in the map projections.

The thesis aims also to follow the proposal of the digital geodetic real map by another proposal for the appropriate map index. The proposed map index should be easy, simple, and global to guarantee one mapping system all over the globe.

1.4 SCOPE OF PRESENTATION

In order to achieve the main objectives of the present investigation, the subsequent material of the thesis is presented in six chapters which can be summarized as follows:

Chapter 2

In this chapter, the basic idea of map projection, map projection history and developments are stated. Some definitions will be used in the thesis are mentioned. The chapter also focused on map projection revolution that is began in the middle of the 20th century and Computer programs technology that had developed in the field (Computer programs were being devised that could create maps from digital data).

The chapter reviewed the most classifications in the science of map projection;

- According to geometry of projection surface
- According to orientation of projection surface.
- According to secant-tangent point.
- According to properties of the map

- According to map generation (geometric or mathematical)
- According to number of projection surfaces.

Distortion in maps and Tissot Indicatrixi and kinds of distortions are also illustrated. Finally, myriahedral projection is explained as new solution to minimize the map distortion.

Chapter 3

This chapter reviewed the mathematical equations that are used in computing the azimuths, distances and areas once from the map coordinates and once more from the geodetic coordinates. The formulae for direct and inverse problems in short and long lines in geodetic computations are stated. The relation between geodetic curvilinear coordinates and geodetic Cartesian coordinates is mentioned. Finally, calculating the area on the ellipsoid is discussed.

Chapter 4

The area is considered small when the curvature of the globe is negligible. This chapter compares the chord and curved distances between two points on the ellipsoid datum; the difference between them is computed and analysed. Geodetic and projected map in two main cases (the Egyptian case with deferent surveying scales and the global case with deferent surveying scales) are studied. The chapter studied the distortion in area on the map and on the used ellipsoid or sphere in multi scales. Finally, the idea of producing a map without projection is introduced.

Chapter5

This chapter explains in details Egypt Quadrant System, kilometric system, millionth map system, ortho-photo map system and geographic map index. The chapter introduced a proposal for globe map index system with deferent surveying scales. The map index proposal is simple and universal.

Chapter 6

This chapter summarizes the presented material of the current study. The main conclusions mentioned for automatic real map production. The recommendations based on the conclusions of the research are mentioned.

Finally, a set of references have been referred to in the appropriate places within the text of this thesis, are included and arranged in an alphabetical order, for the convenience of the reader.

2 MAP PROJECTION BASICS

Mapping is important in surveying and cartography. The fact that the round earth cannot be flattened onto a plane without distortion means that every flat map must have a distortion, however refined or crude. Until a few decades ago, maps were plotted largely by hand. First a graticule of meridians of longitude and parallels of latitude was laid onto the flat plane using geometric construction or with help of a table of coordinates that had been calculated using table of logarithms or a desktop calculator. These calculating techniques are fairly unwieldy to use with the advent of high-speed computers.

The earth is sphere shape; maps are plane. If a any map is to represent only a very small limited area of the earth, such as a few city or village, the curvature of globe will be not noticeable. If, on the other side, a map is to show the globe, the curvature will present a big problem. Some kind of distortion will be necessary, [McDonnell P. W., 1979].

The globe is a true representation of the earth and, provided it is large enough, all parts of the earth's surface can be represented on it in their true shape, relative size and position; but great problems arise when an attempt is made to transfer details of land masses and oceans from the globe spherical surface to the flat surface of a sheet of paper. It is impossible to do it with complete accuracy of shape, relative size and position at the same time; so that the study of map projection resolves itself into a study in compromise. Because the position of any point on the globe can accurately be determined in terms of its geodetic latitude and geodetic longitude, the system of meridians and parallels is the framework or skeleton of the land masses. The study of map projection is the study of the various ways which may be devised for transferring these meridians and parallels from spherical surface of the globe to the plane surface of a sheet of paper. Clearly, it is to make the 'skin' of globe lie flat without stretching in its places.

It is possible to develop a framework of intersecting meridians and parallels so that land masses plotted on them are truly represented in area, but only by the sacrifice of their shape. Conversely, true shape can be preserved only at the expense of area and, even then, one cannot on a plane surface. A map can preserve the shape of small area, strictly speaking of infinitely small areas, and it is then said to be orthomorphic. The particular system or projection adopted for representing parallels and meridians will depend on the purpose for which the final map is required. Some projections show equatorial area well, if not with complete accuracy of shape or area while polar areas are badly distorted. Other preserve area and not shape and so are best use for showing distributions, for example, of races or rainfall or natural vegetation; areas

under coniferous forest may thus be compared visually with that under equatorial forest or hot desert to prairieland. [Roblin, H. S., 1983].

The application of computers in cartography develops every day bigger and bigger, and many countries have their mapped data in digital form. Therefore, the standardization of geographic database transfer is one of the most important tasks for mapmaking as profession. In order to understand and absorb the complete operation of such a transfer, it is of greatest importance to absorb the basics of cartographic theory that it is based on. It includes the terms: real maps (hard copies) and virtual maps (soft copies).

These terms result from the development of analytical cartography, the area being the major activist of the development in theoretical and mathematical cartography basic. Moellering has given a condensed presentation of analytical cartography. The first two basic terms in this area are the terms about virtual and real maps. After many years of research, the definition of virtual and real virtual maps was suggested. The other decisive characteristic is whether the product can be touched. The classes of real and virtual maps obtained, [Lapaine, M. 1999].

To draw parts of the surface of the globe on a paper map or on a screen of computer, the curved horizontal reference surface must be draw onto the two-dimension mapping plane. The geodetic datum for large-scale mapping is usually an oblate ellipsoid or sphere, and for small-scale mapping, Mapping onto a two-dimension mapping plane means transforming the geodetic coordinates (ϕ, λ) for each to a set of two-dimension Cartesian coordinates (x, y) representing positions on the map plane, figure (2-1).

$$(x, y) = f(\phi, \lambda) \qquad (2\text{-}1)$$

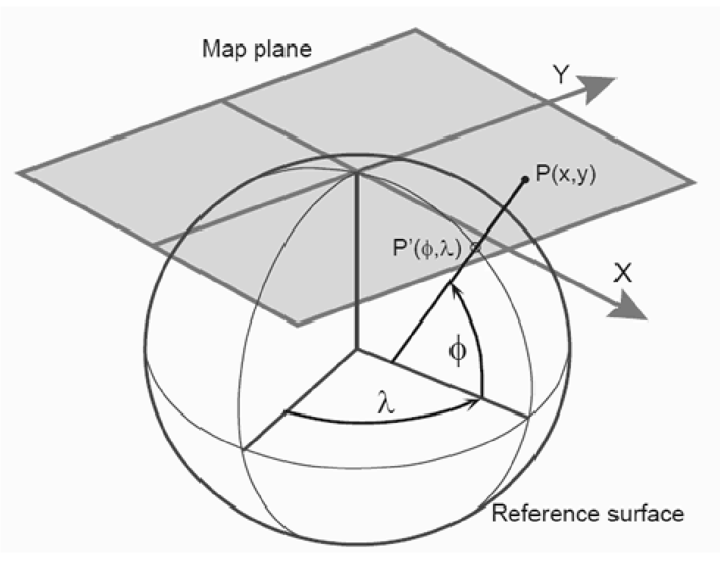


Figure (2-1): Reference surface and map plan.

The actual mapping can't usually be visualized as a true geometric projection, directly onto the mapping plane. This is mostly achieved through mapping mathematical model. A forward mapping equation transforms the geodetic coordinates (ϕ, λ) of a point on the globe surface to a set of map coordinates (x, y), representing the position of the same point on the map plane,

[http://kartoweb.itc.nl/geometrics/mapprojections/body.htm].

The equivalent inverse mapping equation transforms mathematically the map coordinates (x, y) of a point to a set of geographic coordinates (ϕ, λ) on the ellipsoid geodetic datum:

$$(\phi, \lambda) = f\,(x, y) \qquad (2\text{-}2)$$

The fundamental coordinate system for mapping and surveying is a set of geodetic coordinates related to a particular datum. It is then necessary to consider how to arrange the data so that it can be placed on a plan. There are two reasons for doing this. The first, and most obvious, is presentational. Whether the data is to be shown on a paper map or on a computer screen, it is necessary to be presented in two-dimensional format. The second reason for rearranging the geodetic coordinates in two dimensions is computational. Even a simple concept such as the distance between two points becomes excessively complex when expressed in spheroidal formulae, and wherever possible it is more desirable to carry out computations in a simple two-dimensional coordinate system. A projection, then, is defined as an ordered system of meridians and parallels on a projection surface. It should be immediately apparent that it is impossible to convert a sphere or a spheroid into a flat plane without distorting or cutting it. It follows that there is no single method for doing this; so, there are many kinds of map projection.

The target of studying map projections is to create mathematical model for the production of maps and solving the practical and theoretical tasks in navigation, geodesy, cartography, astronomy, geography, and other related sciences.

Basic cartographic formulas enable calculating of distortions in projected map. Namely, while projecting the surface of a globe into the plane of map, it comes to distortions of lengths, surfaces and angles. The choosing type of map projection is necessary to select the most convenient projection for the maps of area and purpose.

In the selected projection one should then drawing the graticule of map. For this purpose, the computer programs now are used for calculating and producing graticules for any part of the globe in any projection type and required scale. These programs enable also the drawing of other contents. A map is difficult drawing of the globe, but it is obtained on the basis of certain formulas. These formulas presume indirect transfer from the natural surface of the earth into its graphic draw in the plane. First, we pass from the globe surface onto the mathematical geodetic surface ellipsoid or sphere, this transfer is realized by means of orthogonal or central projection of datum surface

points onto the mathematical surface by means of geodetic control network enabling correct geographic position and orientation of map contents within the frame of some co-ordinate network on the ellipsoid and then on the map. These projections are called map projections,
[Lapaine, M., 1999]

2.1 DEFINITIONS

We have general definitions and private definitions used in this research, we had identified our private definitions by ** sign

Projection: It is the method of representing points or areas from surface onto another. Each point of one surface is represented by only one point of the second surface.

Map Projection: A method for representing part of the surface of the earth or a celestial sphere or ellipsoid datum on a plane or developable surface. It is the kind of Projection in which one surface is the surface of the earth and the second surface is the map. These two surfaces are not identical and thus there must be a certain relationship to control the representation of these two surfaces.

Datum Surface: The surface from which we are going to project, this is the surface of the ellipsoid or sphere on which point is defined by its coordinate (ϕ , λ)

Projection Surface: This is the surface on which we are going to project the point. This is the surface of the map on the plan sheet on which a point is defined by plane coordinate East & North.

Developable Surface: A developable surface is a simple geometric form capable of being flattened without stretching like plane. The famous developable surface is cone or cylinder.

Undevelopable Surface: The Surface cannot develop as plane sheet, like sphere and ellipsoid of revolution.

The map: The map is a representation of an area symbolic depiction highlighting relationships between elements of that space such as themes, regions, and objects, all put down on a sheet and intended to give a representation of the features on a part of the earth.

Graticules: A network of straight lines or arcs representing a selection of the Earth's meridians and parallels.

2D Map: The projected map represented by 2D coordinates (E, N) on a plane.

2D network: In 2D network; (ϕ, λ) only are used to represent the point on the surface of ellipsoid of the geodetic datum.

3D network: In 3D network, the point is defined in three dimensions as curvilinear coordinates (ϕ, λ, h) or rectangular coordinates (X, Y, Z).

The real map: The conventional cartographic products as map sheets, atlases and globes which have hard touchable reality and are directly visible as cartographic images are called real maps.

The Automatic real map:** The automatic real map is digital map presented by latitude and longitude directly without map projection; these maps depend on digital form in basic entities point in tables form by latitude and longitude coordinates. Calculation of distances, azimuths, and areas will be done using the appropriate geodetic equations by hot keys ad-joint to the digital map; these points are known in geodetic datum like WGS84 and EGD (Egyptian Geodetic Datum). The map can be plotted when a hard copy is needed.

A cartographer: He is a person dealing with drawing map. Cartography is a discipline dealing with, promotion, production, studying of maps and foundation.

Map Scale, (Nominal Scale): When a large area is shown on a sheet of paper, the result is said to be a small-scale map, like 1: 1000 000, 1: 100 000. A large-scale map is the opposite; an example would be the map of small cities on paper sheet, like 1:1000, 1:2 500. Scale is usually given as a fraction or ratio: 1/100,000, or 1:100,000.

Scale factor: The scale factor describes, at each point on the projected map, the amount of distortion in length. This distortion is of course due to maintaining conformality and fulfilling other conditions prescribe for the projection. It should not be confused with the "line scale ", which is concerned with the scale distortion over a finite length of line. The attention here is on two surfaces, the ellipsoid and the map plan surfaces. That the meridians and parallels on the ellipsoid surface are perpendicular. Since the projection is conformal, these two curves are also perpendicular on the map plane, [Youssry, A.M. M., 1984]

$$k = \frac{\text{Distance on the map}}{\text{Distance on the datum sphere or ellipsoid}} \qquad (2\text{-}3)$$

Actual Scale: Actual scale is obtained from nominal scale divided by scale factor.

Distortions: It is defined as the change in angle, shape, distance and area due to the process of Projection.

Positive Distortions:** If the projected map values (distance or area) is greater than geodetic distance or geodetic area that means positive distortion.

Negative Distortions:** If the projected map values (distance or area) are less than geodetic distance or geodetic area that means negative distortion.

Indecatrix of Tissot: An infinite small circle drawn on the surface of the geodetic ellipsoid datum when it is projected on a map sheet it will be represented by an infinite small ellipse called Indecatrix of tissot.

2.2 HISTORY OF MAP PROJECTION

The early maps were drawn by hand; in general scales, shapes and location, for different places were not actual shape. "About 3000 years ago, the Egyptians for the first time drowned a map which was somewhat accepted. This map, which was prepared for income dividing, collection lines of land zones were shown. In about 600 B. C., Thales suggested the idea of a Gnomonic projection. Around 540 B.C., Pythagoras confirmed that the earth is sphere. Gradually due to the works of Aristotle (384-322 B.C.) Eratosthenes (273-192 B.C.), Ptolemy (85-165 A.D.) and others, the ideas of the poles of the Earth, the equator, various climatic regions and drawing of maps using projections came into being. Anaximander (610 -540 B.C.) for the first time prepared a map of the entire world, relying on stories told by travelers, sailors, etc. Strabo stated for the first time that the characteristics of spherical Earth cannot all be represented correctly in a plane diagram, and he was suggested the need for corrections in latitudes and longitudes. The Geographic of Ptolemy contained world map and twenty-six other maps. In the 16th century, publication of maps became a rewarding business. However, as regards distortion in shape and distance, these maps were of the same standard as that of Ptolemy map. The person who liberated map making from the influence of Ptolemy was Gerhard Mercator," [Mukhopadhyay, U., 1999].

Map projections have been Matching the big jumps in computing devises and programming facilities parallel with the development of production of cartography and map in general. The development of several sciences, technical achievements and the daily life needs have gradually began broader and broader requirement for the production of topographic and

thematic maps in many scales and for many purposes, which requested continuously develop of map projections and edit & update of mathematical model basis for maps.

A completely new research topic to study the projection suitable for image satellite and mission confronts science of map projection. In last years, the computer machine, especially the personal computers, have been widely applied to all classification of map projections and have complete changed the look of science of map projection. Examples are the applications of computers to the coordinate calculation, to the automatic generation of the mathematical organization of maps. the computer has helped us in the science of map projection transformation is even big jump for mapping. To meet the requirement of computer mapping, it is an urgent task to study the methods and theory of transformation of map projection to study the processing of spatial information positioning, topographic data and transformation in information systems.[http://www.kartografija.hr/projections_long.pdf.]

2.2.1 The 20th-Century Revolution

Map making is now in the center of a major revolution that had its beginnings in the middle of the 20th century. As with any revolution the changes involve technology, increased and new data, and philosophical factors. World War II was a major motivation in that it created a need for up-to-date maps of widespread areas. The number of maps required was huge and they needed to be created rapidly. In the United States, at that time, a call went out for thousands of people to be trained and employed in map making, photogrammetry, and air photo interpretation.

At the end of the war, geography departments began teaching cartography, which had previously been concentrated in civil engineering. They were especially concerned with "geographic cartography" or thematic cartography rather than surveying and mapping or engineering cartography. However, until the 1950s, geographers considered cartography a tool and a skill, not a science or research area, and little research was done on how maps work. There were few textbooks available. Geographical journals published articles on map projections and the history of maps, but little on symbols and nothing on design.

After the war, Arthur Robinson returned to his studies and his dissertation topic was unusual in that it dealt with map design. It covered such subjects as color, typography, and map structure. Dissertations carried out under Robinson were often psychophysical studies of symbols such as graduated circles and isopleths. In the same period, other cartographers who had been involved in mapping during World War II took teaching positions at universities and cartography began to emerge as a discipline. Two of those cartographers, George Jenks, at the

University of Kansas, and John Sherman, at the University of Washington, and their students also carried out research on how maps function, [Tyner, J. A., 2010].

2.2.2 Technology

"By the 1960s new technology was revolutionizing the field. Computer programs were being developed that could create maps from digital data. The Harvard Laboratory for Computer Graphics introduced SYMAP in the 1960s. Although the maps were crude, the potential could be seen. In the early days the only printers were line printers that operated as automatic typewriters and all symbols on the map were made up of alphanumeric characters. SYMAP maps were of little use for presentation, but they did permit rapid spatial representation and analysis of data. Another major technological impact was remote sensing. Aerial photographs had been widely used during World War II and before, but with the advent of satellites and sensors a wealth of high-resolution imagery became available. The concepts for geographic information systems date to the 1930s when geographical analysis was carried out by placing information on a series of clear plastic layers. Modern GIS utilizes virtual layers in analysis", [Tyner, J. A., 2010].

2.3 CLASSIFICATIONS OF MAP PROJECTION

Map projections may be classified into different classes according to many concepts. According to geometry of projection surface (plane, cone or cylinder), according to orientation of projection surface, and according to condense of projection surface and datum surface, according to properties of the map, according to map generation and according to number of projection surfaces.

2.3.1 According to Geometry of Projection Surface

The operation of creating map projections can be imagined by putting a light source inside the earth on which opaque earth features are placed. Then project the feature outlines onto a two-dimensional plane of paper. Different method of projecting can be produced by surrounding the globe in a cylindrical or cone, or even as a flat surface. Each of these methods produces what is named a map projection family. Therefore, there is a family of cylindrical projections, a family of conical projections, and another called planar projections, [Sutton T., et al., 2009], Figure (2-2).

The surface put “near, in or on” the globe described above is usually a cylinder, a cone, or a plane. Some projection is purely mathematical, it is often placed in one of these three families, conical, cylindrical, and (planar) azimuthal, [Clynch J. R., 2006].

2.3.1.1Cylindrical Projection

If a cylinder covers the globe, touching it along the equator latitude, the projection got by radial projection from the center of the earth is called a cylindrical projection. We get the map by cutting the cylinder along a meridian and laying it out plane surface. After obtaining the projection of each point on the earth on cylinder, the curved surface of the cylinder is developed to obtain the map as plane of almost the whole world, [Mukhopadhyay, U., 1999]. A cylindrical projection maps the globe to a cylinder which is formed by wrapping plane around the globe with axis coinciding with a great circle; this great circle is sometimes referred to as equator. This great circle is sometimes referred to as longitude.

2.3.1.2 Planar Projection (called also Zenithal Projection or Azimuthal Projection).

An azimuthal projection is formed by bringing a plane into contact with the sphere or spheroid and formulating a set of rules for the transfer of features from one surface to the other. Once again, the properties preserved can be distance, area, shape, or others. Because the point of contact between a sphere and a plane is a single point, the scale factor distortion will be circularly symmetric. That is, the scale factor will be proportional to the distance from the center of the projection. An azimuthal projectionist therefore particularly suited to small 'circular' features on the surface of the globe, [Iliffe J., 2003]. In this projection: we project the globe on a plan; it is difficult to project the whole hemisphere on one plan.

2.3.1.3 Conical Projection

Simplest conic projection is tangent to the globe at a small circle (called standard parallel latitude). The meridians are represented onto the conical surface, meeting at the vertex of the cone. Parallel lines of latitude are represented onto the cone surface as arcs, [Ruddier Gens, 2006]. That many different shapes of cone can be selected, all resulting in a different standard parallel. The choice will depend upon which region of the Earth is to be mapped, an appropriate standard parallel being one that passes through the center of the region. The resultant form of the conic projection is that the meridians appear as straight lines converging towards one of the poles. The angle between two meridians is a function of the standard parallel, and can be expressed as:

$$\gamma = \Delta\lambda \sin \phi_{\circ} \qquad (2\text{- }4)$$

Where $\Delta\lambda$ is the difference in longitude of the two meridians and $\phi_{\circ}$ is the latitude of the standard parallel, [Iliffe J. 2003].

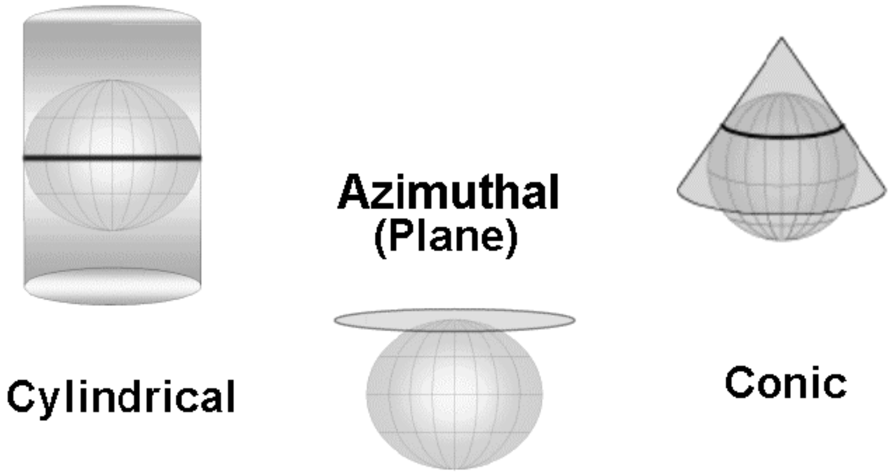


Figure (2-2): Families of projection surfaces.

2.3.2 According to Orientation of Projection Surface

Projections can be described in terms of the orientation of the projection plane's related to the earth surface. This is called the aspect of a map projection. The three possible aspects are oblique normal, and transverse.

2.3.2.1 Normal Projection (in Cone and Cylinder)

In a normal projection, the main orientation of the projection surface is parallel to the Earth's axis (as in the figure (2-3) for the cylinder and the cone). The classical cylindrical projection can be visualized as wrapping a sheet of paper round the earth’s equator to form a cylinder and projecting points on the earth’s surface out onto it, The projection angle is a function of the latitude, of a given point; the function can be defined so as to give an equal area projection or a conformal projection. When the paper is unwrapped, the resulting projection shows parallels as straight horizontal lines and meridians as straights, equally spaced vertical lines. This type of projection has no distortion on the equator and low distortion nearby, and so is particularly suitable for mapping tropical countries, [Johnson A., 2004].

2.3.2.2 Transverse Projection (in Cone and Cylinder)

A transverse projection has its main orientation perpendicular to the Earth's north south axis. In the cylinder, the transverse form of a cylindrical projection involves wrapping the sheet of paper round a meridian (called the central meridian) rather than the equator.

2.3.2.3 Oblique Projection (General)

Oblique projections are general case, non-perpendicular and non- non-parallel. The figure (2-3) provides examples.

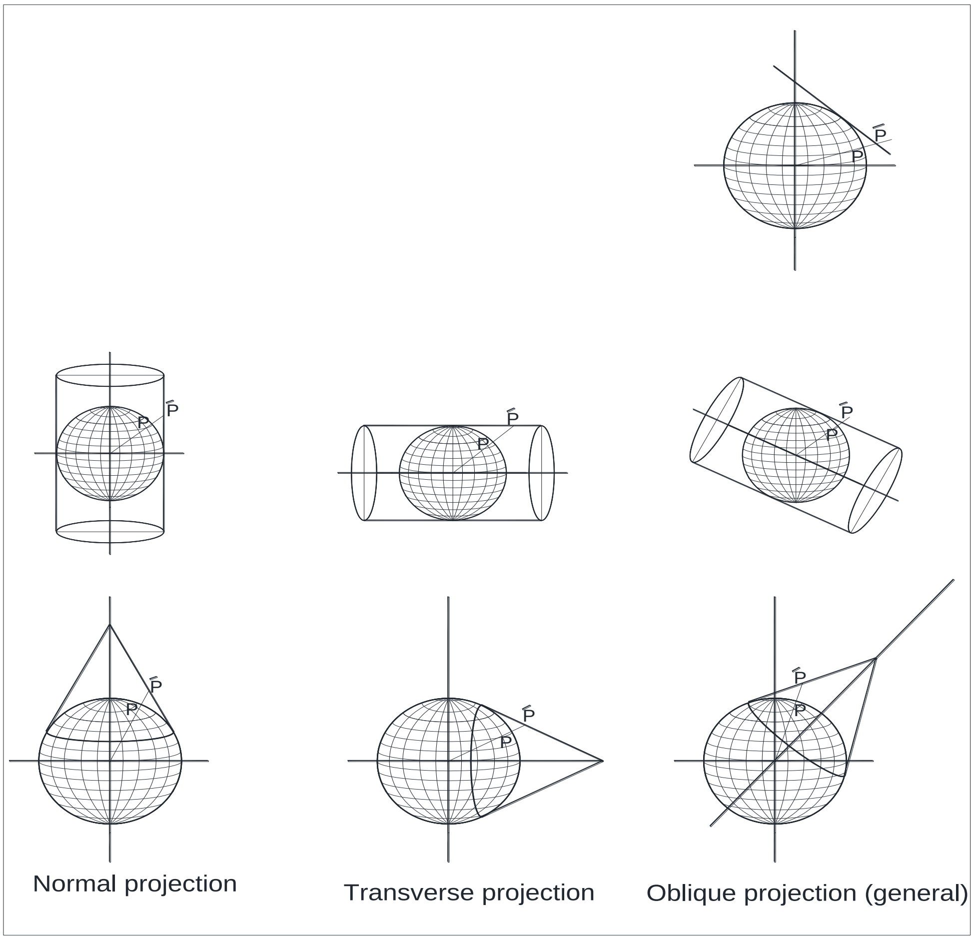


Figure (2-3): Orientation of projection surface.

2.3.3 According to secant-tangent point

2.3.3.1 Tangential Projection

The cylindrical, planar and conical surfaces in the figure (2-4) are all tangent surfaces with datum surface; they touch the reference surface in along a closed line in cone and cylinder, or one-point plane.

2.3.3.2 Secant Projection

Secant projections is got if the surfaces are chosen to be intersect with reference surface, figure (2-4). Then, the reference surface is intersected along one closed line with plane or two closed lines with cone or cylinder. Secant map surfaces are used to reduce distortion because the intersection lines are not distorted on the map.

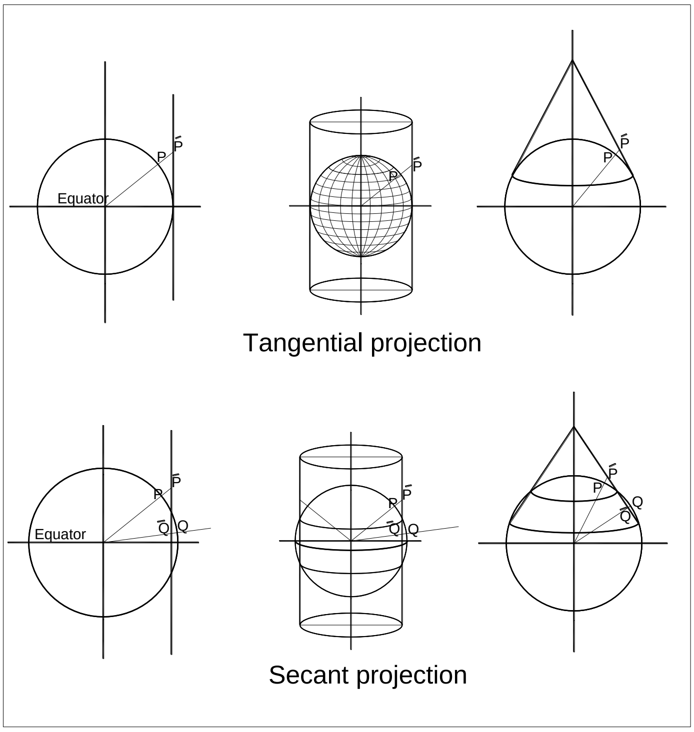


Figure (2-4): Secant-tangent projection surface.

2.3.4 According to Properties of the map

Projections are never absolutely actual representations of the spherical earth on the map, as a result of the map projection process, every map shows distortion of distance, angular conformity, or area. A map projection may one or two of these properties, or may be a compromise that distorts all the properties of distance, area and angular conformity, within acceptable value, [Sutton T., et al, 2009].

2.3.4.1 Conformal Projection (Orthomorphic Projection)

Maintaining the correct angular properties on the map projection as well. A map projection that keep this property of angular value is called an orthomorphic or conformal projection. These projections are used when the keep of angular relationships is important. They are commonly used for meteorological or navigational tasks. It is important to remember that maintaining true angles on a map is difficult for large areas and could be represented only for small area of the globe,] Sutton T., et al, 2009].

In a conformal map projection, the angles between lines in the map are identical to the angles between the original lines on the geodetic datum (sphere or ellipsoid). This means that angles with limit sides and shapes of limit small areas are shown correctly on the map. Figure (2-5) indicates, all angles will be correct, and conformity achieved, if at each point, the scale factor S.F. along the

latitude, drowned by horizontal edge in the figure, is equal to the scale factor along the longitude, drowned by vertical edge in this figure. For the map of a curved ellipsoid surface, this value is a local phenomenon that can change from point to other point and in different directions from the same point. So, the local scale factor of the map at a certain point along a given line is the proportion of arc length elements along the map of the line and along the line itself, [Freeman, T. G., 2000].

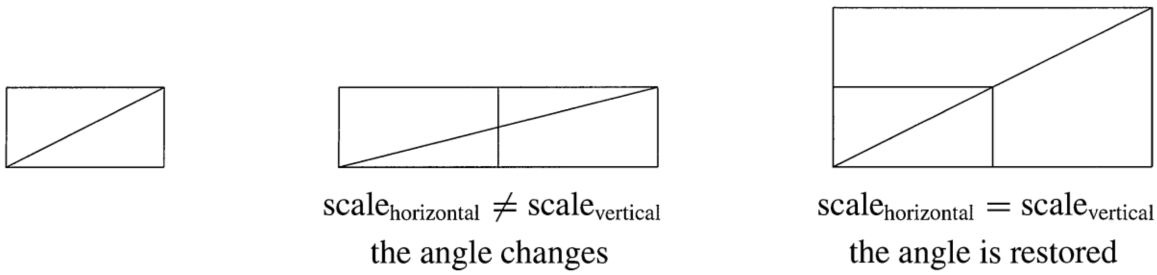


Figure (2-5): How scale factors affect angles.

2.3.4.2 Equal Area Projection

"When a map portrays areas over the entire map, so that all mapped areas have the same proportional relationship to the areas on the earth that they represent, the map is an equal area map. In practice, general reference and educational maps most often require the use of equal area projections. As the name implies, these maps are best used when calculations of area are the dominant calculations you will perform. If, for example, you are trying to analyses a particular area in your town to find out whether it is large enough for a new shopping mall, equal area projections are the best choice. On the one hand, the larger the area you are analyzing, the more precise your area measures will be, if you use an equal area projection rather than another type. An equal area projection results in distortions of angular conformity when dealing with large areas. Small areas will be far less likely to having their angles distorted when you use an equal area projection," [Sutton T., et al, 2009].

2.3.4.3 Equidistant Projection

If the target in projecting a map is to accurately measure distances, a projection that is designed to get distances well should be choose. these projections, called equidistant projections, require that maps kept constant. A map is equidistant when it correctly represents distances from the center of the projection to any other place on the map. These projections are used for seismic mapping, radio and for navigation, [Sutton T., et al, 2009]. No map

projection can be both conformal and equal-area together. A projection can only be equidistant (true to scale) at certain directions or in choose places.

2.3.4.4 True Direction Projection

“The shortest route between two points on a curved surface such as the earth is along the spherical equivalent of a straight line on a flat surface; that is a great circle on which the two points lie. True direction or azimuthal projections are used to rectify some of the great- circle arcs, giving the directions or azimuths of all points on the map correctly with respect to the centre. There are projections for this type that are also conformal, or equal-area, or equidistant, [Snyder J. P., and Reston Va., 2001]. True-direction or azimuthal projections maintain some of the great circle arcs, giving the directions or azimuths of all points on the map correctly with respect to the centre; it is not possible to maintain the azimuth between any other points on the map,” [Kennedy M. and Kopp S., 2000].

2.3.5 According to Map Generation

Some projections of the cylindrical, conic and azimuthal families have a direct geometric central projection (rays projected from a source intercept the Earth and map) or orthogonal projection (rays projected parallel to the map trough the datum), according to perspective laws, "draw" its features on a surface. The latter may be a plane, yielding the map itself, or an intermediate shape like a conical or cylindrical shell. For Indeed, many projections have simply no physical or geometric interpretation, and are described purely by mathematical formulae.

2.3.5.1 Geometrical Projection (Perspective).

In Geometrical projection, the plane and a surface which can be developed, such as a cone or cylinder are selected in such a way that either cut or touch the ellipsoid datum. A point is then selected as the projection centre from which straight lines are connected to points on the ellipsoid and extended until they intersect with the selected mapping surface, [Youssry A.M. M, 1997]. The various systems of projection used in the preparation of maps my generated from point of projection, the globe represented is drawn as seen when viewed from a fixed point, called the "point of sight". This classification belongs orthographic, gnomonic and stereographic globular Projections. These systems are used for maps of the world in for star maps and hemispheres, Figure (2-6a), (2-6b).

a) Gnomonic Projection

In this system the point of sight is supposed to be at the centre of the earth, which may be looked upon as a transparent globe, and the portions of the earth's surface are drawn as seen projected on

the faces of the circumscribing cube, [Threlfall H., 1936]. "The gnomonic projection (also known as central azimuthal projection) is neither conformal nor equal-area. The scale increases rapidly with the distance from the centre. Area, shape, distance and distortions are extreme, but all great circles (orthodromes) - the shortest distances between two places on a sphere - are shown as straight lines." [http://kartoweb.itc.nl/ geometrics /map%20projections body.htm].

b) Stereographic Projection

The **stereographic** projection is a conformal projection. So, meridians and parallels are perpendicular. In the polar the meridians are equally spaced straight lines, the parallels are circles centred at the pole, figure(2-6a&b). The scale is not different along any circle, the ellipses of Indicatrix distortion remain circles (indicating conformity). Areas increase with distance from the projection centre. [http://kartoweb.itc.nl/geometrics/map/projections/body.htm].

c) Orthographic Projection

In Orthographic projection the point of sight is assumed to be at infinite distances from the plane of projection, the projectors are parallel to each other, and are perpendicular to plane of projection. Distortion in area near the projection limit appears more realistic than almost any other projection. In the polar aspect, meridians are lines start from the centre, and the latitude are projected as concentric circles that become closer toward the edge of the globe. Only one hemisphere can be project, [http://kartoweb.itc.nl/geometrics/ map/projections/body.htm].

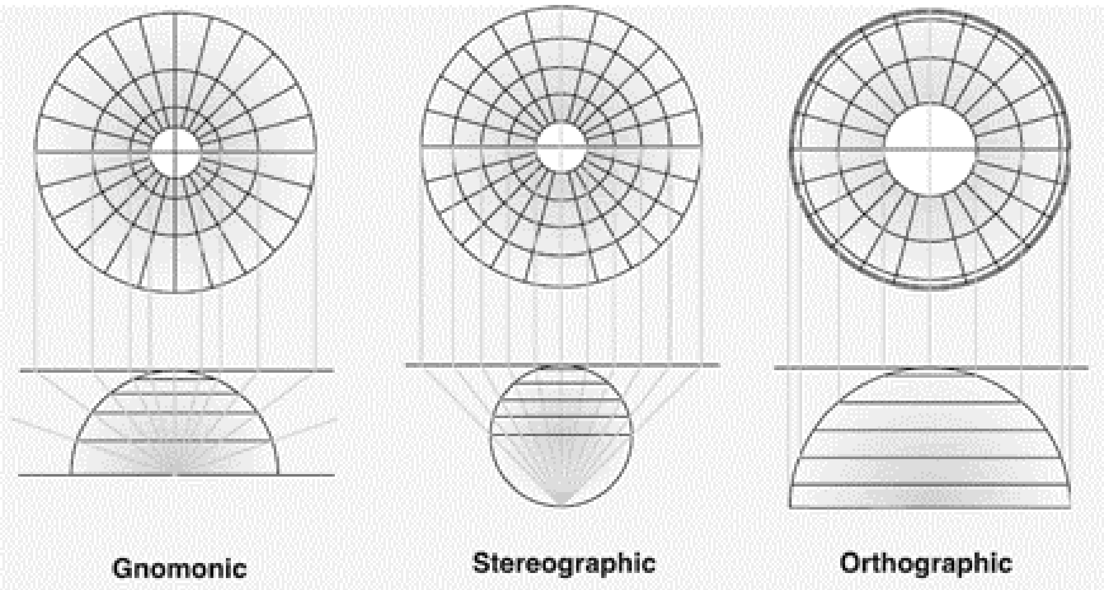


Figure (2-6a): Geometrical projection.

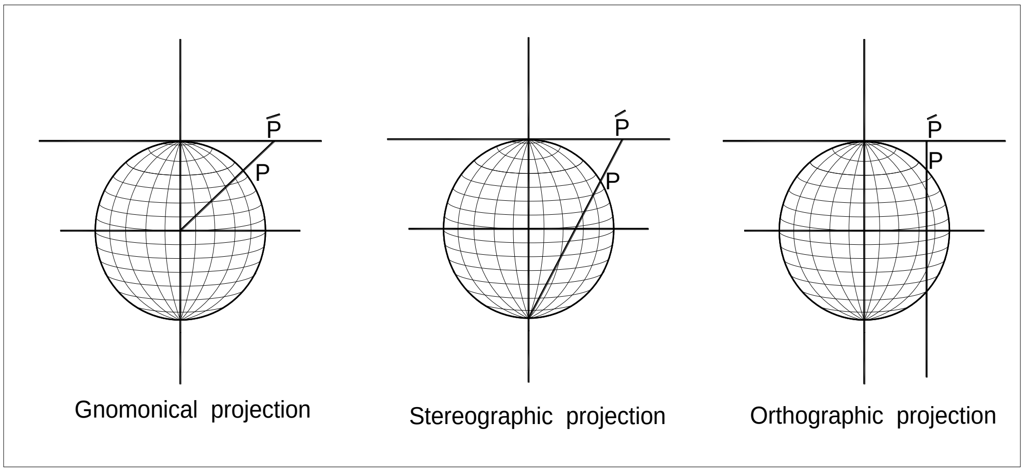


Figure (2-6b): Geometrical projection.

2.3.5.2 Mathematical Projection

In mathematical projection, there is no one particular projection point, instead, a form of equations (E, N) = f (ϕ, λ, a, f) is used to compute the location (E, N) of the point on the map from its position (ϕ, λ) on the earth, [Youssry A. M. M, 1997]. A mathematic equation satisfies a mathematic condition and the rays of projection cannot be imagined.

2.3.6 According to Number of Projection Surfaces

Sometimes the map is projected on one surface, and in some other times is projected on two or more surfaces. The goal is decreasing the distortion as possible.

2.3.6.1 Single Projection Surface

In Mercator transverse projection we use single normal cylinder as wrapping a sheet of paper round the earth's equator to form a cylinder and projecting points on the earth's surface out onto it. Also in planar projection and conical projection, one projected surface is used.

2.3.6.2 Multi Projection Surfaces

In Universal Transverse Mercator projection (UTM), 60 zones with 60 cylinders are used as projected surfaces; also, in planar world map, 6 plane surfaces as cubes are used. In a poly-conic projection, a series of cones are used to reduce distortion, figure (2-7a), figure (2-7b)

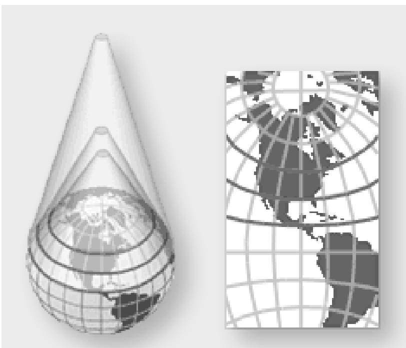


Figure (2-7a): Polyconic projection.

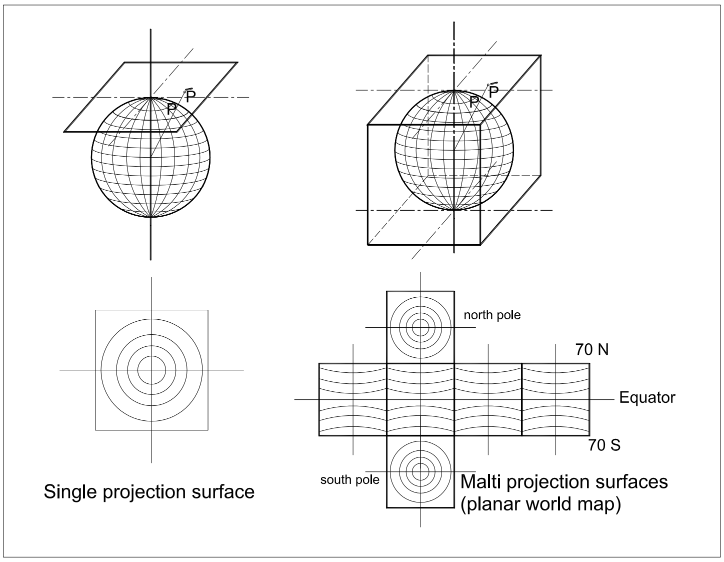
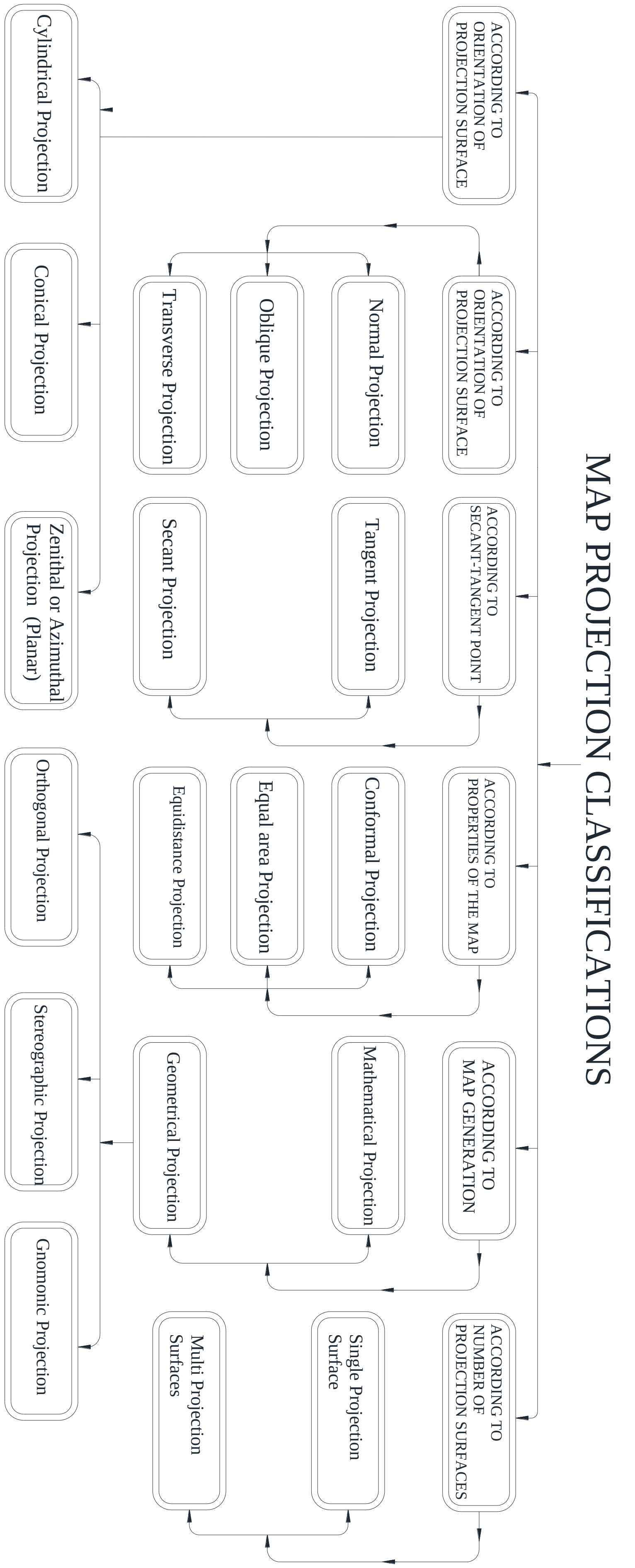


Figure (2-7b): Single and multi-planar projection surfaces.

Remarks about classifications:

Form the previous classifications, all classifications refer to a very important point which is trying to get minimum distortions.

- **In the first classification; the map projection is classified by geometry of projection surface (plane, cone or cylinder). The plane does not surround the geodetic datum. Cone and cylinder are the best in this point.**
- **In the second classification; the map projection is classified by orientation of projection surface, (normal, transverse and oblique). The choosing of classification depends on the convergence of the geodetic datum surface and projection surface.**
- **In the third classification; the map projection is classified by secant-tangent point. The choice satisfies minimum distortion at required region.**
- **In the fourth classification; the map projection is classified by properties of the map (equidistant, True Direction, equal area and angular conformity). The choice achieves minimum distortion at the target of the map, if the target is area measurement, we choose equal area map projection.**
- **In the fifth classification; the map projection is classified by map generation [geometric (gnomonic, stereographic and orthographic) and mathematical].**
- **In the sixth classification; the map projection is classified by number of projection surfaces (single or multi), multi projection surfaces made to reduce the distortion. Poly- conic projection is less distortion than conical projection.**


MAP PROJECTION CLASSIFICATIONS
ACCORDING TO ORIENTATION OF PROJECTION SURFACE
Cylindrical Projection
Conical Projection
Zenithal or Azimuthal Projection (Planar)
ACCORDING TO ORIENTATION OF PROJECTION SURFACE
Normal Projection
Oblique Projection
Transverse Projection
ACCORDING TO SECANT-TANGENT POINT
Tangent Projection
Secant Projection
ACCORDING TO PROPERTIES OF THE MAP
Conformal Projection
Equal area Projection
Equidistance Projection
ACCORDING TO MAP GENERATION
Mathematical Projection
Geometrical Projection
Orthogonal Projection
Stereographic Projection
Gnomonic Projection
ACCORDING TO NUMBER OF PROJECTION SURFACES
Single Projection Surface
Multi Projection Surfaces

Figure(2-8)

2.4 MAP DISTORTION

Projection of map without distortions would correctly represent angles, shapes, distances, areas, and directions, in each part on the map. unluckily, any map projection is affected with some type of distortion. There is simply no way to flatten out a piece of spherical or ellipsoidal surface without deformation some parts of the surface more than other parts. The value and which types of map distortions will have, depends largely upon the dimension the area being mapped and the kind of the map projection that has been choose.

Map distortion may be defined as the changing in the angles, shapes, distances, and areas due to the process of projection. Some projections minimize distortions in some of one or two properties at the expense of maximizing distortion in others. Others attempt to only accept distort all of these properties. each projection has its own some of advantages and disadvantages.

The only position on a map where there is no distortion effect is along the intersection of the map with the surface of the geodetic datum ellipsoid or sphere. Fortunately, depending upon the kind of projection used, at least one of the four properties can generally be preserved. A conformal projection primarily maintains shape, an equal-area projection primarily maintains area and an equidistant projection primarily maintains distance.

since there is no map projection that preserves correct scale all over the map, it could be very important to know the value to which the scale varies on a part of the map. On a world map, the scale variations are clear where any continent are wrongly sized or out of shape and the graticules do not intersect at right angles or are not spaced uniformly (latitude and longitude).

2.4.1 Tissot Indicatrixi

“Scale distortions can be measured and shown on a map by ellipses of distortion. The ellipse of distortion, also known as Tissot's Indicatrix, shows the shape of an infinite small circle with a fixed scale on the Earth as it appears when plotted on the map. Every circle is plotted as circle or an ellipse or, in extreme cases, as a straight line. The size and shape of the ellipse shows how much the scale is changed and in what direction. The indicatrices on the map in the figure below have varying degrees of flattening, but the areas of the indicatrices everywhere on the map are the same, which means that areas are represented correctly on the map.

The distortion property of the map projection is therefore equal-area (or equivalent). When the indicatrices are circles everywhere on the map, the angles and consequently shapes (of small areas) are shown correctly on the map. The distortion property of the map projection is therefore conformal

(e.g. the Mercator projection).. Tissot's indicatrix has been used extensively to enhance understanding of projections in specialized cartographic literature (Snyder and Voxland 1989).

The indicatrix is also used as an educational tool in explaining map projection distortion in many contemporary textbooks in cartography (e.g., Dent 1999; Jones 1997; Kraak and Ormeling1996; and Robinson et. al. 1995) ", [Mulcahy K. A. & Clarke K. C., 2002].

The circle on the geodetic datum surface is represented as an ellipse on the projection map. The axes of the ellipse called as the coefficients of the maximum (a) and minimum (b) distance distortions, figure (2-9) [Ipbuker C. & Ulugtekin N., 2001].

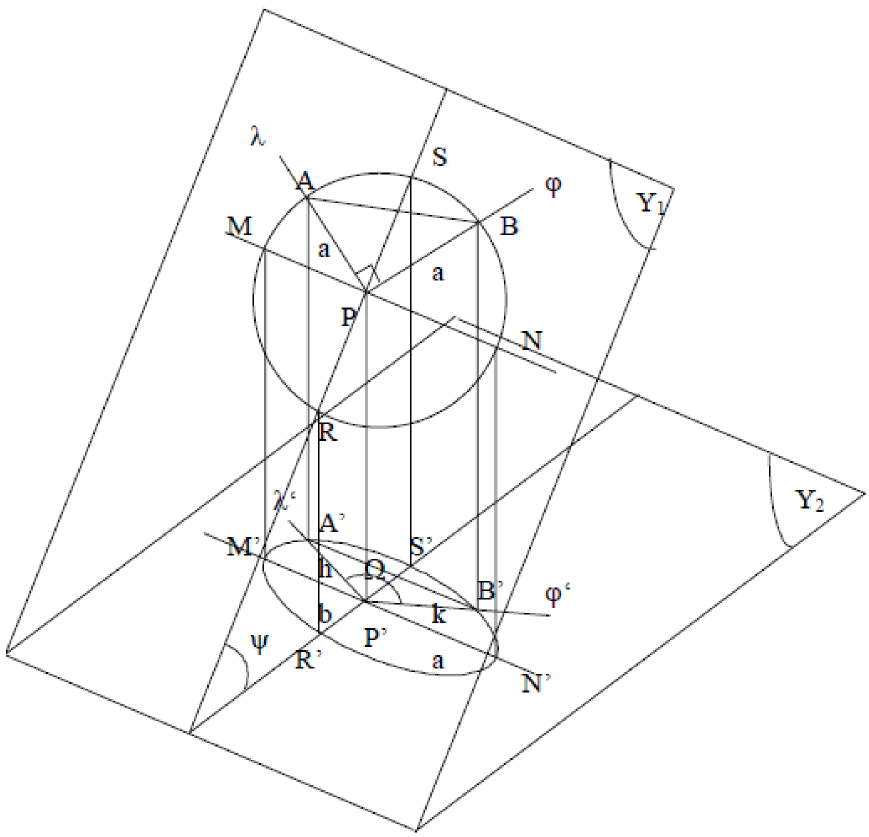


Figure (2-9): Tissot's indicatrix.

2.4.2 Distortion Types in Maps

2.4.2.1 Distortion in Equal Area Projection (cylindrical as example)

The cylindrical equal-area projection represents areas correctly by actual value without distortion, but it does have rather noticeable shape distortions towards the north and south poles. Parallels are circles unequally spaced and farthest apart near the equator Because of the distortions it is of little use for world maps, figure (2-10).

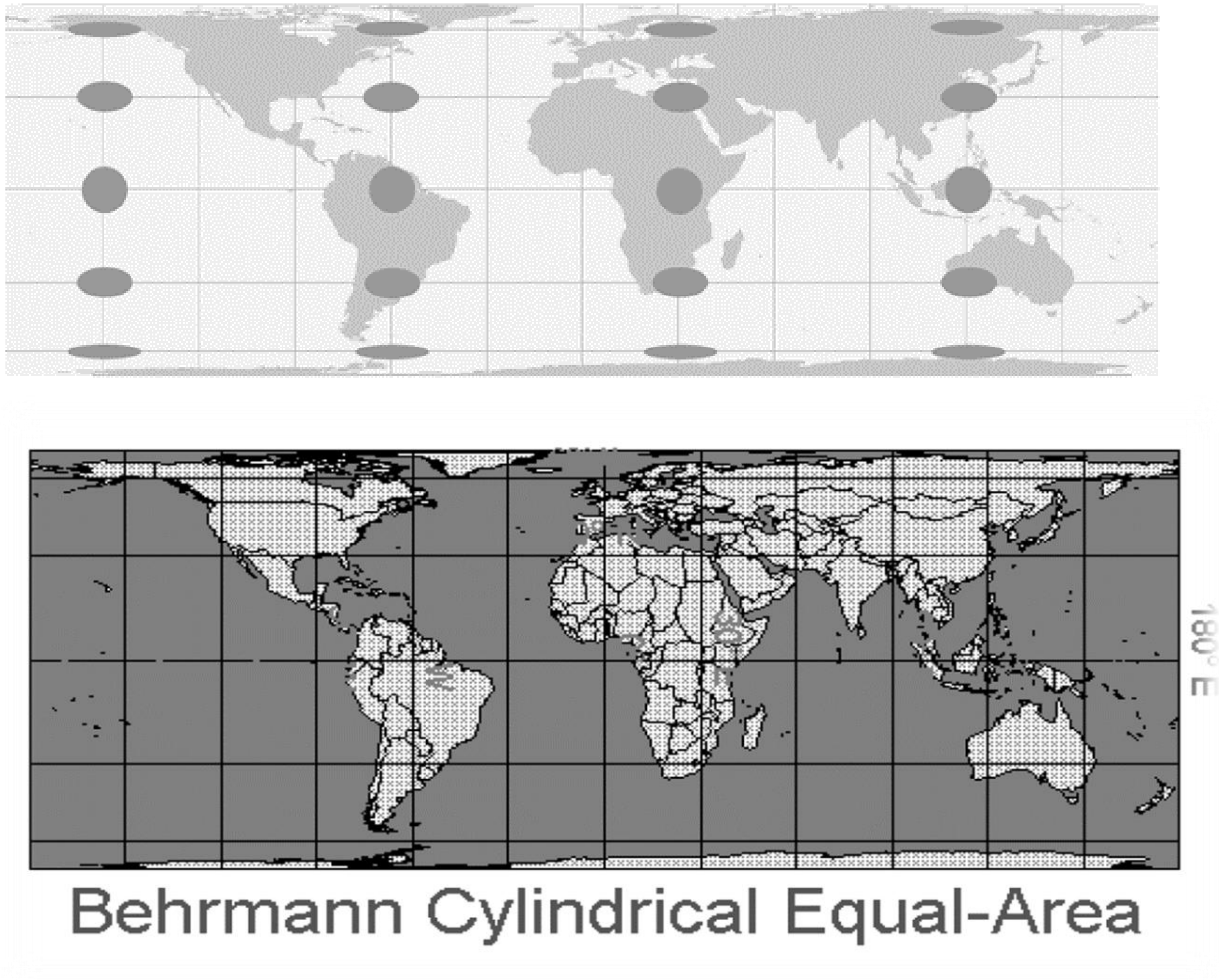


Figure (2-10): Tissot_indicatrix in equal area projection.

2.4.2.2 Distortion in conformal projection: (cylindrical as example)

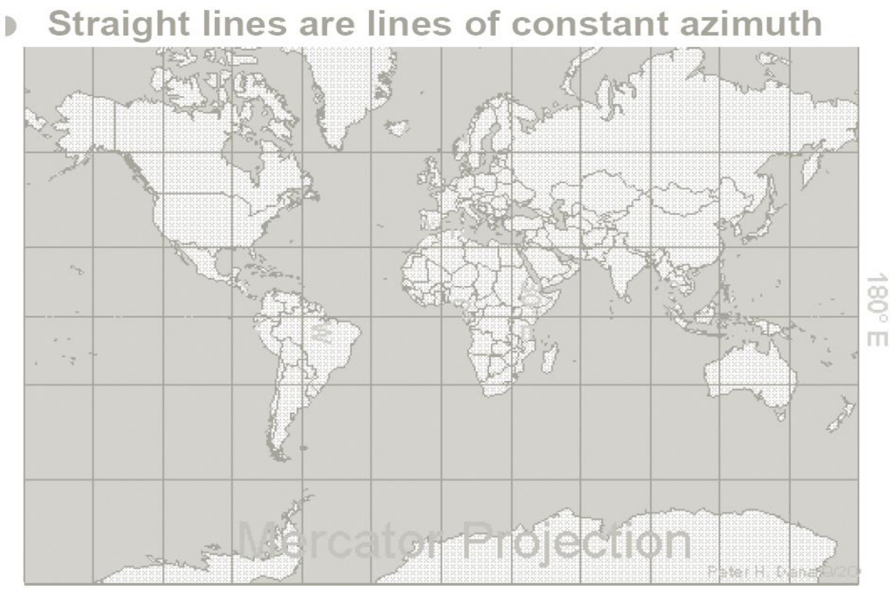
a) Cylindrical Conformal: Mercator

The Mercator projection is a conformal projection and it is normal cylindrical projection. graticules are straight lines intersecting at right angles, a requirement for conformity property. The parallel spacing increases with distance from the equator, Meridians are equally spaced, figure (2-11).

“This projection represents the tropical countries as very small in comparison to mid- and high-latitude countries. An advantage of this projection, however, is that it gives true compass bearings between any two points. This makes the Mercator projection very useful in world navigation and was used by early sailors in their explorations and discoveries of new continents,” [map projection, http://www.edu.gov.mb.ca/k12/cur/socstud/frame_found_sr2/tns/tn-6.pdf]

.

Clearly, the pole cannot be shown on this projection. On the globe, the pole is a point, but on Mercator projection, it would be represented as a straight line of the same length as the equator and would, therefore, be infinitely exaggerated. To maintain the property of orthomorphism, the scale along the meridian at the pole would also have to be infinitely exaggerated and so the pole would be infinitely distant from the equator.



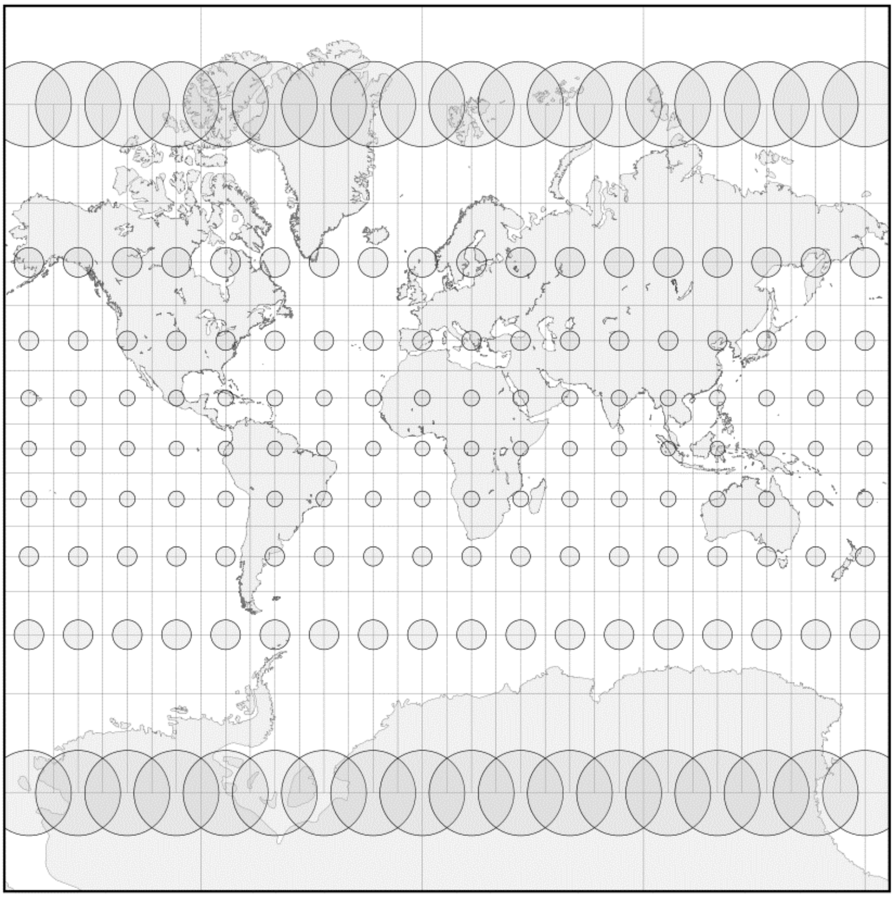


Figure (2-11): Tissot_indicatrix in conformal cylindrical projection.

One would require a sheet of paper infinitely long in order to represent it. Because the parallels are stretched progressively pole ward as the secant of latitude (parallel length on projection/parallel length on globe = $2\pi r/2\pi r \cos \phi = \sec \phi$) and because at any point on the projection this east-west stretching is balanced by an equal north south stretching, area must be exaggerated at any point as the square of the secant of the latitude. In figure (2-12), to illustrate progressive exaggeration of area with increasing distance from the equator. Each black square shows the correct representation of area, i.e. at the equator. The large squares show the actual re representation of area at latitude indicated, [Roblin H. S., 1983].

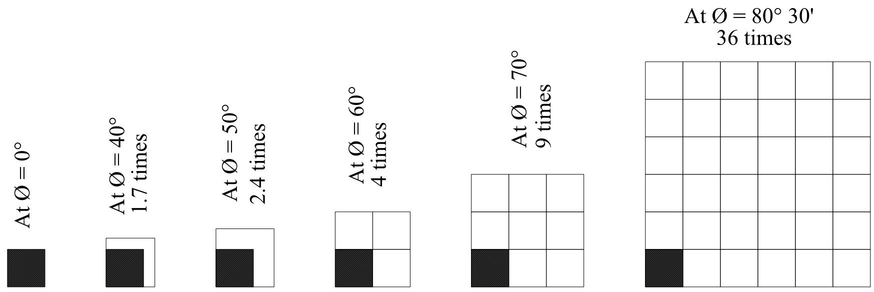


Figure (2-12): Black Square represents the correct area; the large squares represent the distort area.

Except near the equator, as a special case of lambert - this is lambert conical Projection orthomorphic projection in which the origin is on the equator, so $\phi_0 = 0$ - this projection is no general use to surveyors, but it is the basis of other more useful projections as UTM, [Bomford G. ,1983].

b) Cylindrical Conformal: Transverse Mercator & Universal Transverse Mercator

Except for surveys of polar region, which are best treated in stereographic projection, the Transverse Mercator Projection on the reference ellipsoid onto a plane (also known as the Gauss- Kruger Projection) has superseded all other conformal projection for geodetic purposes. A Transverse Mercator Projection is a projection on a cylinder tangent (or secant) to the ellipsoid along the meridian (called the central meridian) to enforce the conformity condition. The central meridian is used as the origin of the map E coordinate, and that of the map N coordinates is the equator, [Maen I, Habib, 1997].

“The Transverse Mercator (TM) projection is a conformal cylindrical projection with the cylinder rotated 90 degrees with respect to the regular Mercator projection. The cylinder is tangent to a central meridian of longitude around its entire circumference. The central meridian and equator are straight lines, but all other meridians and parallels are complex curves. Scale is constant along any meridian. Scale change along parallels is insignificant near the central meridian, but increases rapidly away from it, so the Transverse Mercator projection is useful only for narrow bands along the central meridian. Informs the basis for the Universal Transverse Mercator Coordinate System, and is

primarily used for large-scale (1:24,000 to 1:250,000) quadrangle maps. The central meridian can be mapped at true scale (Central Scale parameter = 1.0), or at a slightly reduced constant scale (for example, the value 0.9996 used in the UT system). In the latter case pair of meridians bracketing the central one maintains true scale, and the mean scale for the entire map is closer to the true scale", [Smith R. B., 2011].

- **Transverse Mercator Projection Formula**

The derivation of the following formula is found in, [Sherif M. M., 2004].

$$Y = Nr\,[(\lambda\, r)\cos\phi + ((\lambda r)^{3}\cos^{3}\phi / 6)(1-\tau^{2}+\eta^{2}) + ((\lambda r)^{5}\cos^{5}\phi / 120)(5 - 18\tau^{2} + \tau^{4} + 14\eta^{2} - 58\tau^{2}\eta^{2} + 13\eta^{4}) + \ldots\ldots] \qquad (2\text{-}5)$$

$$X = MA + Nr\,[((\lambda r)^{2}/2)\sin\phi\cos\phi + ((\lambda r)^{4}/24)\sin\phi\cos^{3}\phi\,(5-\tau^{2}+9\eta^{2}+4\eta^{4}) + ((\lambda r)^{6}/720)\sin\phi\cos^{5}\phi\,(61-58\tau^{2}+\tau^{4}+270\eta^{2}-330\tau^{2}\eta^{2}+445\eta^{4}) + \ldots\ldots] \qquad (2\text{-}6)$$

Where: $\lambda\, r = \lambda - \lambda_0$, longitude difference from central meridian in radians

$Nr = a / (1- e^{2}\sin 2\phi)^{0.5}$

$\tau = \tan\phi$

$\eta^{2} = e^{2}\cos^{2}\phi / (1 - e^{2})$

$MA = a\,(Ao\,\phi r - A1\sin^{2}\phi + A2\sin^{4}\phi - A3\sin^{6}\phi)$

Where: $Ao = 1 - (1/4)e^{2} - (3/64)e^{4} - (5/256)e^{6} - \ldots\ldots$

$A1 = (3/8)e^{2} + (3/32)e^{4} + (45/1024)e^{6} + \ldots\ldots$

$A2 = (15/256)e^{4} + (45//1024)e^{6}$

$A3 = (35/33072)e^{6}$

The terms used in the last equation are sufficient to yield X and Y values that are accurate to 1 cm for zones 3^{o} of longitude around the central meridian (i. e. zones width 6^{o}).

- **The Scale Factor for line**

Scale factors for the Transverse Mercator Projection can be computed by the following equation, [Deakin, R.E., 2006]

$$K = k_0\left[1 + \frac{E_1'^2 + E_1'E_2' + E_2'^2}{6r_m^2}\left\{1 + \frac{E_1'^2 + E_1'E_2' + E_2'^2}{36r_m^2}\right\}\right] \qquad (2\text{-}7)$$

Where:

K: is the scale factor of line 1-2

k_O: is the scale factor at the Central Meridian (C.M.).

E'_1 and E'_2: is the east or west distance of a survey point from central meridian.

E, N are projected coordinates (Easting and Northing).

$$r^2_m \quad = M*N*k^2_0 \tag{2-8}$$

and M and N are the meridian and prime vertical radii of curvature of the ellipsoid at middle latitude of the line.

$$M = \frac{a(1-e^2)}{(1-e^2 * \sin^2 \phi)^{3/2}}$$

$$N = \frac{a}{(1-e^2 * \sin^2 \phi)^{1/2}}$$

- **Distortion in Azimuths**

Geodetic Azimuth. = Grid Azimuth +Convergence Angle (γ) + (t-T) Correction (2-9)

It is the difference between grid north and geodetic one and it is given as:

$\gamma = (\lambda r) \sin [1+ ((\lambda r)2 \cos^2\phi (1+3\eta^2)) /3 + ((\lambda r)^4 \cos 4\phi (2 - \tau^2)) / 15]^3$ (2-10)

(t –T) correction is the angular difference between the projected survey line (arc) and straight grid line (chord), Figures (2-13) & (2-14)

$$t\text{-}T = \frac{(N_2-N_1)*(2E_1+E_2)}{6 M_r N_r} \tag{2-11}$$

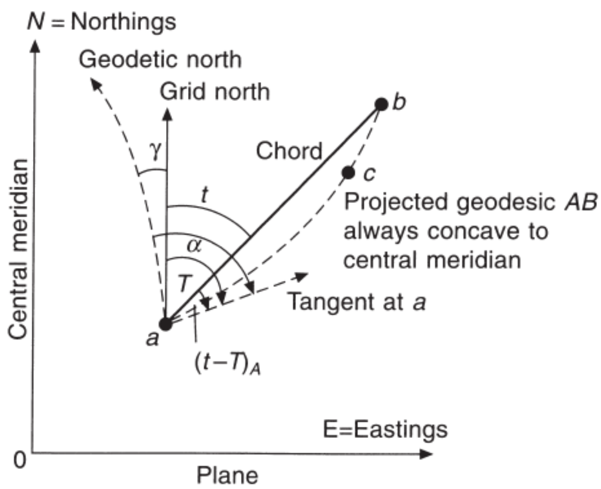


Figure (2-13): Distortion in Azimuths.

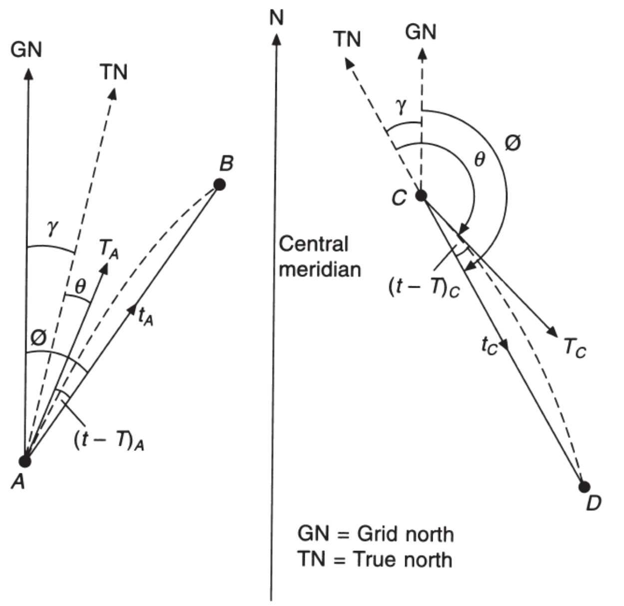


Figure (2-14): Sign of t-T correction.

"Universal Transverse Mercator (UTM) is a special globe case from TM, UTM is a worldwide system of TM projections. It comprises 60 zones, each 6◦ wide in longitude, with central meridians at 3◦, 9◦, etc. The zones are numbered from 1 to 60, starting with 180◦ to 174◦ W as zone 1 and proceeding eastwards to zone 60. Therefore, the central meridian (CM) of zone n is given by CM = 6n◦ – 183◦. In latitude, UTM system extends from 84◦ N to 80◦ S, with the polar caps covered by a polar stereographic projection. The scale factor at each central meridian is 0.9996 to counteract the enlargement ratio at the edges of the strips. The false origin of nothings is zero at the equator for the northern hemisphere and 106 m at the equator for the southern hemisphere. The false origin for easting is 500 000 m west of the zone central meridian," figure (2-15). [Schofield W. and Breach, 2007],

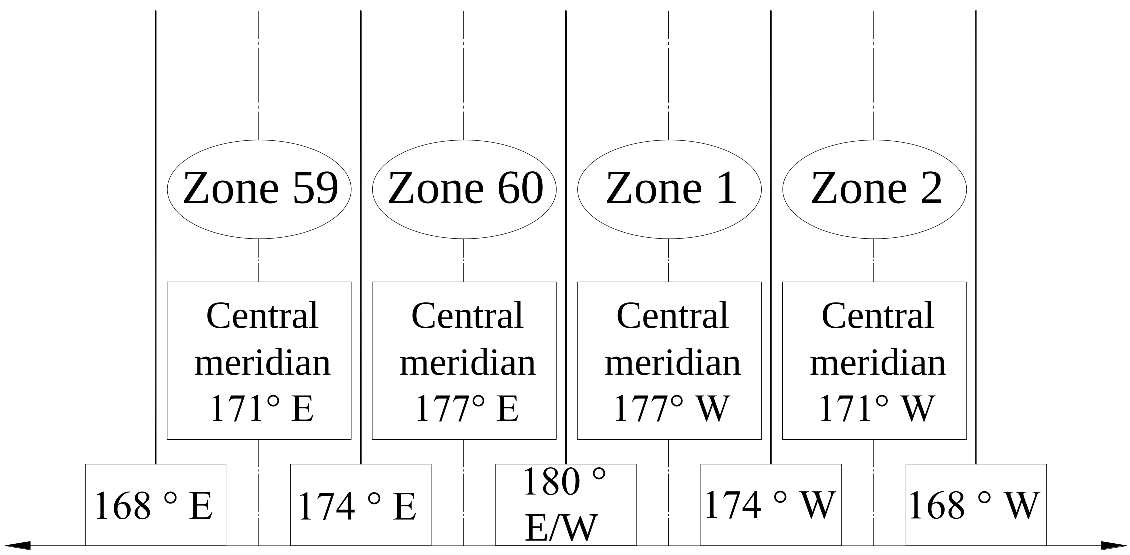


Figure (2-15): The Universal Transverse Mercator zone system.

2.4.2.3 Distortion in Equidistant Projection

a) The equidistant projection (cylindrical as example)

The equidistant cylindrical projection, has a true scale along all meridians. Meridians are spaced at the same distances as the parallels, forming a grid of equal rectangles. Both shape and area are reasonably well preserved with the exception of the Polar Regions, figure (2-16).

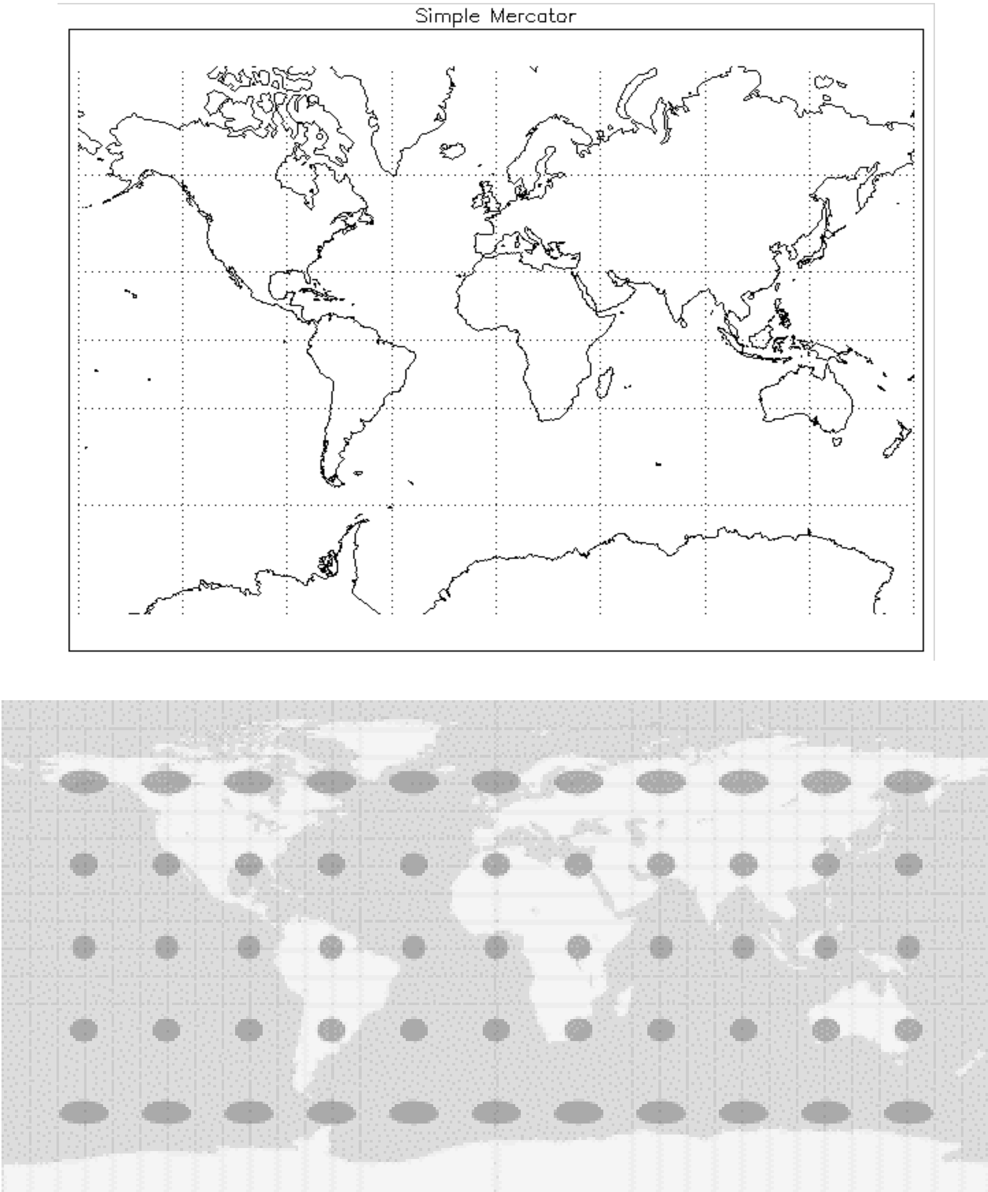


Figure (2-16): Tissot_indicatrix in equidistant cylindrical projection.

b) Distortion in modified simple cylindrical projection (sinusoidal)

To construct a sinusoidal projection of the whole world, one simply draws a standard central meridian and then constructs straight horizontal standard parallels at equal intervals, figure (2-17). Because the lengths of the parallels vary with cosine Ø, the meridians are sine waves (or cosine waves), hence the name sinusoidal, [McDonnell P. W., 1979].

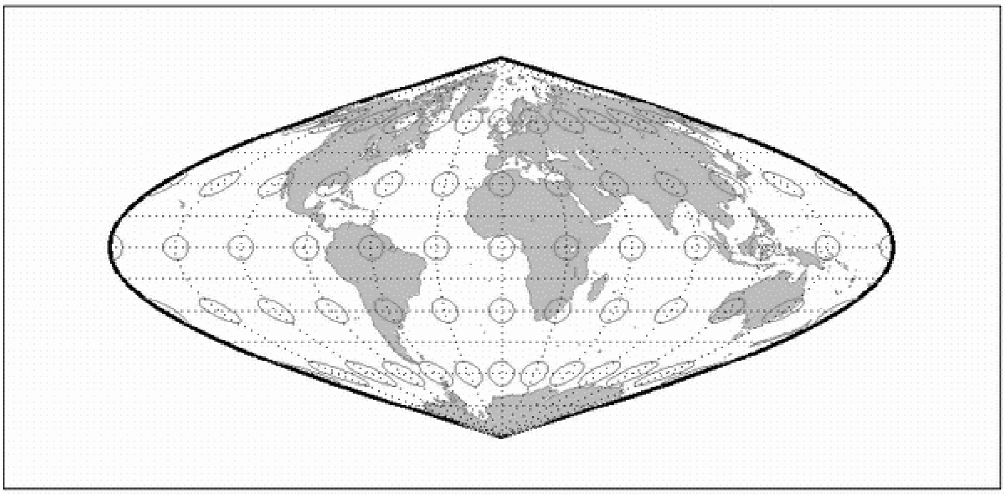


Figure (2-17): Tissot indicatrix in simple cylindrical projection (sinusoidal).

2.5 MYRIAHEDRAL PROJECTIONS

“A layman might wonder why map projection is a problem at all. A map of a small area, such as a district or city, is almost free of distortion. So, to obtain a map of the earth without distortion one just has to stitch together a large number of such small maps. The term myriahedral is assigned for the resulting class of projections. A myriahedron is a polyhedron with a myriad of faces. The Latin word myriad means ten thousand or innumerable. We project the surface of the earth on such a myriahedron, we label its edges as folds or cuts, and fold it out to obtain a float map.

2.5.1 Method of Myriahedral Projections

The earth’s surface is projected on a polyhedral mesh, label edges as cuts or folds, and unfold the mesh. Assume that the faces of the mesh are small compared with the radius of the globe, such that area and angular distortion are almost negligible. A mesh can be considered as a (planar) graph G = (V, E), consisting of a set of vertices V and undirected edges E that connect vertices. Consider the dual graph H = (V’, E’), where each vertex denotes a face of the mesh, and each edge corresponds to an edge of the original graph, but now connecting two faces instead of two vertices, Figure (2-18). After labeling edges as folds and cuts, two sub graphs H_f and H_c are obtained, where all edges of each sub graph are labeled the same.

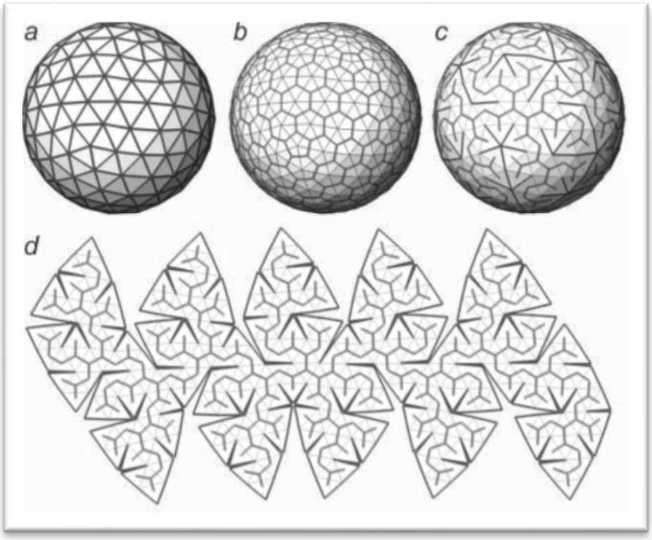


Figure (2-18): (a) Mesh G; (b) Dual mesh H; (c) Cuts and folds; (d) Foldout.

Where W_{ϕ} and W_{λ} are overall scaling factors, and ϕ_o and λ_o denote where a maximal strength is desired. Different values for these lead to a number of familiar looking projections, Figure (2-19).

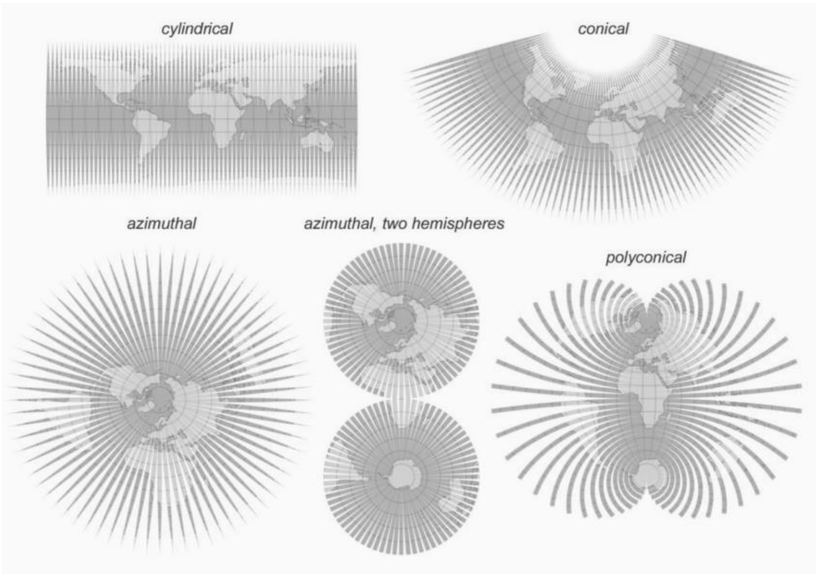


Figure (2-19): Graticular projections, derived from a 5u graticule. 2592 polygons: a) cylindrical; b) conical; c) azimuthal; d) azimuthal, two hemispheres; e) polyconical.

The relation between a spatially varying weight w and the decision where to cut and fold can be understood by considering Prim's algorithm. Suppose, without loss of generality, that we start at a maximum of w and proceed to attach the edges with the highest weight. At some point, edges at the boundary will have approximately the same weight and, after a number of additions, a ring of faces is added, with cuts in between neighboring faces in this ring. Hence,

edges aligned with contours of w typically turn into folds, whereas edges aligned with gradients of w turn into cuts. Each strip is almost free of angular or area distortion, however, a large number of interrupts occur with varying widths.

These gaps visualize, just like the Tissot indicatrix, the distortion that occurs when a non-interrupted map is used, and can be used to explain the basic problem of map projection. If we want to close these gaps, the strips must be broadened. However, to maintain an equal area, they have to be shortened, and to maintain the same aspect ratio they have to be lengthened, which is not possible simultaneously. Also, it is clearly visible that mapping a point (such as a pole) to a line leads to a strong distortion. When the number of strips is increased, the gaps are less visible, and the distortion is shown via the transparency of the map Figure (2-20)", [Wijk, J. V. 2008)],

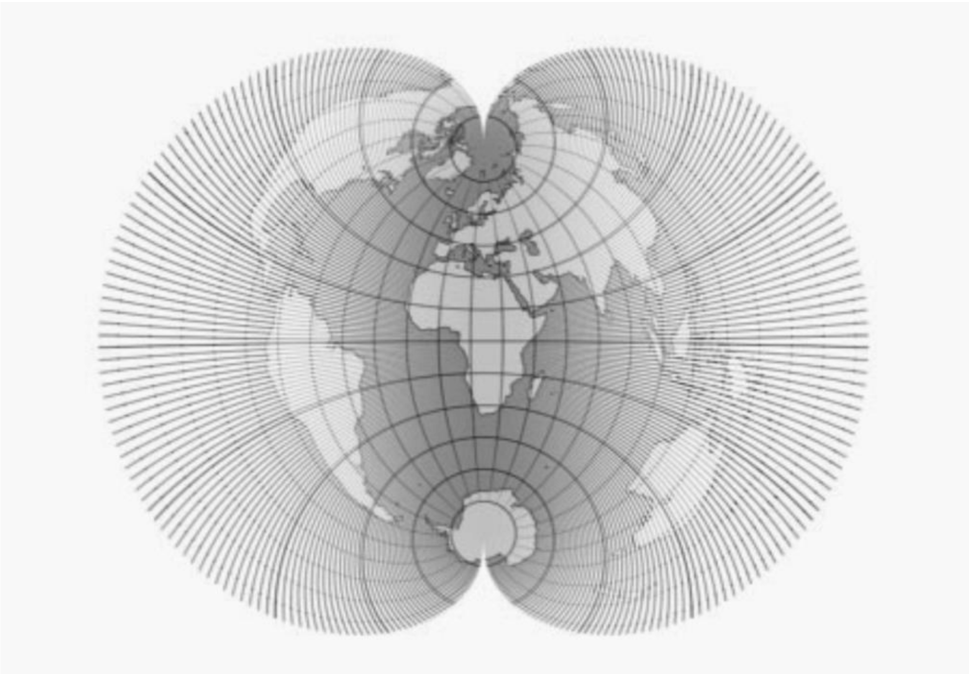


Figure (2-20): Polyconic projection, derived from a 1u graticule, 64 800 polygons.

Wijk (professor of Visualization at the Technische Universiteit Eindhoven) mentioned many projections for this idea, which seems as myriahedral (polyhedral) in the previous figures and many proposals. He explains how to do division, arrangement and combination of these plates, to the extent that distributed plates cover the earth as a sphere. We conclude from his theorem, if the projection surfaces increase the distortion will decrease. From this; we can say that if the surfaces increase to indefinitely, the distortion will vanish. That means, we will return to work on the surface of the earth as sphere or ellipsoid that is the aim of our thesis.

3. MATHEMATICS USED IN MAP AND GEODETIC DATUM

The surveying elements to be introduced to the user are azimuths or bearings, distances, and areas. These elements could be obtained by computing them from either map (projected) coordinates or from geodetic coordinates. The earth as a planet is geometrically represented as an ellipsoid or a sphere where geodetic computations should be followed. In small areas and as a special case, the work area can be considered as a plan and plan metric computations could be followed. In the past not everybody can deal with the geodetic coordinates, so map projection has been introduced to facilitate dealing with metric maps. Nowadays computers and computer programming enabled us to deal easily with geodetic computations and geodetic maps. In this chapter, the mathematical formulae used in investigating and verifying the proposed map are presented. Those formulae are programmed in this thesis as software functions to be used in producing the proposed map.

3.1 COMPUTATIONS USING MAP COORDINATES

In the case of working with map coordinates, the used equations in computing the surveying elements as distance, bearing, and area has simple form. In a plan, the distance and bearing between any two points, such as P1 and P2 in figure (3-1) can be simply computed as follows;

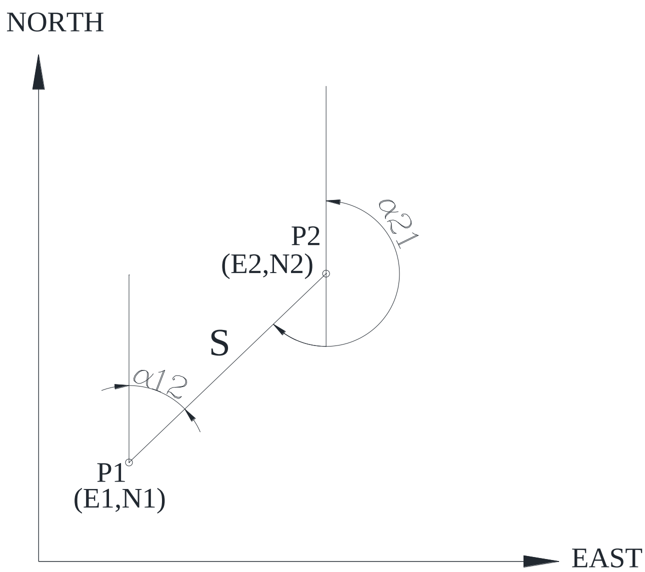


Figure (3-1): Plan distance and bearing between two points.

3.1.1 Map Distance between Two Points

$$S_{12} = \sqrt{(E_2-E_1)^2 + (N_2-N_1)^2} \qquad (3\text{-}1)$$

It is clear that the distance is a positive value and the fore and back distances of the same line are equal, i.e. $S_{12}= S_{21}$.

3.1.2 Map Azimuth (Bearing) between Two Points

The bearing can be directly computed, if the line is fixed by the coordinates of its terminal points. The bearing of a line from point P_1 to P_2 α_{12}, is given by:

$$\alpha_{12} = Tan^{-1}((E_2-E_1) / (N_2-N_1)) \qquad (3\text{-}2)$$

[Mikhail, E. M., and Anderson. M., 1998]

And the back azimuth can be obtained as:

$$\alpha_{21} = \alpha_{12} \pm 180° \qquad (3\text{-}3)$$

Remark: After calculating the distance and bearing from a projected map. then they must be corrected by correction equations of distortion to this projection, for distortion correction of UTM projection see sec 2.4.2.2b

3.1.3 Area Computations using map coordinates

Any triangle has three sides of lengths a, b, c. Its area can be obtained from the following equation:

$$Area = \sqrt{S(S-a)(S-d)(S-c)}$$; Where S= (a + b + c)/2 (3-4)

Or $A = \frac{1}{2} a.b.SinC$; Where C is the angle between sides a and b (3-5)

And the area of any polygon of (n) vertices can be obtained using:

$$Area = ½ [N_1 (E_2-E_n) +N_2 (E_3-E_1) +N_3 (E_4-E_2) +\ldots.. + N_n(E_1-E_{n-1})] \qquad (3\text{-}6)$$

Or

$$Area = ½ [(N_1+ N_2) (E_1-E_2) + (N_2+ N_3) (E_2-E_3) + (N_3+ N_4) (E_3-E_4) +\ldots.. + (N_n+ N_1) (E_n-E_1)]. \qquad (3\text{-}7)$$

3.2 COMPUTATIONS USING GEODETIC COORDINATES (MATHEMATICS PROGRAMMED IN PROUCUNG THE PROPOSED MAP)

3.2.1 Reduction of Spatial Distances

Electronic Distances Measurement (EDM) yields spatial distances ***(l)*** between two points. These distances may either be used directly or in computing geodetic coordinates ϕ, λ, h as in three-dimensional geodesy, or they may be reduced to the ellipsoid datum to calculate chord distances ***(l_o)*** or ellipsoidal geodesic distances ***(S)***. The ellipsoidal arc between the two points is approximated by a circular arc of radius R, that is the average ellipsoidal radius of curvature along the line,

[Shaker, A. A., 1990b].

$$S = 2R.Sin^{-1}\left(\frac{l_o}{2R}\right) \qquad (3\text{-}8)$$

Where $l_o = \left[\dfrac{l^2{}_{12} - \Delta h_{12}^2}{(1+\dfrac{h_1}{R_\alpha})(1+\dfrac{h_2}{R_\alpha})} \right]^{1/2}$ (3-8a)

3.2.2 Direct Geodetic Problem

Geodetic computations mainly concernthe solution of both direct and inverse geodetic problems on the reference ellipsoid. Direct geodetic problem deals with geodetic position computations of a certain point from the known geodetic coordinates of another point connected to it with known distance and direction.

Given: $\phi_1, \lambda_1, S_{12}, \alpha_{12}$

Where: ϕ_1, λ_1 are the geodetic coordinates of the known point 1.

S_{12}, α_{12}: are the geodetic distance and azimuth to point 2 of unknown coordinates.

Required: $\phi_2, \lambda_2, \alpha_{21}$

3.2.2.1 Short line formulae

1- Compute the first approximate value for $\Delta\phi$ from equation (3-9) by setting $\alpha_m = \alpha_{12}$

2- and $M_m = M_1$.

$$\Delta\phi'' = \rho'' \frac{(S_{12} * \cos\alpha_m)}{M_m} \quad (3\text{-}9)$$

Where ρ'' = 206264.8" is the number of arc seconds in one radian.

3- Compute the first approximate value for ϕ_m from equation (3-10).

$$\phi_m = \phi_1 + \Delta\phi/2 \quad (3\text{- }10)$$

4- Compute the first approximate value for $\Delta\lambda$ from equation (3-11) using the last computed value of ϕ_m.

$$\Delta\lambda'' = \rho'' \frac{(S_{12} * \sin\alpha_m)}{N_m * \cos\phi_m} \quad (3\text{-}11)$$

In which M and N are the meridian and prime vertical radii of curvature of the ellipsoid at point 1.

$$M_1 = \frac{a(1-e^2)}{(1-e^2 * \sin^2 \phi_1)^{3/2}} \tag{3-12}$$

$$N_1 = \frac{a}{(1-e^2 * \sin^2 \phi_1)^{1/2}} \tag{3-13}$$

Where: e is the first eccentricity of the ellipsoid and expressed as:

$$e = \frac{(a^2 - b^2)^{1/2}}{a} \tag{3-14}$$

in which a and b are the semi major, semi minor axis of thereference geodetic datum.

5- Compute the first approximate value for $\Delta\alpha$ from equation (3-15)

$$\Delta\alpha'' = \Delta\lambda'' Sin\phi_m \tag{3-15}$$

6- Compute α_m from equation (3-16).

$$\alpha_m = \alpha_{12} + \Delta\alpha/2 \tag{3-16}$$

7- Get better value for $\Delta\phi$ from equation (3-9) by using α_m.

8- The iterative procedure (step 2 to 6) can be continued using successive approximate values for $\Delta\phi$, $\Delta\lambda$ and $\Delta\alpha$ until the desired precision limit is reached.

9- Finally, the desired coordinates of point 2, ϕ_2, and λ_2 can be calculated by adding the values of $\Delta\phi$, $\Delta\lambda$ to the coordinates of point 1, ϕ_1, and λ_1, [Mahmoud, S. M., 2004].

3.2.2.2 Long line formulae

a. Long line formulae (McCawnon-iterative formulae)

There are many solutions to the direct problem that are require no iteration and quite accurate. Papers of interest include those of McCaw (1930), referenced in Rainsford (1955), a report by Sodano and Robinson (1963) that expands a report of Sodano (1963), and a thesis by Singh (1980) that discusses a non-iterative procedure based on some McCaw procedures. For the purposes of this text we examine first the principals involved with the McCaw solution with more detailed discussion being found in Ganshin (1969, p.86) or Singh (1980).

McCaw's solution for direct geodetic problem uses an auxiliary sphere for calculational purposes. But this sphere is used such that a point on theellipsoid geodetic datum with geodetic latitude ϕ, has a corresponding point on the sphere with the same latitude. and this correspondence the longitude on the sphere will differ the corresponding longitude on the

ellipsoid difference on the sphere must be different than on the ellipsoid, also, this correspondence the azimuths difference on the sphere must be different than on the ellipsoid. The equations of the original McCaw solution were re-cast by Rainsford (1955) and put into the following computational form given ϕ_1, λ_1 a_{21} and s: [Rapp, R. H., 1993]

Being given ϕ_1, λ_1 the solution proceeds as follows:

$$Tan\beta = (1-e^2)^{\frac{1}{2}} Tan\phi_1 \tag{3-17}$$

Where, β is the reduced latitude of the point. Now we can compute U

$$U^2 = e'^2 Cos^2\alpha = \frac{(a^2-b^2)}{b^2} Cos^2\alpha \tag{3-18}$$

Then we can compute the next coefficients

$$C_0 = 1 - \frac{3}{4}U^2 + \frac{39}{64}U^4 - \frac{133}{256}U^6 + \frac{7491}{16384}U^8, \tag{3-19a}$$

$$C_2 = \frac{3}{8}U^2 - \frac{3}{16}U^4 + \frac{111}{1024}U^6 - \frac{141}{2048}U^8, \tag{3-19b}$$

$$C_4 = \frac{15}{256}U^4 - \frac{15}{256}U^6 + \frac{405}{8192}U^8, \tag{3-19c}$$

$$C_6 = \frac{35}{3072}U^6 - \frac{105}{6144}U^8, \tag{3-19d}$$

$$C_8 = \frac{315}{131072}U^8 \tag{3-19e}$$

$$D_2 = \frac{3}{4}U^2 - \frac{3}{8}U^4 + \frac{213}{1024}U^6 - \frac{255}{2048}U^8, \tag{3-20a}$$

$$D_4 = \frac{21}{128}U^4 - \frac{21}{128}U^6 + \frac{1599}{12288}U^8, \tag{3-20b}$$

$$D_6 = \frac{151}{3072}U^6 - \frac{453}{6144}U^8, \tag{3-20c}$$

$$D_8 = \frac{1097}{65536}U^8 \tag{3-20d}$$

Then we can compute *k*, *G_1 and K*

$$k^2 = \frac{(1-U^2)}{(1+e'^2)} \tag{3-21}$$

$$Tan(G_1) = \frac{K.Tan\phi_1}{Cos\alpha_{12}} \tag{3-22}$$

$$K = \frac{\left(1 - U^2\right)^{\frac{1}{2}}}{b} \qquad (3\text{-}23)$$

Easy to compute γ, γ_1, γ_2 and γ_m than G, G_1 and G_m

$$\gamma = K.C_0.S_{12} \qquad (3\text{-}24)$$

$$\gamma_1 = G_1 - C_2 Sin(2G_1) + C_4 Sin(4G_1) - C_6 Sin(6G_1) + C_8 Sin(8G_1) \qquad (3\text{-}25)$$

$$\gamma_2 = \gamma_1 + \gamma \qquad (3\text{-}26)$$

$$2.\gamma_m = \gamma_1 + \gamma_2 \qquad (3\text{-}27)$$

$$G = \gamma + D_2 Sin(\gamma) Cos(2\gamma_m) + D_4 Sin(2\gamma) Cos(4\gamma_m) + D_6 Sin(3\gamma) Cos(6\gamma_m) \qquad (3\text{-}28)$$

$$G_2 = G_1 + G \qquad (3\text{-}29)$$

$$G_m = \frac{G_1 + G_2}{2} \qquad (3\text{-}30)$$

$$Sin\phi_2 = \frac{Sin(G_2).Cos\alpha}{k} \qquad (3\text{-}31)$$

$$Cos\alpha_{21} = kCot(G_2) / Cot\phi_2 \qquad (3\text{-}32)$$

$$Cos\lambda = \frac{Cos(G) - Sin\phi_1 Sin\phi_2}{Cos\phi_1 Cos\phi_2}, \qquad Sin\lambda = \frac{Sin(G)\sqrt{k^2 - Cos^2\alpha_{12}}}{kCos\phi_2} \qquad (3\text{-}33)$$

$$(\lambda - L) = f.Sin\alpha\left[E_0 G - E_2 Sin(G) Cos(2G_m) + E_4 Sin(2G) Cos(4G_m) - E_6 Sin(3G) Cos(6G_m) + ...\right] \qquad (3\text{-}34)$$

Where $E_0 = 1 - \frac{1}{4} f\left(1 + f + f^2\right)Cos^2\alpha + \frac{3}{16} f^2\left(1 + \frac{9}{4} f\right)Cos^4\alpha - \frac{25}{128} f^3 Cos^6\alpha$ (3-35a)

$$E_2 = \frac{1}{4} f\left(3 + 5f + 7f^2\right)Cos^2\alpha - f^2\left(1 + \frac{49}{16} f\right)Cos^4\alpha + \frac{365}{256} f^3 Cos^6\alpha \qquad (3\text{-}35b)$$

$$E_4 = \frac{5}{32} f^2\left(1 + \frac{13}{4} f\right)Cos^4\alpha - \frac{95}{256} f^3 Cos^6\alpha \qquad (3\text{-}35c)$$

$$E_6 = \frac{35}{768} f^3 Cos^6\alpha \qquad (3\text{-}35d)$$

And finaly,

$$\lambda_2 = \lambda_1 + \lambda - (\lambda - L) \qquad (3\text{-}36)$$

b. Long line Bolbol's direct formula for geodesic

This formula was derived by Saad Bolbol, M.Sc. thesis in 1978 at the Department of Geodetic Science, University of Oxford. From given geodesic length, forward azimuth and geodetic coordinates of the first point, the direct solution gives the back azimuth and geodetic coordinates of the second end. If S is the geodesic distance and $\Delta\theta$ is the central angle of geodesic distance at the center of the earth. If $\alpha_{12}$is the forward azimuth and φ_1 and φ_2 are the geodetic latitudes of two ends of the geodesic distance. If β_1 and β_2 are the reduced latitudes, α_{21} is the back azimuth and $\lambda_1$and λ_2 are the two longitudes of the two ends of the geodesic line. The values a, b, e are the semi major, semi minor and eccentricity of the reference ellipsoid. Then Bolbol's formula for calculating the direct solution is as follows:

$$(\alpha_{21}+\alpha_{12}) = Tan^{-1}([Sin\{(\beta_2-\beta_1)/2\} / (Cos\{(\beta_2+\beta_1)/2\})]*Cot\,\Delta L/2) \quad (3\text{-}37)$$

$$(\alpha_{21}-\alpha_{12}) = Tan^{-1}([Cos\{(\beta_2-\beta_1)/2\} /(Sin\{(\beta_2+\beta_1)/2\})]*Cot\,\Delta L/2) \quad (3\text{-}38)$$

Sum of equations (3-37) and (3-38) = α_{21}

Difference of equations (3-37) and (3-38) = α_{12}

$$\beta_o = Cos^{-1}(Cos\beta_1 Sin\alpha_{12}) = Cos^{-1}(-Cos\beta_2 Sin\alpha_{21}) \quad (3\text{-}39)$$

Where β_o is the parametric latitude (latitude perpendicular on the geodesic line)

$$\theta_1 = Sin^{-1}(Sin\beta_1 / Sin\beta_o) \quad (3\text{-}40)$$

$$Where\ \beta = Tan^{-1}((b/a)\,Tan\varphi) \quad (3\text{-}41)$$

The calculations of the geodetic coordinates of the end of geodesic line:

$$\Delta\theta'' = \alpha\,(s/b) - \beta\,Sin\,\Delta\theta\,Cos\,2\sigma + \gamma\,Sin\,2\Delta\theta Cos\,4\sigma + Б\,Sin\,3\Delta\theta Cos\,6\sigma \quad (3\text{-}42)$$

$$\text{Where } 2\sigma = 2\theta_1 + \Delta\theta \quad (3\text{-}43)$$

$$\text{And } Sin\,\Delta\theta_1 = Sin\,\beta_1 / (Cos^{-1}(Cos\beta_1 Sin\,\alpha_{12})) \quad (3\text{-}44)$$

$$S' = b\,(A_o\,\Delta\theta_r + B_o Sin\,\Delta\theta Cos4\sigma + C_o Sin\,2\Delta\theta Cos\,4\sigma) \quad (3\text{-}45)$$

$$d\Delta\theta = \alpha\,((s-s')/b) \quad (3\text{-}46)$$

$$d\Delta\theta'' = d\Delta\theta'' - (\beta Cos\,(\Delta\theta + 2\sigma)\,d\Delta\theta) + (\gamma Cos(2\Delta\theta+4\sigma)\,2d\Delta\theta) \quad (3\text{-}47)$$

$$\Delta\theta'' = \Delta\theta'' + d\,\Delta\theta'' \quad (3\text{-}48)$$

The geodetic latitude φ_2 of the second end of the geodesic line can be calculated as follows:

$$Sin\beta_2 = Sin\beta_1 Cos\,\Delta\theta + Cos\beta_1 Sin\,\Delta\theta\,Cos\alpha_{12} \quad (3\text{-}49)$$

$$And\ \ Tan\,\varphi_2 = (a/b)\,Tan\beta_2 \quad (3\text{-}50)$$

The back azimuth α_{21}: $$Sin\alpha_{21} = (Cos\,\beta_1 Sin\,\alpha_{12}) / Cos\,\beta_2 \quad (3\text{-}51)$$

The difference of longitude is obtained as:

$$\Delta\lambda = Cos^{-1} (-Cos\alpha_{12}Cos\alpha_{21} + Sin\alpha_{12}Sin\alpha_{21}Cos\Delta\theta) + Cos\,\beta_o\,(A\,\Delta\theta + B\,Sin\,\Delta\theta\,Cos\,2\sigma$$

$$+ C\,Sin\,2\Delta\theta\,Cos4\sigma + D\,Sin\,3\Delta\theta Cos\,6\sigma) \quad (3\text{-}52)$$

Where $Cos\beta_o = Cos\beta_1 Sin\alpha_{12}$ (3-53)

Then $\lambda_2 = \lambda_1 + \Delta\lambda$ (3-54)

Where:

$$A_o=1+(k^2/4)-(3k^4/64)+(5k^6/256)-(175k^8/16384) \quad (3\text{-}55a)$$

$$B_o=(k^2/4)-(k^4/16)+(15k^6/256)-(35k^8/2045) \quad (3\text{-}55b)$$

$$C_o=(k^4/128)-(3k^6/512)+(35k^8/8192) \quad (3\text{-}55c)$$

$$D_o=(3k^6/1536)-(5k^8/6144) \quad (3\text{-}55d)$$

Where k is modulus

and $k = e'\,sin\,\beta_o$ (3-55e)

$$\alpha=(1/A_o)\,\rho'' \quad (3\text{-}56)$$

$$\beta=(B_o/A_o)\,\rho'' \quad (3\text{-}57)$$

$$\gamma=(C_o/A_o)\,\rho'' \quad (3\text{-}58)$$

And

$$Б=(D_o/A_o)\,\rho \quad (3\text{-}59)$$

$$A=e^2\,((1/2)+(e^2/8)+(e^4/16)+(5e^6/128)+(7e^8/256))-(e^2\,sin^2\beta_o/16)(1+e^2+15e^4)+$$

$$((3/128)\,e^6Sin^4\beta_o\,(1+15e^2/8))-((25e^8/2048)\,Sin^6\beta_o) \quad (3\text{-}60a)$$

$$B=(((e^4/16)Sin\beta)(1+e^2+(15e^4/16)))-((e^6Sin^4\beta o/32)(1+(15e^2/8)))+(75e^8Sin^6\beta_o/4096) \quad (3\text{-}60b)$$

$$C=((e^6Sin^4\beta_o(1+(15e^2/8)))-(15e^8Sin^6\beta_o/4096) \quad (3\text{-}60c)$$

$$D=5e^8Sin^8\beta_o/12288 \quad (3\text{-}60d)$$

For more accurate data the parameters can be expanded, [Bolbol S., 2018].

3.2.3 Inverse Geodetic Problem

Inverse geodetic problem in geometric geodesy involves the computations of distance and direction between any two points of known coordinates. There are many solutions for solving the direct and

inverse geodetic problems, these solutions (formulae) are classified according to the distance for which they are valid, and so there are two types of formula:

The distance and azimuth of a line connecting two points can be computed from their known geodetic coordinates (ϕ, λ) taking the form:

$$S_{12}, \alpha_{12}, \alpha_{21} = f\ (\phi_1, \lambda_1, \phi_2, \lambda_2) \quad (3\text{-}61)$$

3.2.3.1 Inverse Short line formulae

a) Gauss Mid-Latitude

Here short line formulae such as Gauss Mid-Latitude Formulae will be explained, these formulae based on spherical approximation of the earth. They involve more approximation than long line formulae such as (puissant formulae) and hence are expected to be simpler. Consequently, they are valid to the computation for lines less than 40 km located at latitudes less than 70°, [Nassar, 1986]. Since ϕ_m is known can be computed from (3-10), $\Delta\lambda$ can be computed from equation (3-11). Then α_m can be calculated from next equation (3- 62).

$$\alpha_m = \arctan \frac{[N_m * \cos\phi_m * (\Delta\lambda / \Delta\phi)]}{M_m} \quad (3\text{-}62)$$

Now α_{12} and α_{21} can be computed using equations (3-63) and (3- 64)

$$\alpha_{12} = \alpha_m - \Delta\,\alpha/2 \quad (3\text{-}63)$$

$$\alpha_{21} = \alpha_m + \Delta\,\alpha/2 + 180^o \quad (3\text{-}64)$$

Finally, the distance S_{12} can be computed using (3-65).

$$S_{12} = (\Delta\varphi / \rho'') * (M_m / Cos\ \alpha_m) \quad (3\text{-}65)$$

Remark: The solution S_{12} is not correct if the two point in the same latitude because $\Delta\phi = 0.0$

b) Puissant’s formulae

Theseformulae are named after the French mathematician who is credited with their development. Their derivation is based on asp
herical approximation; thus, they are generally considered to be correct to l ppm at 100 km, beyond which they break down rapidly (40 ppm at 250 km when latitude = 60°) [Bamford, 1971]. Thus, we say that Puissant's Formula is a "short" line formula, [Krakiwsky E.J. & Thomson D.B., 1995].

When the geodetic coordinates are available which the case here is, only the inverse geodetic problem is needed. Recall here that the solution of the inverse geodetic problem implies the

determination of the geodesic distance S_{12}, the forward azimuth, α_{12} and the backward azimuth α_{21} from the given geodetic coordinates (ϕ, λ) of points 1 and 2. This can be computed by following the next steps, [Nassar, M. M. 1984]:

$$S_{12}\cos\alpha_{12}=\frac{\Delta\phi^{\backslash\backslash}}{\rho}\left[\frac{M_1}{1-\frac{3e^2\sin\phi_1\cos\phi_1}{2(1-e^2\sin^2\phi_1)}\left(\frac{\Delta\phi^{\backslash\backslash}}{\rho}\right)}\right]+\frac{S_{12}^2\tan\phi_1\sin^2\alpha_{12}}{2N_1} \tag{3-66}$$

$$+\frac{S_{12}^3\cos\alpha_{12}\sin^2\alpha_{12}(1+3\tan^2\phi_1)}{6N_1^2}$$

$$S_{12}\sin\alpha_{12}=\frac{\Delta\lambda^{\backslash\backslash}}{\rho}\times\frac{N_2}{\sec\phi_2}+\frac{S_{12}^3}{6N_2^2}\sin\alpha_{12}-\frac{S_{12}^3}{6N_2^2}\sin^3\alpha_{12}\sec^2\phi_2 \tag{3-67}$$

1. Compute an approximate value α_{12}^0 as follows:

$$\alpha_{12}^0=\tan^{-1}\left[\frac{first\,term\,of\,(3-67)}{first\,term\,of\,(3-66)}\right] \tag{3-68}$$

2. Compute the first approximate value S_{12}^0 from either Equation (3-66) or (3-67)

$$S_{12}^0=\frac{firsttermof\,(3-67)}{\sin\alpha_{12}^0}=\frac{firsttermof\,(3-66)}{\cos\alpha_{12}^0} \tag{3-69}$$

3. Use α_{12}^0 and S_{12}^0 to evaluate the full expression of the right-hand side of both equations
4. (3-66) and (3- 67), from which more accurate value for the azimuth α_{12} can be obtained as follows:

$$\alpha_{12}=\tan^{-1}\left[\frac{RHS\,of\;Equation(3-67)}{RHS\,of\;Equation(3-66)}\right] \tag{3-70}$$

5. Use the computed value of α_{12} in step 3 to get more accurate value for S_{12} using the following equation:

$$S_{12}=\frac{RHS\,of\;Eq.(3-66)}{\cos\alpha_{12}}=\frac{RHS\,of\;Eq.(3-67)}{\sin\alpha_{12}} \tag{3-71}$$

[Krakiwsky E. J. & Thomson D.B., 1995]

6. Perform the iteration upon steps 3 and 4 until the change in the value of α_{12} and S_{12} is negligible. This value may be in the order of 0.001" to 0.00001" and 0.0001m respectively for most applications, and is taken here on our case as 0.00001 " for azimuths.

To compute the backward azimuth α_{21}, we should firstly compute $\Delta\alpha$ as follows:

$$\Delta\alpha = \Delta\lambda \sin\phi_m \sec\left(\frac{\Delta\phi}{2}\right) + \frac{\Delta\lambda^3}{12}\left[\sin\phi_m \sec\left(\frac{\Delta\phi}{2}\right) - \sin^3\phi_m \sec^3\left(\frac{\Delta\phi}{2}\right) + ...\right] \qquad (3\text{-}72)$$

Then compute α_{21} from the following equation:

$$\alpha_{21} = \alpha_{12} \pm 180 + \Delta\alpha \qquad (3\text{-}73)$$

3.2.3.2 Inverse Long Line Formulas

a. Inverse Long Line Formulae (Bessel's Formulae)

The calculation of the distance is direct once the auxiliary spherical triangle has been solved with good precision. The geodesic distance on ellipsoid and the azimuth can be computed from two known geodetic points, [Richard H. Rapp, 1976] & [Krakiwsky E. J. & Thomson D.B., 1995]:

$$S_{12} = b(B_o\sigma + B_2 Sin\sigma Cos2\sigma_m + B_4 Sin2\sigma Cos4\sigma_m + B_6 Sin3\sigma Cos6\sigma_m + B_8 Sin4\sigma Cos8\sigma_m + ...) \qquad (3\text{-}74)$$

In equation (3- 74), we have the following coefficients:

$$B_0 = 1 + \frac{1}{4}U^2 - \frac{3}{64}U^4 + \frac{5}{256}U^6 - \frac{175}{16384}U^8, \qquad (3\text{-}75a)$$

$$B_2 = -\frac{1}{4}U^2 + \frac{1}{16}U^4 - \frac{15}{512}U^6 + \frac{35}{2048}U^8, \qquad (3\text{-}75b)$$

$$B_4 = -\frac{1}{128}U^4 + \frac{3}{512}U^6 - \frac{35}{8192}U^8, \qquad (3\text{-}75c)$$

$$B_6 = -\frac{1}{1536}U^6 + \frac{5}{6144}U^8, \qquad (3\text{-}75d)$$

$$B_8 = -\frac{5}{65536}U^8, \qquad (3\text{-}75e)$$

And U computed from equation (3-18)

$$U^2 = e'^2 Cos^2\alpha = \frac{(a^2 - b^2)}{b^2} Cos^2\alpha$$

The maximum effect on the geodesic distance (S) of any term in U is less than 0.5 millimeter, [Rainsford H. F,1960]

Also, from the triangle (P1'- pole- P2') we can apply the spherical law of Cosines to get:

$$Cos\sigma = Sin\beta_1 Sin\beta_2 + Cos\beta_1 Cos\beta_2 Cos\Delta L, \qquad (3\text{-}76)$$

σ is the angular distance between the two points on the auxiliary sphere.

This formula weakly computes σ when σ is very small, so that the next equation is recommended (Sodano, 1963) when (Cosσ) is close to one or when both (Sinσ) and Cosσ are to be used in next calculations.

$$Sin\sigma = \left[(Sin\Delta L Sin\beta_2)^2 + (Sin\beta_2 Cos\beta_1 - Sin\beta_1 Cos\beta_2 Cos\Delta L)^2\right]^{\frac{1}{2}} \qquad (3\text{-}77)$$

σ_m is the angular distance on auxiliary sphere from the equator to mid-point of line 1 to 2, measured along the great circle passing through point 1 and 2and given by:

$$Cos2\sigma_m = Cos\sigma - \frac{2Sin\beta_1 Sin\beta_2}{Cos^2\alpha} \qquad (3\text{-}78)$$

Where β_i is the reduced latitude for point i, and it is given by:

$$Tan\beta_i = (1-e^2)^{\frac{1}{2}} Tan\phi_i\,, \qquad (3\text{-}79)$$

And

$$\Delta L = \Delta\lambda + fSin\alpha(A_0\sigma + A_2 Sin\sigma Cos2\sigma_m + A_4 Sin2\sigma Cos4\sigma_m + A_6 Sin3\sigma Cos6\sigma_m + ...) \quad (3\text{-}80)$$

In which:

$$A_0 = 1 - \frac{1}{4} f\left(1 + f + f^2\right)Cos^2\alpha + \frac{3}{16} f^2\left(1 + \frac{9}{4} f\right)Cos^4\alpha - \frac{25}{128} f^3 Cos^6\alpha \qquad (3\text{-}81a)$$

$$A_2 = \frac{1}{4} f\left(1 + f + f^2\right)Cos^2\alpha - \frac{1}{4} f^2\left(1 + \frac{9}{4} f\right)Cos^4\alpha + \frac{75}{256} f^3 Cos^6\alpha \qquad (3\text{-}81b)$$

$$A_4 = \frac{1}{32} f^2\left(1 + \frac{9}{4} f\right)Cos^4\alpha - \frac{15}{256} f^3 Cos^6\alpha \qquad (3\text{-}81c)$$

$$A_6 = \frac{5}{768} f^3 Cos^6\alpha \qquad (3\text{-}81d)$$

The A coefficients are given as functions of (*f*) since they converge more fast than when given as functions of (e^2). The maximum value of any term in f4 (i.e.f3 in the A's) is less than 0.00001" even for a line half round the world, thus the equation (3-81d) can be erase [Rainsford, H. F. ,1960].

(f) is the flattening of the used ellipsoid datum and (b) is the semi - minor axis of that ellipsoid.

And α is the azimuth of geodesic line on the equator and is computed from

$$Sin\alpha = \frac{Sin\Delta L Cos\beta_1 Cos\beta_2}{\sin\sigma} \qquad (3\text{-}82)$$

And the geodetic azimuths α_{12} and α_{21} can be computed as

$$Sin\alpha_{12} = \frac{Sin\alpha}{Cos\beta_1} \qquad (3\text{-}83)$$

$$Sin\alpha_{21} = \frac{Sin\alpha}{Cos\beta_2} \tag{3-84}$$

Somewhat more the best equations are recommended bySodano (1963) for azimuth determinations:

$$Tan\alpha_{12} = \frac{Sin\Delta L Cos\beta_2}{Sin\beta_2 Cos\beta_1 - Cos\Delta L Sin\beta_1 Cos\beta_2} \tag{3-85}$$

$$Tan\alpha_{21} = \frac{Sin\Delta L Cos\beta_1}{Sin\beta_2 Cos\beta_1 Cos\Delta L - Sin\beta_1 Cos\beta_2} \tag{3-86}$$

Proper quadrant determinations for the azimuths can be made by using arc tangent subroutines where the input parameters are (Sinα, and (Tanα).

It is obvious from equations (3-74), (3-83), (3-84), (3-85) and (3-86) that S_{12}, α_{12}, and α_{21} cannot be computed directly. The steps of this procedure can be written as follows:

1) The first step is to compute β_1 and β_2(reduced latitudes) using equation (3-79).
2) The difference in longitude on the sphere ΔL is considered, as first approximation, equals the difference in longitude on the ellipsoid Δλ, where $\Delta\lambda = \lambda_1 - \lambda_2$
3) Then, from reduced sphere we can compute the arc length σ using equation (3-76) or (3-77).
4) σ_m is computed using equation (3-78).
5) the value of α is computed using equation (3-82)
6) A new better value for ΔL is computed from equation (3-80) using the computed values of σ, σ_m and λ.
7) Steps from 3 to 6 are repeated until the change in the value of ΔL is negligible. To difference about 0.00001". The sought value of S_{12} computed using equation (3-74). Also, the required azimuths α_{12}, α_{21} as well as their proper quadrant can be now computed using equations (3-83), (3-84), or (3-85), (3-86), [Awad, M. E. M., 1997].

b. Inverse Geodesic Long Line Formulae (Bolbol's Formulae)

Many formulae have been derived for computing the geodesic distance with different accuracy. Bolbol's formula was derived at Department of geodetic science, The University of Oxford, which gives accuracy as one millimeter for 10,000km as follows:

Where: The geodesic length S is:

$$S = b\{A_0 \Delta\theta + (B_0 + F_0/2 + G_0) Sin\,\Delta\theta\,Cos\,2\sigma - (C_0/2 + H0/4) Sin\,2\Delta\sigma Cos\,4\sigma$$
$$+ (D_0/3 + F_0/6) Sin\,3\Delta\theta\,Cos\,6\sigma - (E_0/4 + H_0/8) Sin\,4\Delta\theta Cos\,8\sigma + (G_0/10) Sin\,5\Delta\theta\,Cos\,10\sigma \tag{3-87}$$

$$A_0 = 1 + (k^2/4) - (3k^4/64) + (5k^6/256) - (175k^8/16348) + (441k^{10}/65536) - (14553k^{12}/3145728) \tag{3-88a}$$

$B_0 = (k^2/4) – (k^4/16) + (k^6/256) – (35k^8/2048) + (k^{10}/65536) – (186k^{12}/1572864)$ (3-88b)

$C_0 = (k^4/64) + (3k^6/256) – (35k^8/4096) + (105k^{10}/16384) – (19467k^{12}/6291456)$ (3-88c)

$D_0=(3k^6/512) – (5k^8/2048) – (147k^{10}/65536) – (1449k12/786432)$ (3-88d)

$E_0 = (5k^8/4096) + (35k^{10}/65536) – (945k^{12}/1572864)$ (3-88e)

$F_0 = (21k^{10}/65536) – (567k^{12}/786432)$ (3-88f)

$G_0 = (7k^{10}/65546) – (189k^{12}/786432)$ (3-88g)

$H_0 = (189k^{12}/1572864)$ (3-88h)

$k = e\ Sin\beta_0$ (3-88i)

$\theta = Sin^{-1} (Cos\varphi'_1 Cos\varphi'_2 Sin (\Delta L/2) + Sin \{(\varphi'_2 – \varphi'_1)/2\})^{1/2}$ (3-89)

φ' is the reduced latitude

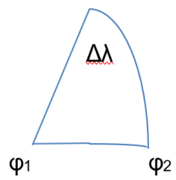
Where$\varphi' = Tan^{-1} \{(1-e)^{1/2} Tan\varphi\}$ (3-90)

$Cos\ 2б = (2\ Sin\varphi'_1 Sin\varphi'_2 / Sin^2\beta_0) – Cos\theta$ (3-91)

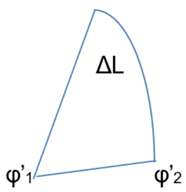
$Cos\ 4\theta = 2\ Cos^2\ 2б\ -1$ (3-92)

$\beta_0 = Sin^{-1} \{1 – (Sin\varphi'_1 Cos\varphi'_2 Sin\Delta L / Sin\theta)^2 \}^{1/2}$ (3-93)

The solution of ellipsoidal triangle is by auxiliary spherical triangle as in figure (3-2)



Ellipsoidal triangle



Equivalent spherical Triangle

Figure (3-2): Ellipsoidal and spherical triangles.

The difference between ΔL and Δλ is denoted by ΔX and calculated as the following formula, (derived by the author at The University of Oxford 1978): [Bolbol S., 2018].

$\Delta X = \{(K_1 + B)\ \theta_0 + (C + D)\ Sin\theta_0 + [(F+ E)\ G)\ A]\} / K_2$ (3-94)

Where θ_0 is computed as θ with changing ΔL by Δλ

$K_1 = (16H – e''^2) / (16\ e^2\ H^2 + e'^2)$ (3-95a)

$K_2 = (16\ Sin\ 1'') / (e' (16\ e^2 H^2 + e'^2))$ (3-95b)

$K_3 = (16\ e^2\ H^2 + e'^2) / e'^2$ (3-95c)

$K_4 = 2e'^2 / (16\, e^2 H^2 + e'^2)$ (3-95d)

$K_5 = 16\, e^2 H^2 / (16\, e^2 H^2 + e'^2)$ (3-95e)

$K_6 = 16\, e^2 H^2 / e'^2$ (3-95f)

e' is the second eccentricity = $(a^2 - b^2) / b^2$ (3-96)

$A = (Cos\varphi'_1\, Cos\varphi'_2 Sin\Delta\lambda) / Sin\theta_0$ (3-97a)

$B = A^2$ (3-97b)

$C = Cos\theta_0\, (1 - B) / K_3$ (3-97c)

$D = - K_4 Sin\varphi'_1 Sin\varphi'_2$ (3-97e)

$E = - K_5 Sin\varphi'_1 Sin\varphi'_2$ (3-97f)

$F = C\, K_6$ (3-97g)

$G = \theta_0^2 / Sin\, \theta_0$ (3-97h)

The most accurate and accepted formula for calculating azimuth is Cunningham's formula, which is closed formula.

$$\Omega_{12} = \frac{Tan\varphi_2}{(1+E)Tan\varphi_1} + e^2 \left(\frac{(1+E)Tan^2\varphi_2}{(1+E)Tan^2\varphi_1} \right)^{1/2} \quad (3\text{-}98)$$

$$Cot\, A_{12} = (\Omega_{12} - \cos\, \Delta\lambda)\, Sin\, \varphi_1 Cosec\, \Delta\lambda \quad (3\text{-}99)$$

Where

$E = e^2 / (1 - e^2)$, (3-100)

and $e^2 = (a^2 - b^2) / a^2$ & e is the eccentricity (3-101)

a and b are the semi major and semi minor axes of reference ellipsoid

A_{12} is the geodesic and normal section azimuth from point 1 to point 2

Similar:

$$\Omega_{21} = \frac{Tan\varphi_1}{(1+E)Tan\varphi_2} + e^2 \left(\frac{(1+E)Tan^2\varphi_1}{(1+E)Tan^2\varphi_2} \right)^{1/2} \quad (3\text{-}102)$$

$A_{21} = (\Omega_{21} - \cos\, \Delta\lambda)\, \sin\, \varphi_2\, Cosec\, \Delta\lambda$ (3-103)

A_{21} is the geodesic and normal section azimuth from point 2 to point 1

3.2.4 Relation between Geodetic Curvilinear Coordinates and Geodetic Cartesian Coordinates

For the spatial determination of points with respect to the ellipsoid of geodetic datum we often use geodetic coordinates: latitude ϕ, longitude λ and ellipsoidal height h above or under the surface of

reference datum. The relationship between the Cartesian rectangular coordinates *X, Y, Z.* of a space point and curvilinear coordinates ϕ, λ, h are well known. The inverse problem of computing ϕ, λ and h from *X, Y, Z* has been considered by many authors.

By referring to Figure (3-3), the well-known relations between the two systems can be written as; $(\phi, \lambda, h) \rightarrow (X, Y, Z)$

The geodetic coordinates (ϕ, λ, h) are related to the equivalent Cartesian rectangular coordinates (X, Y, Z) by

$$X = (N + h)Cos\phi Cos\lambda \tag{3-104a}$$

$$Y = (N + h)Cos\phi Sin\lambda \tag{3-104b}$$

$$Z = [N(1-e^2) + h]Sin\phi \tag{3-104c}$$

Where N = the radius of curvature in the prime vertical and

h = the ellipsoidal height.

Now we come to the inverse process, that is; $(X, Y, Z) \rightarrow (\phi, \lambda, h)$

This procedure is computed by iteration process. The solution used by here was given by Vincenty(1979). It is a combination between two solutions Bartelme and meissl(1975) and Bowring(1976), its procedure is simply given as follows:

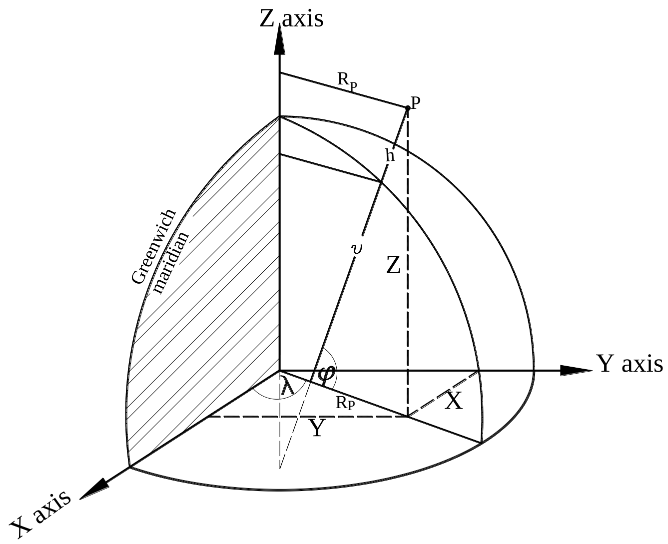


Figure (3-3): Relation between the geodetic Coordinates and geodetic Cartesian Coordinates.

First, the geodetic longitude is calculated by

$$Tan\lambda = Y/X \tag{3-105a}$$

Put $R_P = \sqrt{(X^2 + Y^2)}$

And $Tan\,\theta = (Z / R_P)\,(a / b)$

Then $Tan\phi = (Z + e^2\, b\, Sin^3\theta) / (R_P - e^2 a\, Cos^3\,\theta)$ (3-105b)

The previous formula works without iteration. The expression for tan θ is principle only correct at spheroidal level, but it involves an error of less than 0.1* 10-6 in f for any possible height [Shaker, 1982]. The expression for the height may be written by referring to Figure (3- 4), as follows:

$$h = \sqrt{(R_P - R_E)^2 + (Z - Z_E)^2} \quad \text{(3-105c)}$$

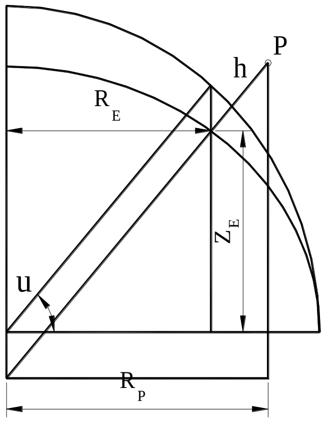


Figure: (3-4): Expression for the height.

$$h = \sqrt{((R_P - a \, Cos \, u)^2 + (Z - b \, Sin \, u)^2)} \quad \text{(3-106)}$$

Where $Tan \, u = (b/a) \, Tan \, \phi$ [Arafa, A. S. A., 2005],

3.2.5 Area Calculation on ellipsoid Datum

The calculation of the area of ellipsoidal polygons is an important problem in geodesy and cartography. There are some methods known for the calculation of the area of geodetic polygons but they are limited to small figures.

a) Calculation of the areas of geodetic spherical polygons using spherical trapezoids

The first one is based on the approximation of polygon by elementary trapezoids limited by parallels and meridians.The area of spherical trapezoid limited by two parallels and two meridians might be written bydoubleintegration on sphere surfers. A narrow zone located at any latitude will have a width of (Rdϕ)and a radius of R Cosϕ. Now the area is (R Cosϕdλ) (Rdϕ) and the total area desired therefore is

$$\boxed{\int_{\phi 1}^{\phi 2}\int_{\lambda 1}^{\lambda 2} R^2 Cos\phi d\phi d\lambda = (\lambda_2 - \lambda_1)\int_{\phi 1}^{\phi 2} R^2 Cos\phi d\phi = (\lambda_2 - \lambda_1)R^2 (Sin\phi_2 - Sin\phi_1)} \quad \text{(3-107)}$$

Remark: the method of calculation of trapezoid on **sphere** in this equation (3-107) cannot be compared with an area of two spherical triangles because this form is surrounded by two longitudes and two latitudes, latitude does not represent a great circle to determine the border limit of spherical

triangles. The trapezoid on a shape surface is special case for area pounded by multi points; i.e the points have difference in latitude and longitude values (random distribution), figure (3-5).

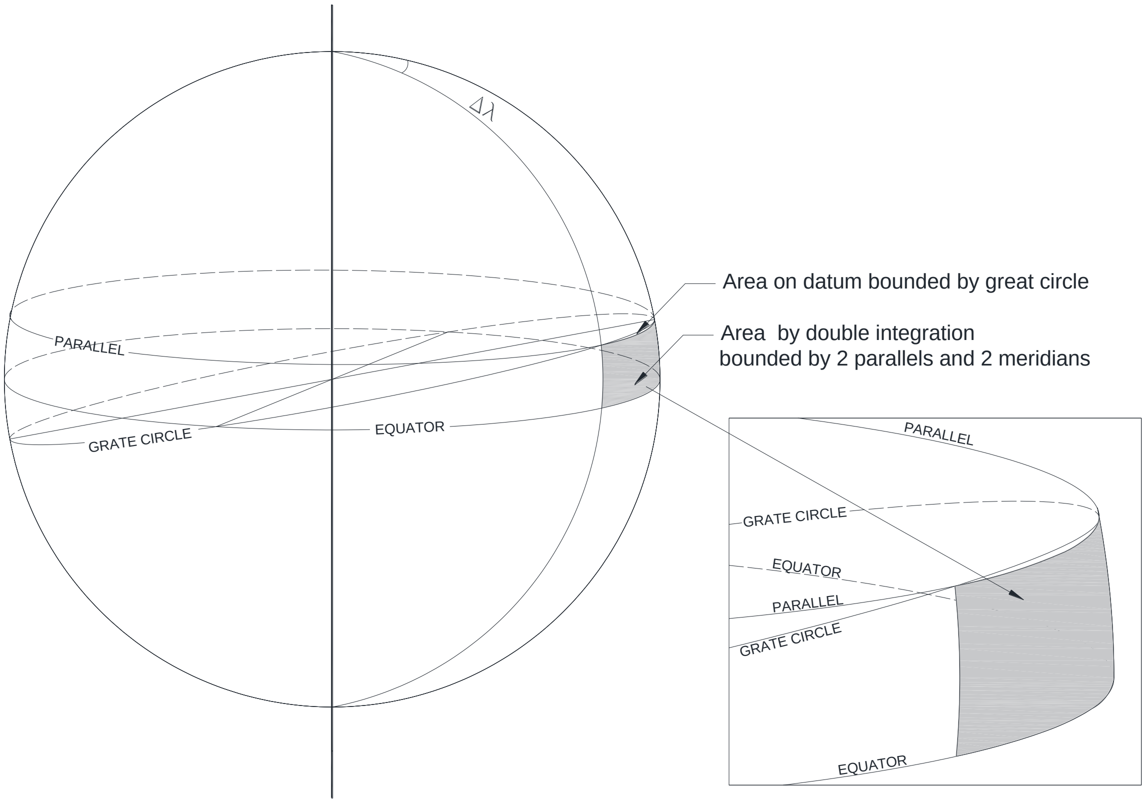


Figure (3-5): Area on datum is bounded by great circle arcs.

b) Calculation of the areas of ellipsoidal geodetic polygons using elementary triangles through spherical excess. (very sensitive method)

This method is based on the division of geodetic polygons into elementary triangles. We assume that vertices of ellipsoidal triangles represent vertices of spherical triangles, i.e. $\phi = \mathrm{B}$ $\lambda = \Lambda$. Radii of spheres are calculated separately for each triangle from formulas

$$R_i = \sqrt{M_i N_i} \tag{3-108}$$

$$R = \frac{1}{3}(R_1 + R_2 + R_3) \tag{3-109}$$

Where M, N – radii of curvature calculated at vertices of triangles. We solve the spherical triangles by using the formulas of spherical trigonometry. Then, having the spherical excess, we calculate the triangle areas. The sum of the areas of the triangles is the approximate area of the geodetic polygon.

$$\varepsilon = \frac{F}{R^2} \tag{3-110}$$

Or $$F = \frac{\varepsilon''.R^2}{\rho} \tag{3-111}$$

Where ϵ is the spherical excess and F is the area of spherical triangle, [Shaker, A. A., 1990a]& [Jackson,J.E. 1980].

From the geodetic coordinates, distances and azimuths (forward and backward) can be computed on ellipsoid datum; the internal angles can be computed by using those azimuths; Then the spherical excess can also be computed by using the parameters of the used geodetic datum in addition to the latitude. For each triangle, the average of Gaussian mean radii will be computed at 3 vertexes of the triangle. Finally, the area will be obtained on the geodetic datum as two spherical triangles using the previse equation.

Remark1: The value of ρ =206264.8062 produced the area of map 1: 100000 = 4,103,066,640.899 m2; and the value of ρ = 206265 produced the area = 4,103,062,785.7892 mt^2.

Remark2: This equation (3-111) is sensitive for the value ϵ, When it is equal to 1", the area be 197,225,082.252 mt^2, to get the area by few meters in this equation, we need to calculate ϵ by 8 number after decimal point 0.000 000 01".
If we apply this value 0.000 000 01 in equation (3-111), the corresponding area of spherical triangle is approximately 2 mt^2. Then we want the calculation to the spherical triangle by appreciation 0.000 000 01" for ϵ or less. **From those remarks, this method is very bad to calculate area of spherical triangle.**

c) Calculation of the areas of ellipsoidal geodetic polygons using elementary triangles through area of corresponding plane triangle (The best method)

The solution of spherical triangles is simplified if one utilizes legendre's theorem which is stated as follows: "if the sides of a plane triangle are equal to the corresponding sides of spherical triangle, then the angles of the plane triangle will be equal to the corresponding angles of the spherical triangle minus one third of the spherical excess", figure (3-6) [Rapp, R. H., 1982] & [Zakatof P. S., 1962]

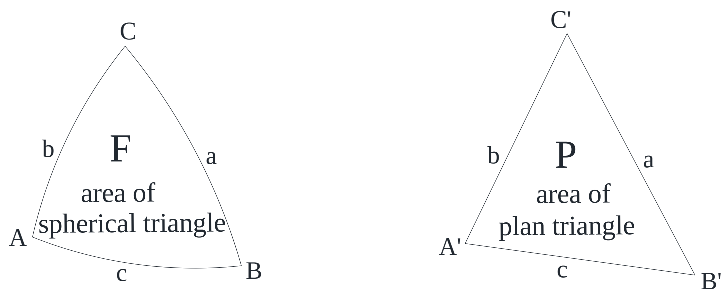


Figure (3-6): Spherical triangle and corresponding plan triangle.

$$A + B + C = 180^{\circ} + \frac{P}{R^2} \tag{3-112}$$

$$\begin{aligned} A - A' &= \frac{\varepsilon}{3} \\ B - B' &= \frac{\varepsilon}{3} \\ C - C' &= \frac{\varepsilon}{3} \end{aligned} \tag{3-113}$$

Equation (3-112) and (3-113) are approximation only. More precise derivations yield the following extended equation, [Rapp, R. H., 1982].

$$\begin{aligned} A - A' &= \frac{P}{3R^2}\left[1 + \frac{a^2 + 7b^2 + 7c^2}{120R^2}\right] + \ldots \\ B - B' &= \frac{P}{3R^2}\left[1 + \frac{7a^2 + b^2 + 7c^2}{120R^2}\right] + \ldots \\ C - C' &= \frac{P}{3R^2}\left[1 + \frac{7a^2 + 7b^2 + c^2}{120R^2}\right] + \ldots \end{aligned} \tag{3-114}$$

If we sum these equations we have:

$$A + B + C = 180^{\circ} + \frac{P}{R^2}\left[1 + \frac{a^2 + b^2 + c^2}{24R^2}\right] \tag{3-115}$$

So that upon comparison with equation (3-114) the spherical excess of the triangle is:

$$\varepsilon = \frac{P}{R^2}\left[1 + \frac{a^2 + b^2 + c^2}{24R^2}\right] \tag{3-116}$$

Hanigan (1970) gives the following extended version of equation (3-116):

$$\varepsilon = \frac{P}{R^2}\left[1 + \frac{m^2}{8R^2} + \frac{8n^4 + 9m^4}{1920R^4}\right] \tag{3-117}$$

Where $m^2 = \dfrac{a^2 + b^2 + c^2}{3}$; $n^4 = \dfrac{a^4 + b^4 + c^4}{3}$; $\dfrac{1}{R^2} = \dfrac{1}{3}\left(\dfrac{1}{R_A^2} + \dfrac{1}{R_B^2} + \dfrac{1}{R_C^2}\right)$

Where e.g. R_A= Gaussian Mean Radius at point A., [Rapp, R. H., 1982].

At this point we note that the area, P of the plane triangle can be rigorously given by equation (3-4):

$$P = \sqrt{S(S-a)(S-d)(S-c)}$$

Where S= (a + b + c) / 2

From equations (3- 111) and (3-117) we have:

$$\frac{F}{R^2} = \frac{P}{R^2}\left[1 + \frac{m^2}{8R^2} + \frac{8n^4 + 9m^4}{1920R^4}\right], \tag{3-118}$$

Or finally in simple form in short lines

$$F = P\left[1 + \frac{m^2}{8R^2}\right] \tag{3-119}$$

And in complete form,

$$F = P\left[1 + \frac{m^2}{8R^2} + \frac{8n^4 + 9m^4}{1920R^4}\right] \tag{3-120}$$

From equation (3-119) and (3-120) we have area of ellipsoidal triangle through area of corresponding plane triangle using geodetic sides distances. This method does not depend on a value of spherical excess; it starts from plane area (plane area computed stable from geodetic distances) and it is corrected by spherical factor in this equation.

From the previous points, the best method for ellipsoidal area is the last method area by using ellipsoidal distances (it is using in our program to get the areas on the datum free from distortion.)We will use the previous equation for calculate the ellipsoidal triangle area; we will divide any polygon to number of triangles, Quadrant shape as two triangles, Pentagon as three triangles and so on.

4. REAL GEODETIC MAP WITHOUT PROJECTION

The earth as a planet has a curved surface. In geodesy, that curved surface is geometrically represented by an ellipsoid or a sphere. This means that the geodetic computations are the default and it should be followed. In the surveying field and when small areas are considered, the plan surveying computations are followed. The area is considered small when the curvature of the earth does not appear, i.e. when the difference between the curved area and its plan surface is not significant. When viewing an image of a small area in Google Earth, it looks like a flat area although the curvature of the earth exists.

In the past not everybody can deal with the geodetic coordinates, so map projection has been introduced to facilitate dealing with metric maps. Nowadays computers and computer programming enabled us to deal easily with geodetic computations and geodetic maps. In this chapter, the proposed computerized real geodetic map is introduced. The computations which have been done to clear the idea of the proposed map and their results are tabulated and illustrated.

4.1. ELLIPSOIDAL VERSUS PLAN DISTANCES

The chord and curved distances between the same two points are computed with varying the distances from 1000m till 6000, 000m. The difference between chord and its arc distance for the same two points on the earth is very small in short lines. Table (4-1) shows the relation between chords and its arc distances as parts from great circles computed using equation (3-8) (minimum distance between two points on sphere) of the earth as a sphere with R=6,371,000m.

From the values in the table;

- Difference between arc and chord distances reached 1mm at distance 10 km.
- Difference between arc and chord distances reached 10cm after distance 45 km.
- Difference between arc and chord distances reached 1m at distance 100 km.

When using the smallest scale map 1:100000 which covers 60 km * 40 km while the table shows differences of 22cm and 6.5cm at distances 60km and 40km respectively. Difference between Distances of 60000.22m and 60000m both drawn at scale 1:100000 will not be noticeable to the user eye. Therefore, using the geodetic coordinates directly in mapping will not show difference with mapping the same area using plan coordinates. i.e. differences between curves and their corresponding straights will not appear on the map.

Table (4-1). Relation between Chord and Arc distances with Earth Radius =6,371,000 m.

Chord Dis. (m).	Arc Length (m)	Scale Factor	Central Angle (D MM SS)
1000	1000.000001	1.000000001	00° 00’ 32.38”
2000	2000.000009	1.000000004	00° 01’ 04.75”
3000	3000.000028	1.000000009	00° 01’ 37.13”
4000	4000.000065	1.000000016	00° 02’ 09. 50”
5000	5000.000128	1.000000026	00° 02’ 41.88”
6000	6000.000222	1.000000037	00° 03’ 14.25”
7000	7000.000352	1.00000005	00° 03’ 46.63”
8000	8000.000526	1.000000066	00° 04’ 19.00”
9000	9000.000748	1.000000083	00° 04’ 51.38”
10000	10000.00103	1.000000103	00° 05’ 23.76”
15000	15000.00346	1.000000231	00° 08’ 05.63”
20000	20000.00821	1.000000411	00° 10’ 47.51”
25000	25000.01604	1.000000642	00° 13’ 29.39”
30000	30000.02772	1.000000924	00° 16’ 11.27”
35000	35000.04401	1.000001258	00° 18’ 53.15”
40000	40000.06570	1.000001642	00° 21’ 35.03”
45000	45000.09354	1.000002079	00° 24’ 16.09”
50000	50000.12832	1.000002566	00° 26’ 58.78”
60000	60000.22173	1.000003696	00° 32’ 22.54”
100000	100001.0266	1.000010266	00° 53’ 57.59”
200000	200008.2132	1.000041066	01° 47’ 55.38”
300000	300027.7233	1.000092411	02° 41’ 53.57”
400000	400065.7274	1.000164318	03° 35’ 52.36”
500000	500128.4058	1.000256812	04° 29’ 51.95”
600000	600221.9530	1.000369922	05° 23’ 52.53”
1000000	1001029.390	1.00102939	09° 00’ 08.90”
5000000	5138119.346	1.027623869	46° 12’ 29.58”
6000000	6247303.299	1.041217216	56° 11’ 00.05”
9009954.61	10007543.400	1.110720735	90° 00’ 00.00”

4.2. GEODETIC AND PROJECTED MAPS, THE EGYPTIAN CASE WITH DIFFERENT SURVEYING SCALES

The geodetic coordinates in Egypt is related to the Egyptian Datum (EGD30) and the used Ellipsoid is Helmert 1906. The Egyptian map projection system (ETM) uses Transverse Mercator projection. In this part of the research, the differences in the distances and azimuths in the map and at the surface of the reference ellipsoid are computed at various scales 1:1000, 1:2500, 1: 10000,

1: 25000, 1: 50000, and 1:100000.

The computations are done in Egypt's Red Belt once at the central meridian of the zone (λ=31^{o} E) where minimum distortion exists (Group 1), figure (4-1), and once more at the edge of the zone (λ=33^{o} E) where maximum distortion is there (Group 2), figure (4-2)

- Figure (4-3) and figure (4-4) show, in scale 1:1000, Group (1) at central meridian and Group (2) at zone edge, dimensions of 600 m x 400 m which is represented in map sheet of 60 cm x 40 cm.
- Figure (4-5) and figure (4-6) show, in scale 1:2500, Group (1) and Group (2), dimensions of 1500 m x 1000 m which is represented in map sheet of 60 cm x 40 cm.
- Figure (4-7) and figure (4-8) show, in scale 1:10000, Group (1) and Group (2), dimensions of 6000 m x 4000 m which is represented in map sheet of 60 cm x 40 cm.
- Figure (4-9) and figure (4-10) show, in scale 1:25000, Group (1) and Group (2) dimensions of 15000 m x 10000 m which is represented in map sheet of 60 cm x 40 cm.
- Figure (4-11) and figure (4-12) show, in scale 1:50000, Group (1) and Group (2) dimensions of 30000 m x 20000 m which is represented in map sheet of 60 cm x 40 cm.
- Figure (4-13) and figure (4-14) show, in scale 1: 100000, Group (1) and Group (2) dimensions of 60000 m x 40000 m which is represented in map sheet of 60 cm x 40 cm.

Table (4-2) shows the geodetic coordinates (EGD30) of the corner points of maps of different scales and the corresponding projected values in the Egyptian Transverse Mercator (ETM); the study is done for deferent scale maps. The geodetic azimuths and distances of sides of the maps was computed by inverse geodetic problem, see sec 3.2.3, the map azimuths and distances need corrections by scale factor to get the geodetic values, see sec 2.4.2.2b.

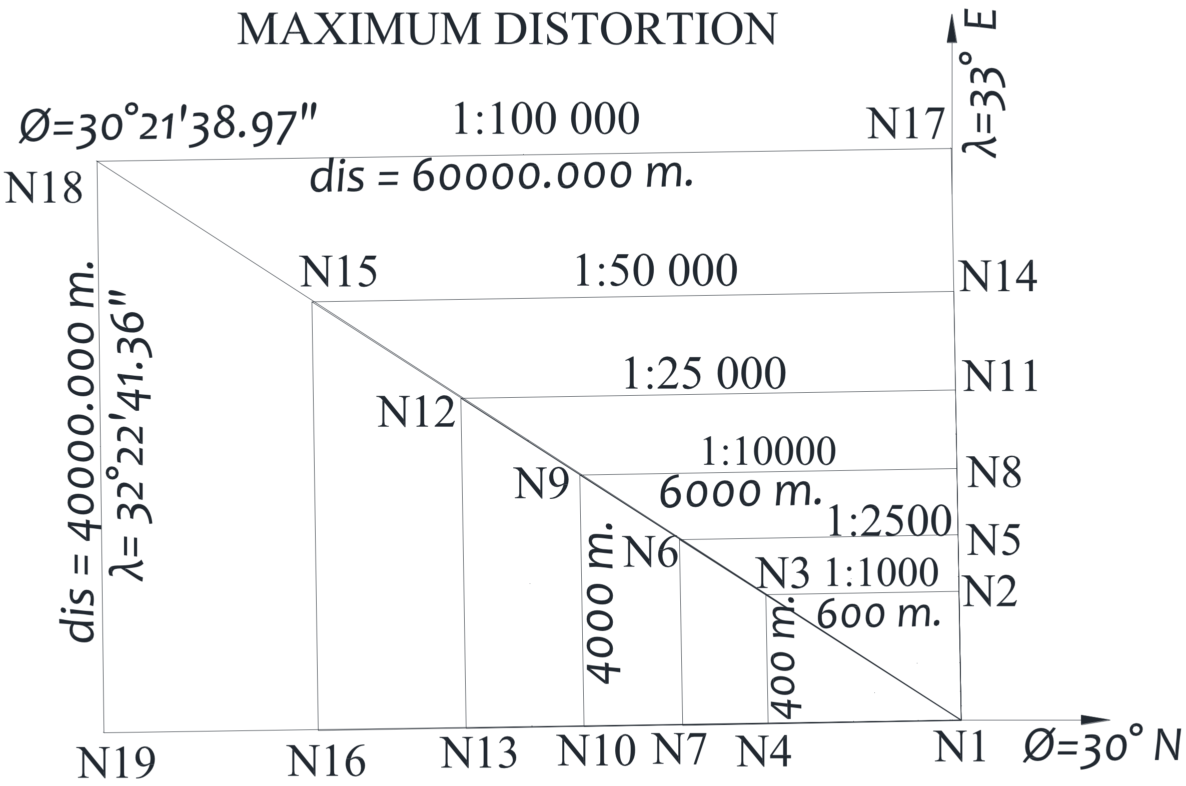


Figure (4-1): Group (1) of maps at central meridian of Egypt's Red Belt.

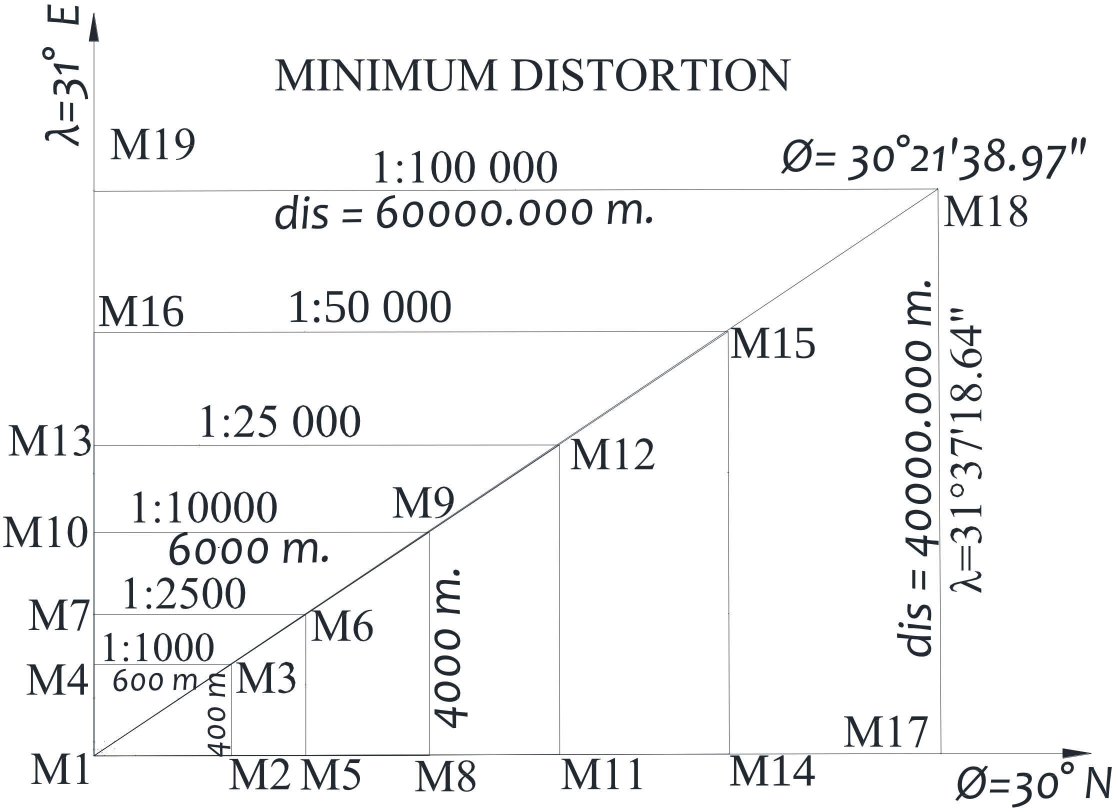


Figure (4-2): Group (2) of maps at zone edge of Egypt's Red Belt.

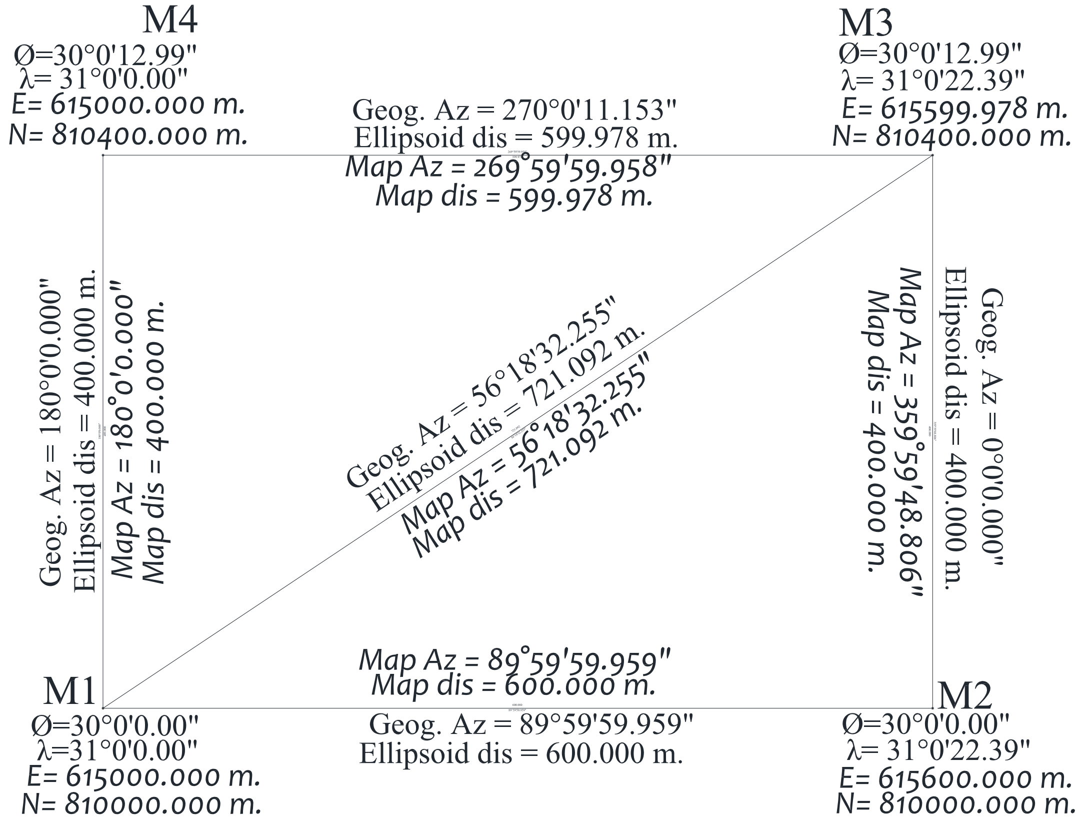


Figure (4-3): Map scale 1: 1000 in Group (1) in Egyptian case.

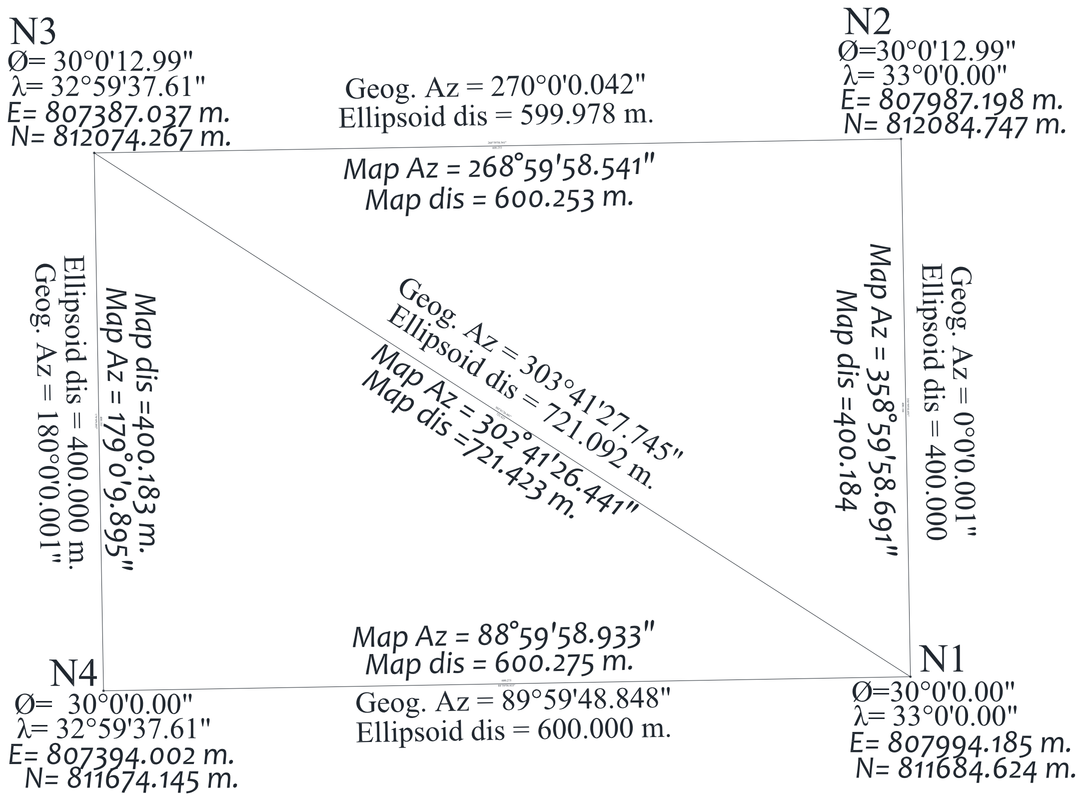


Figure (4-4): Map scale 1: 1000 in Group (2) in Egyptian case.

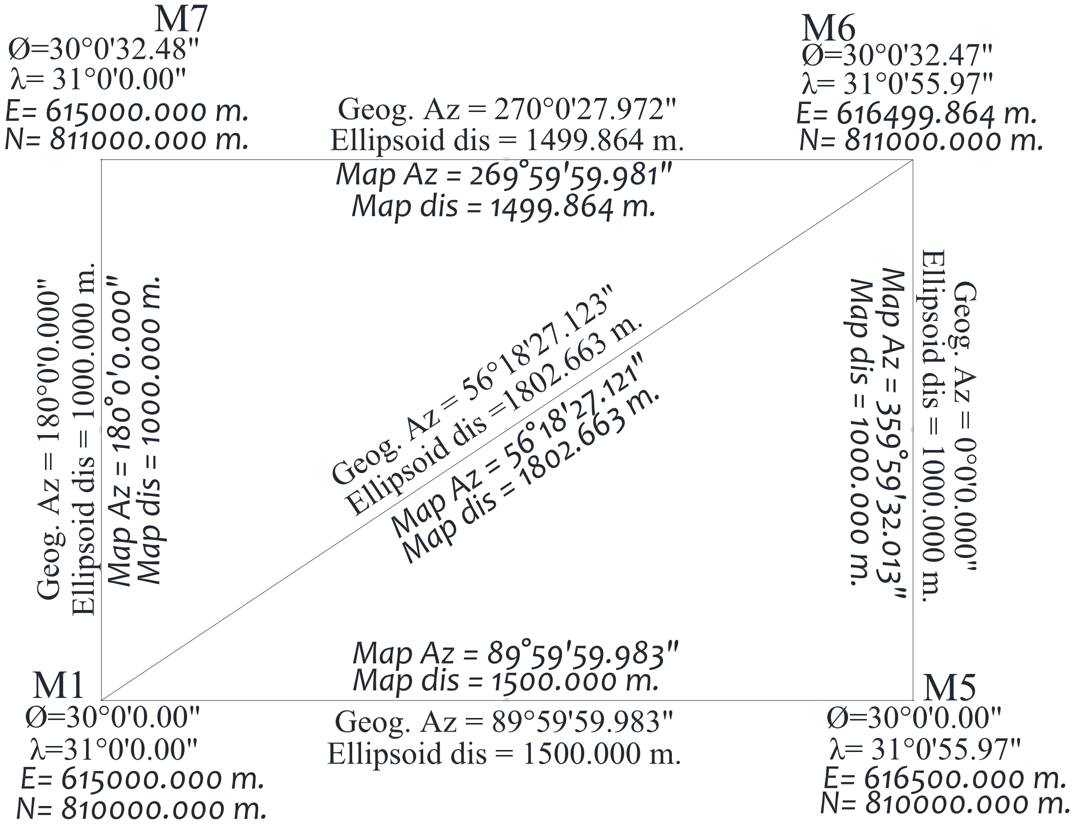


Figure (4-5): Map scale 1: 2500 in Group (1) in Egyptian case.

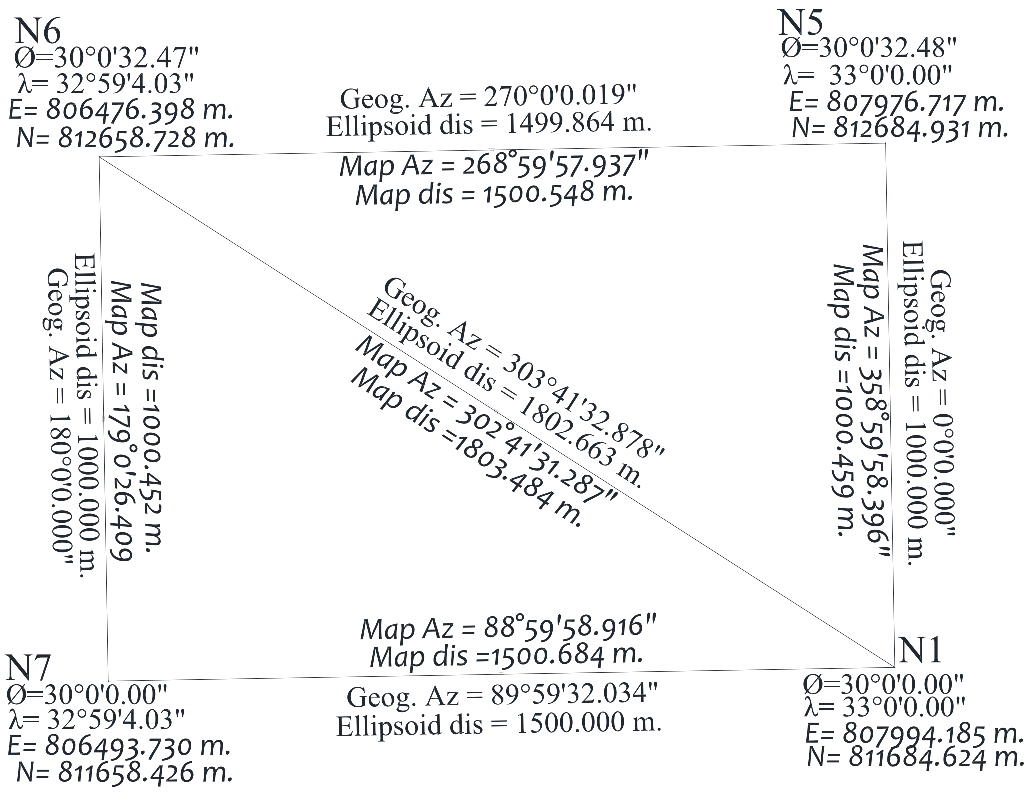


Figure (4-6): map scale 1: 2500 in Group (2) in Egyptian case.

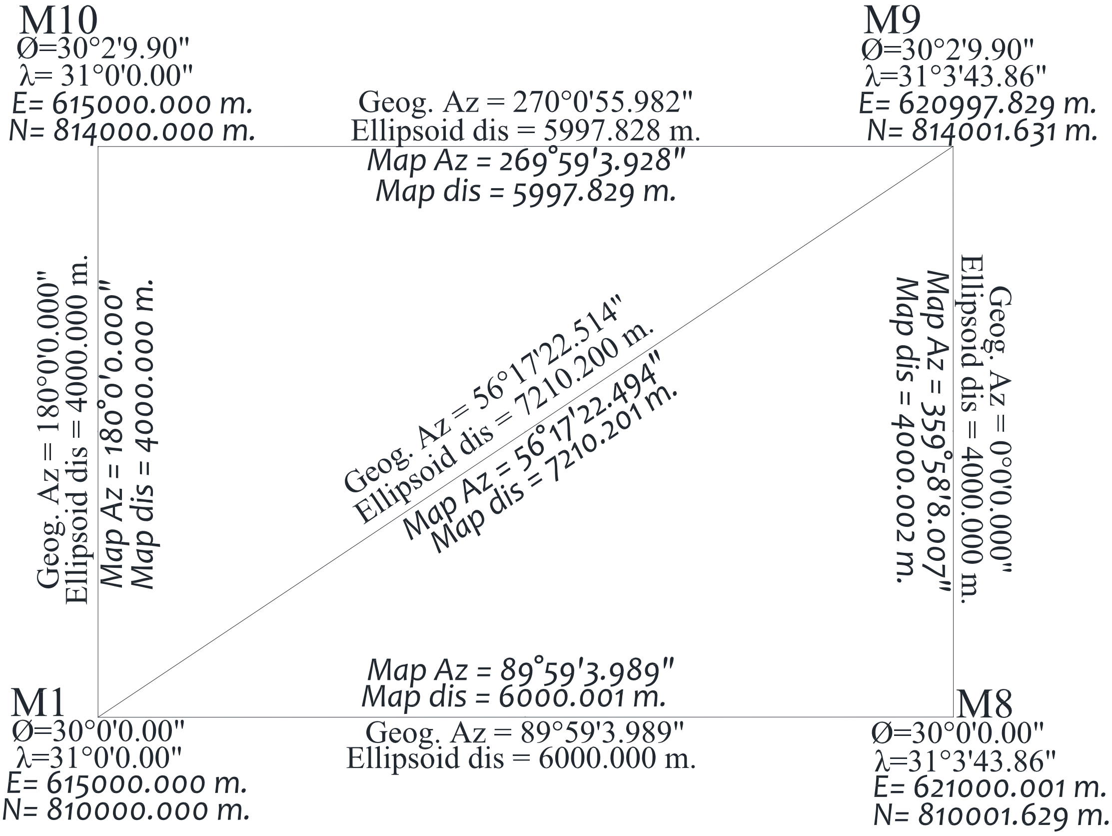


Figure (4-7): Map scale 1: 10000 in Group (1) in Egyptian case.

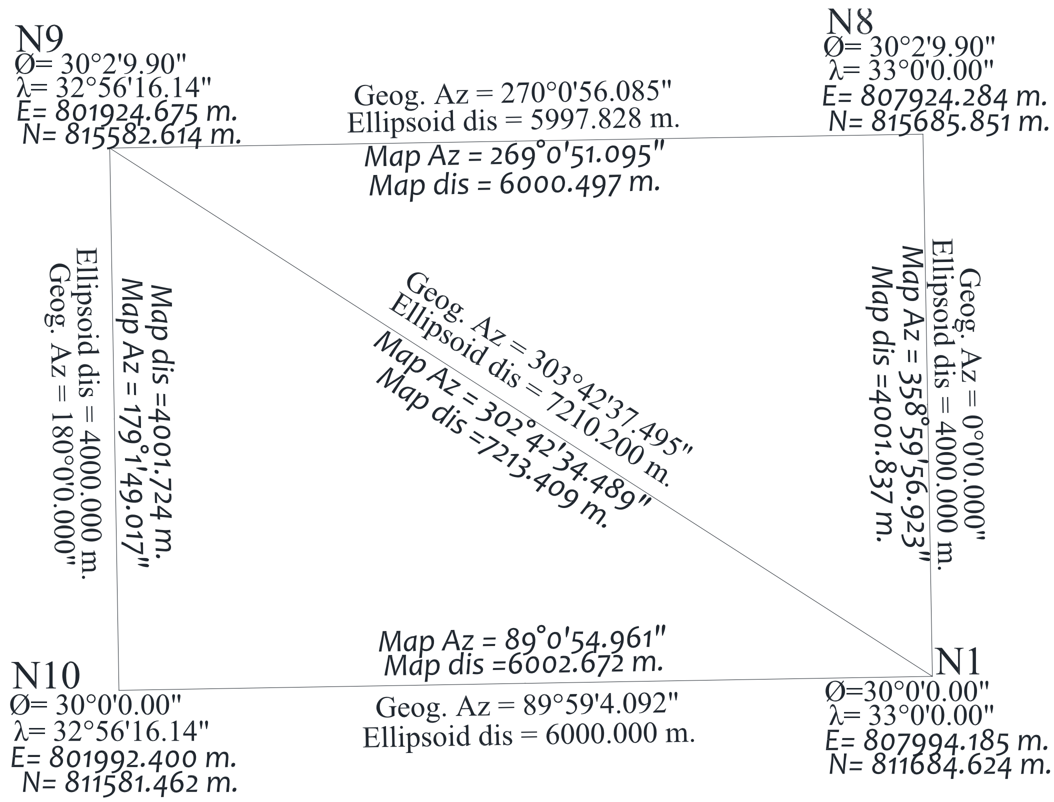


Figure (4-8): Map scale 1: 10000 in Group (2) in Egyptian case.

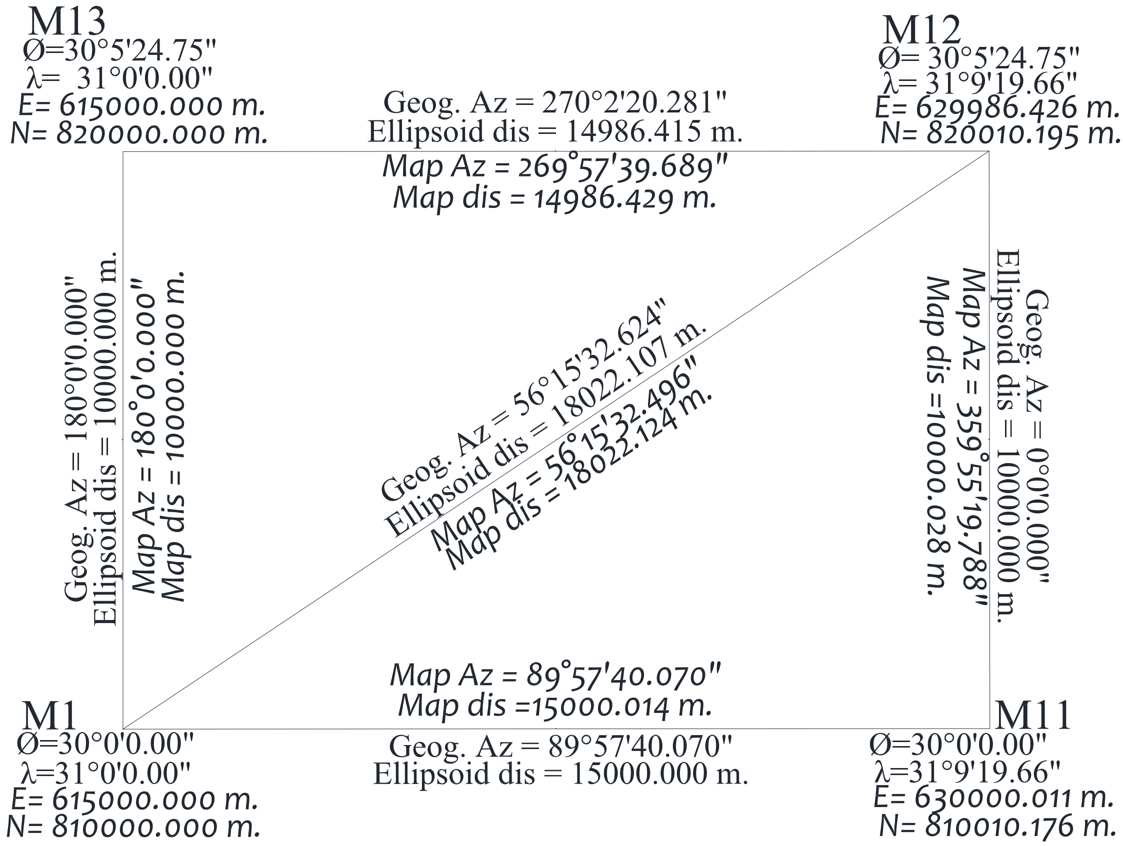


Figure (4-9): Map scale 1: 25000 in Group (1) in Egyptian case.

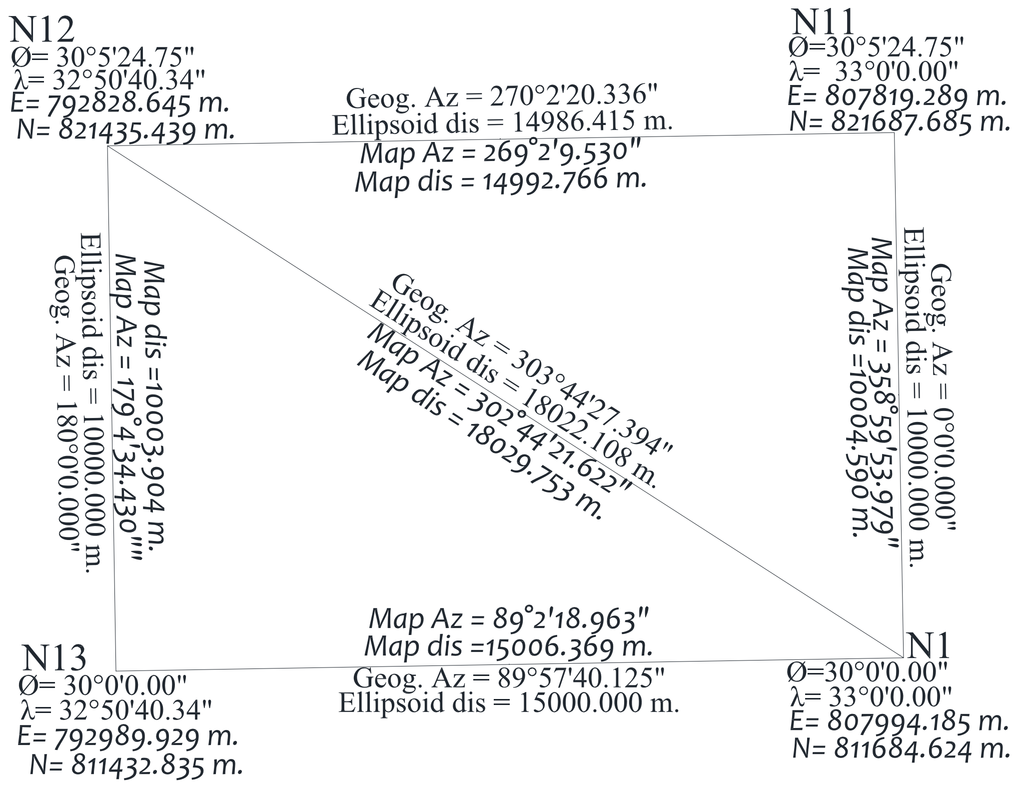


Figure (4-10): Map scale 1: 25000 in Group (2) in Egyptian case.

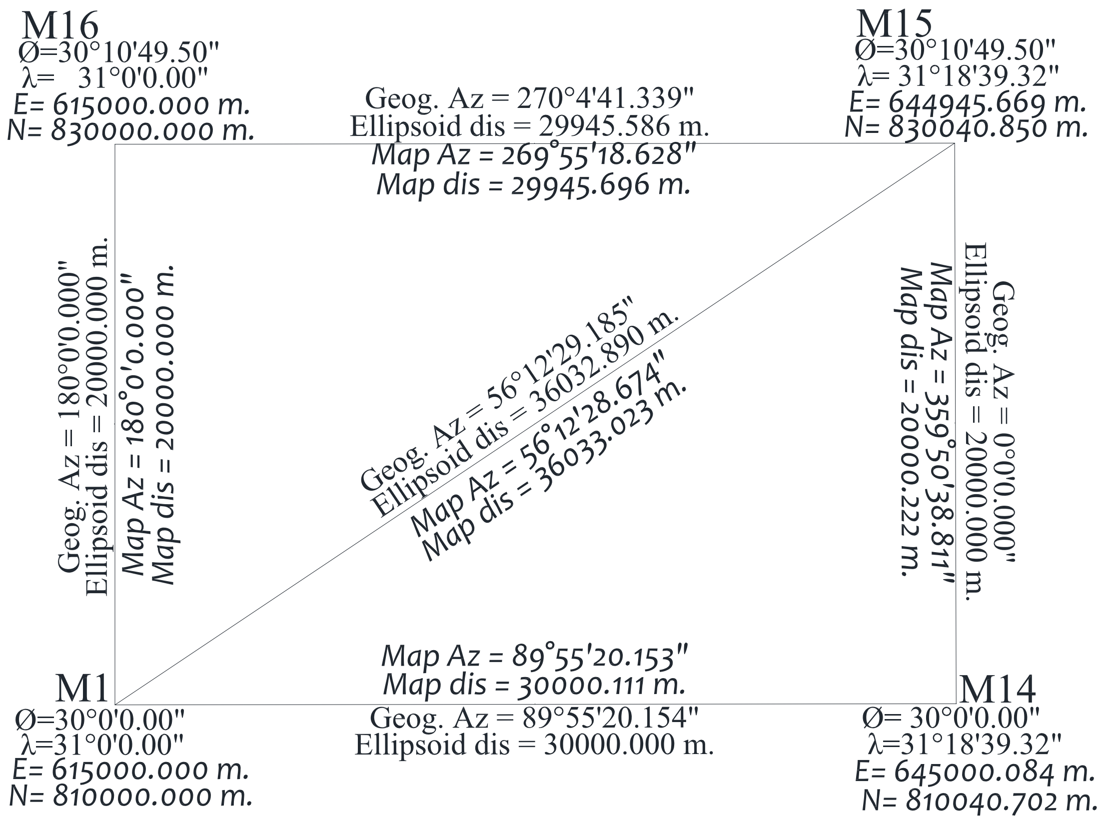


Figure (4-11): Map scale 1: 50000 in Group (1) in Egyptian case.

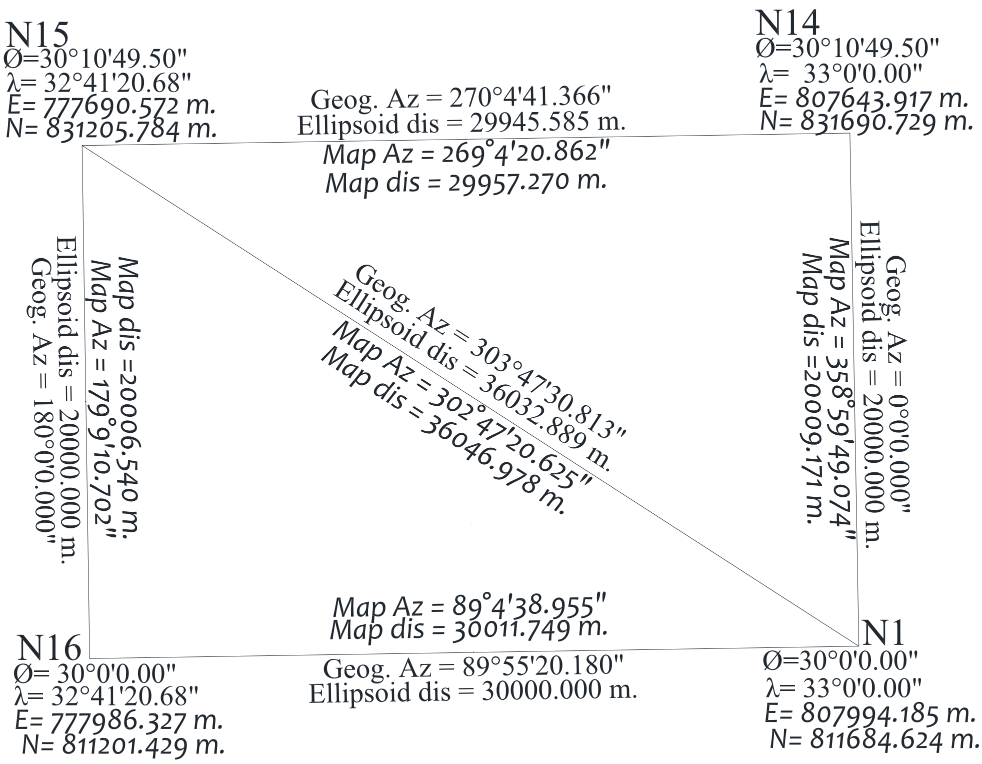


Figure (4-12): Map scale 1: 50000 in Group (2) in Egyptian case.

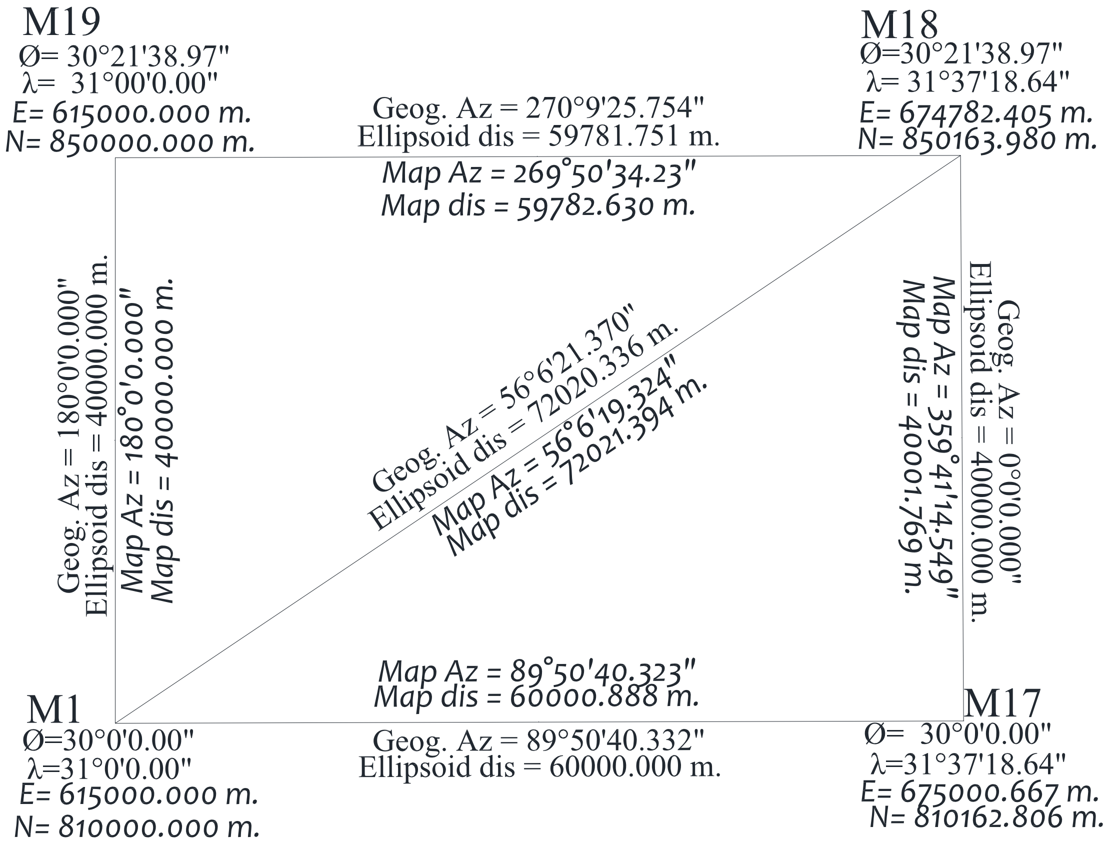


Figure (4-13): Map scale 1: 100000 in Group (1) in Egyptian case.

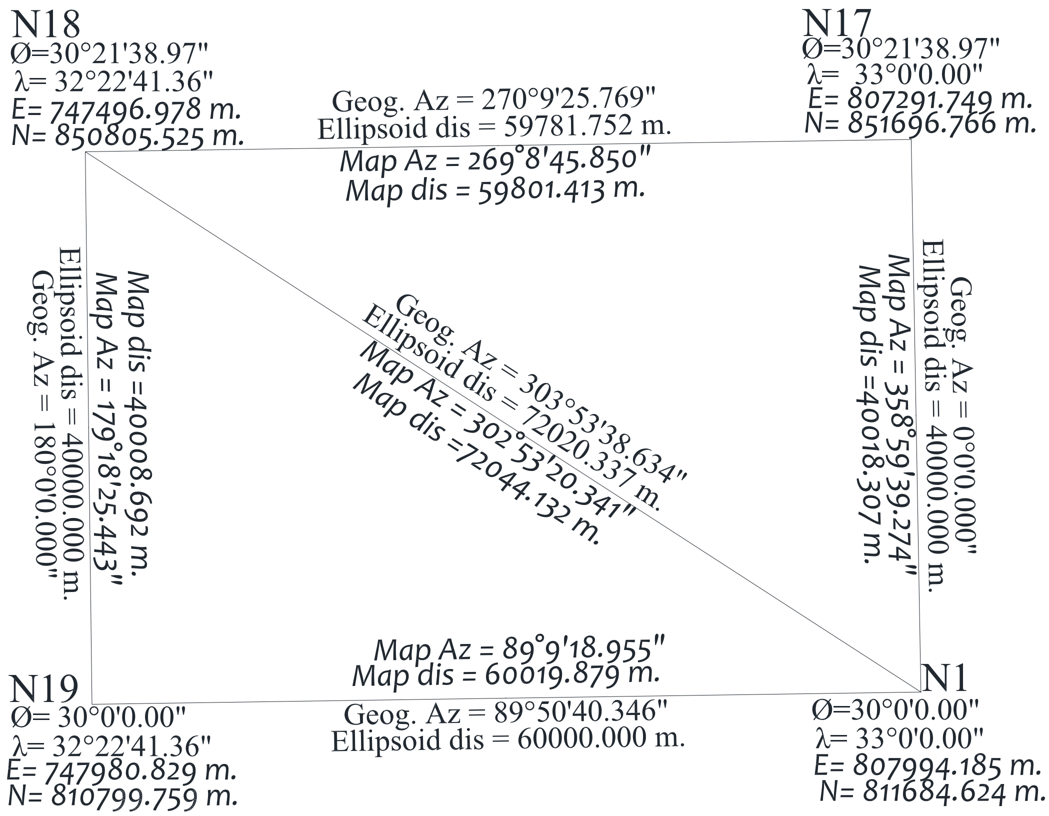


Figure (4-14): Map scale 1: 100000 in Group (2) in Egyptian case.

Table (4-2). Geodetic and projected coordinates of maps corner points, Group1 at central meridian and Group2 at zone edge (Egyptian Case).

	Point	Latitude (dd.mmss)	Longitude (dd.mmss)		East(m)	North(m)
Group1 at the central meridian of the Red Belt zone (λ=31° E)	M1	30° 00’00”	31° 00’00”		615000.000	810000.000
	M2	30° 00’00”	31° 00’22.39”		615600.000	810000.000
	M3	30° 00’12.99”	31° 00’22.39”		615599.978	810400.000
	M4	30° 00’12.99”	31° 00’00”		615000.000	810400.000
	M5	30° 00’00”	31° 00’55.97”		616500.000	810000.000
	M6	30° 00’32.47”	31° 00’55.97”		616499.864	811000.000
	M7	30° 00’32.48”	31° 00’00”		615000.000	811000.000
	M8	30° 00’00”	31° 03’43.86”		621000.001	810001.629
	M9	30° 02’09.9”	31° 03’43.86”		620997.829	814001.631
	M10	30° 02’09.9”	31° 00’00”		615000.000	814000.000
	M11	30° 00’00”	31° 09’19.66”		630000.011	810010.176
	M12	30° 05’24.75”	31° 09’19.66”		629986.426	820010.195
	M13	30° 05’24.75”	31° 00’00”		615000.000	820000.000
	M14	30° 00’00”	31° 18’3932”		645000.084	810040.702
	M15	30° 10’49.5”	31° 18’39.32”		644945.669	830040.850
	M16	30° 10’49.5”	31° 00’00”		615000.000	830000.000
	M17	30° 00’00”	31° 37’18.64”		675000.667	810162.806
	M18	30° 21’38.97”	31° 37’18.64”		674782.405	850163.980
	M19	30° 21’38.97”	31° 00’00”		615000.000	850000.000
Group2 at the edge of the Red Belt zone (λ=33° E)	N1	30° 00’00”	33° 00’00”		807994.185	811684.624
	N2	30° 00’12.99”	33° 00’00”		807987.198	812084.747
	N3	30° 00’12.99”	32° 59’37.61”		807387.037	812074.267
	N4	30° 00’00”	32° 59’37.61”		807394.002	811674.145
	N5	30° 00’32.48”	33° 00’00”		807976.717	812684.931
	N6	30° 00’32.47”	32° 59’04.03”		806476.398	812658.728
	N7	30° 00’00”	32° 59’04.03”		806493.730	811658.426
	N8	30° 02’09.9”	33° 00’00”		807924.284	815685.851
	N9	30° 02’09.9”	32° 56’16.14”		801924.675	815582.614
	N10	30° 00’00”	32° 56’16.14”		801992.400	811581.462
	N11	30° 05’24.75”	33° 00’00”		807819.289	821687.685
	N12	30° 05’24.75”	32° 50’40.34”		792828.645	821435.439
	N13	30° 00’00”	32° 50’40.34”		792989.929	811432.835
	N14	30° 10’49.5”	33° 00’00”		807643.917	831690.729
	N15	30° 10’49.5”	32° 41’20.68”		777690.572	831205.784
	N16	30° 00’00”	W0432° 41’20.68”		777986.327	811201.429
	N17	30° 21’38.97”	33° 00’00”		807291.749	851696.766
	N18	30° 21’38.97”	32° 22’41.36”		747496.978	850805.525
	N19	30° 00’00”	32° 22’41.36”		747980.829	810799.759

The data and results in pervious figures - from figure (4 -3) to figure (4 -14) - are collected in table (4-3) and table (4-4). Concerning the deference between ellipsoidal and map distances. The differences seem significant as absolute values but they are not noticeable as drawn in the map. It means one cannot notice a difference between geodetic and plan metric maps for the same area.

In table (4-4) of map scale 1:2500, distortion value of 46 cm in 1000 m is obtained. This is a big value especially when precise EDM is used in measuring distances in the field. The user does not know about distortion and the surveyor himself should bay attention while dealing with projected map and Total Station in the field. This problem can vanish by using geodetic Total Station in the field and geodetic map. In the 1:2500 map itself, 46 cm difference in 1000m will appear as (46cm/2500) which is not noticeable. More about geodetic total station, one can refer to [Saad, A.A., 2002].

Distortion is variable in map from point to another; to resolve this issue practically we take an average value of distortion in limited region. The problem is more complex in case of international and intercontinental projects such international roads and petroleum pipelines. Again, the problem can vanish by using the proposed geodetic mapping system especially in the presence of WGS84 as global geodetic coordinate system and GNSS as global observation tools.

- **Distortion in distances**

Concerning maps adjacent to the central meridian of the used projection zone

- There are no differences between ellipsoidal and map distances in maps of scales 1:1000, 1:2500, and 1:10000.
- The maximum difference between ellipsoidal and map distances is under 3cm in 1:25000 map.
- The maximum difference between ellipsoidal and map distances is 22cm in 1:50000 map. This difference will be drawn as (22cm/50000) in the map which is not noticeable.
- The maximum difference between ellipsoidal and map distances is1.77m in 1:100000 map. This difference will be drawn as (177cm/100000) in the map which is not noticeable.

Concerning maps adjacent to the edge of the zone:

- The maximum difference between ellipsoidal and map distances is 33cm in 1:1000 map.
- The maximum difference between ellipsoidal and map distances is 82cm in 1:2500 map.
- The maximum difference between ellipsoidal and map distances is 3.21m in 1:10000 map.
- The maximum difference between ellipsoidal and map distances is 7.65m in 1:25000 map.
- The maximum difference between ellipsoidal and map distances is 14.09m in 1:50000 map.
- The maximum difference between ellipsoidal and map distances is 23.80m in 1:100000map.

In all scales the differences, drawn in the map, are not noticeable. This means again that the proposed geodetic map will not differ from its corresponding projected one.

- **Distortion in Azimuths**

Concerning maps adjacent to the central meridian of the used projection zone:

- The maximum difference between geodetic azimuth and map bearing is 11.2” in 1:1000 map which is not noticeable in dimension on map, the maximum difference in mm can be computed by side map*sin (azimuth difference) = 60cm*sin (11.2”) = 0.03 mm
- The maximum difference between geodetic azimuth and map bearing is 28” in 1:25000 map at side 60cm, the equivalent distance value equals 0.08 mm which is not noticeable, also the equivalent value is 0.3 mm in 1:10000 map, 0.8 mm in 25000 map, 1.6 mm in 50000 map and 3.25 mm in100000 map,

Concerning maps adjacent to the edge of the zone:

- The maximum difference between geodetic azimuth and map bearing for all map scales is 1° 00’18.3” at the edge zone at maximum distortion, the equivalent value is nearly 12 mm. in the automatic real map the geodetic azimuths will be used.

Table (4-3). Geodetic (EGM30) and projected (ETM) data in different scales at central meridian of the zone, Egyptian Case.

Map	From	To	Geodetic Azimuth.	Geodetic Dis. (m)		Map Bearing	Map dis. (m)		Diff. bet. Dis. (m)
1 : 1000	M1	M2	89°59'59.959"	600.000		89°59'59.959"	600.000		0.000
	M2	M3	0°0'0.000"	400.000		359°59'48.806"	400.000		0.000
	M3	M4	270°0'11.153"	599.978		269°59'59.958"	599.978		0.000
	M4	M1	180°0'0.000"	400.000		180°0'0.000"	400.000		0.000
	M1	M3	56°18'32.255"	721.092		56°18'32.255"	721.092		0.000
1 : 2500	M1	M5	89°59'59.983"	1500.000		89°59'59.983"	1500.000		0.000
	M5	M6	0°0'0.000"	1000.000		359°59'32.013"	1000.000		0.000
	M6	M7	270°0'27.972"	1499.864		269°59'59.981"	1499.864		0.000
	M7	M1	180°0'0.000"	1000.000		180°0'0.000"	1000.000		0.000
	M1	M6	56°18'27.123"	1802.663		56°18'27.121"	1802.663		0.000
1:10000	M1	M8	89°59'3.989"	6000.000		89°59'3.989"	6000.001		-0.001
	M8	M9	0°0'0.000"	4000.000		359°58'8.007"	4000.002		-0.002
	M9	M10	270°0'55.982"	5997.828		269°59'3.928"	5997.829		-0.001
	M10	M1	180°0'0.000"	4000.000		180°0'0.000"	4000.000		0.000
	M1	M9	56°17'22.514"	7210.200		56°17'22.494"	7210.201		-0.001
1:25000	M1	M11	89°57'40.070"	15000.000		89°57'40.070"	15000.014		-0.014
	M11	M12	0°0'0.000"	10000.000		359°55'19.788"	10000.028		-0.028
	M12	M13	270°2'20.281"	14986.415		269°57'39.689"	14986.429		-0.014
	M13	M1	180°0'0.000"	10000.000		180°0'0.000"	10000.000		0.000
	M1	M12	56°15'32.624"	18022.107		56°15'32.496"	18022.124		-0.017
1:50000	M1	M14	89°55'20.154"	30000.000		89°55'20.153"	30000.111		-0.111
	M14	M15	0°0'0.000"	20000.000		359°50'38.811"	20000.222		-0.222
	M15	M16	270°4'41.339"	29945.586		269°55'18.628"	29945.696		-0.110
	M16	M1	180°0'0.000"	20000.000		180°0'0.000"	20000.000		0.000
	M1	M15	56°12'29.185"	36032.890		56°12'28.674"	36033.023		-0.133
1:100000	M1	M17	89°50'40.332"	60000.000		89°50'40.323"	60000.888		-0.888
	M17	M18	0°0'0.000"	40000.000		359°41'14.549"	40001.769		-1.769
	M18	M19	270°9'25.754"	59781.751		269°50'34.230"	59782.630		-0.879
	M19	M1	180°0'0.000"	40000.000		180°0'0.000"	40000.000		0.000
	M1	M18	56°6'21.370"	72020.336		56°6'19.324"	72021.394		-1.058

Table (4-4). Geodetic (EGD30) and projected (ETM) data in different scales at the edge of the zone, Egyptian Case.

Map	From Pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis. (m)		Diff. bet. Dis. (m)
	N1	N2	0°0'0.001"	400.000		358°59'58.691"	400.184		-0.184
	N2	N3	270°0'0.042"	599.978		268°59'58.541"	600.253		-0.275
	N3	N4	180°0'0.001"	400.000		179°0'9.895"	400.183		-0.183
1:1000	N4	N1	89°59'48.848"	600.000		88°59'58.933"	600.275		-0.275
	N1	N3	303°41'27.745"	721.092		302°41'26.441"	721.423		-0.331
	N1	N5	0°0'0.000"	1000.000		358°59'58.396"	1000.459		-0.459
	N5	N6	270°0'0.019"	1499.864		268°59'57.937"	1500.548		-0.684
	N6	N7	180°0'0.000"	1000.000		179°0'26.409"	1000.452		-0.452
1:2500	N7	N1	89°59'32.034"	1500.000		88°59'58.916"	1500.684		-0.684
	N1	N6	303°41'32.878"	1802.663		302°41'31.287"	1803.484		-0.821
	N1	N8	0°0'0.000"	4000.000		358°59'56.923"	4001.837		-1.837
	N8	N9	270°0'56.085"	5997.828		269°0'51.095"	6000.497		-2.669
	N9	N10	180°0'0.000"	4000.000		179°1'49.017"	4001.724		-1.724
1:10000	N10	N1	89°59'4.092"	6000.000		89°0'54.961"	6002.672		-2.672
	N1	N9	303°42'37.495"	7210.200		302°42'34.489"	7213.409		-3.209
	N1	N11	0°0'0.000"	10000.000		358°59'53.979"	10004.590		-4.590
	N11	N12	270°2'20.336"	14986.415		269°2'9.530"	14992.766		-6.351
	N12	N13	180°0'0.000"	10000.000		179°4'34.430"	10003.904		-3.904
1:25000	N13	N1	89°57'40.125"	15000.000		89°2'18.963"	15006.369		-6.369
	N1	N12	303°44'27.394"	18022.108		302°44'21.622"	18029.753		-7.645
	N1	N14	0°0'0.000"	20000.000		358°59'49.074"	20009.171		-9.171
	N14	N15	270°4'41.366"	29945.585		269°4'20.862"	29957.270		-11.685
	N15	N16	180°0'0.000"	20000.000		179°9'10.702"	20006.540		-6.540
1:50000	N16	N1	89°55'20.180"	30000.000		89°4'38.955"	30011.749		-11.749
	N1	N15	303°47'30.813"	36032.889		302°47'20.625"	36046.978		-14.089
	N1	N17	0°0'0.000"	40000.000		358°59'39.274"	40018.307		-18.307
	N17	N18	270°9'25.769"	59781.751		269°8'45.850"	59801.413		-19.662
	N18	N19	180°0'0.000"	40000.000		179°18'25.443"	40008.692		-8.692
1:100000	N19	N1	89°50'40.346"	60000.000		89°9'18.955"	60019.879		-19.879
	N1	N18	303°53'38.634"	72020.337		302°53'20.341"	72044.132		-23.795

4.3. GEODETIC AND PROJECTED MAPS, THE GLOBAL CASE WITH DIFFERENT SURVEYING SCALES

The computations on another datum WGS84 (World Geodetic System 1984) and UTM (Universal Transverse Mercator) are done. In zone number 31of UTM, two main groups of maps are chosen for the study, one group at the middle of the zone and the other at the zone border. The differences in the distances and azimuths at the surface of the ellipsoid and the map are studied on various scales 1:1000, 1:2500, 1: 10000, 1: 25000, 1: 50000, and 1:100000.

The computations are done at equator (G1 & G2), latitude 30ºN (G3 & G4), latitude 60º N (G5 & G6), latitude 70º N(G7&G8), and latitude 80º N) G9 & G10). In UTM, zone width is equal to 6 degrees; the distribution of Groups is in figure (4-15). The maps study in G1 and G2 at equator describe in figure (4-16) and figure (4-17).

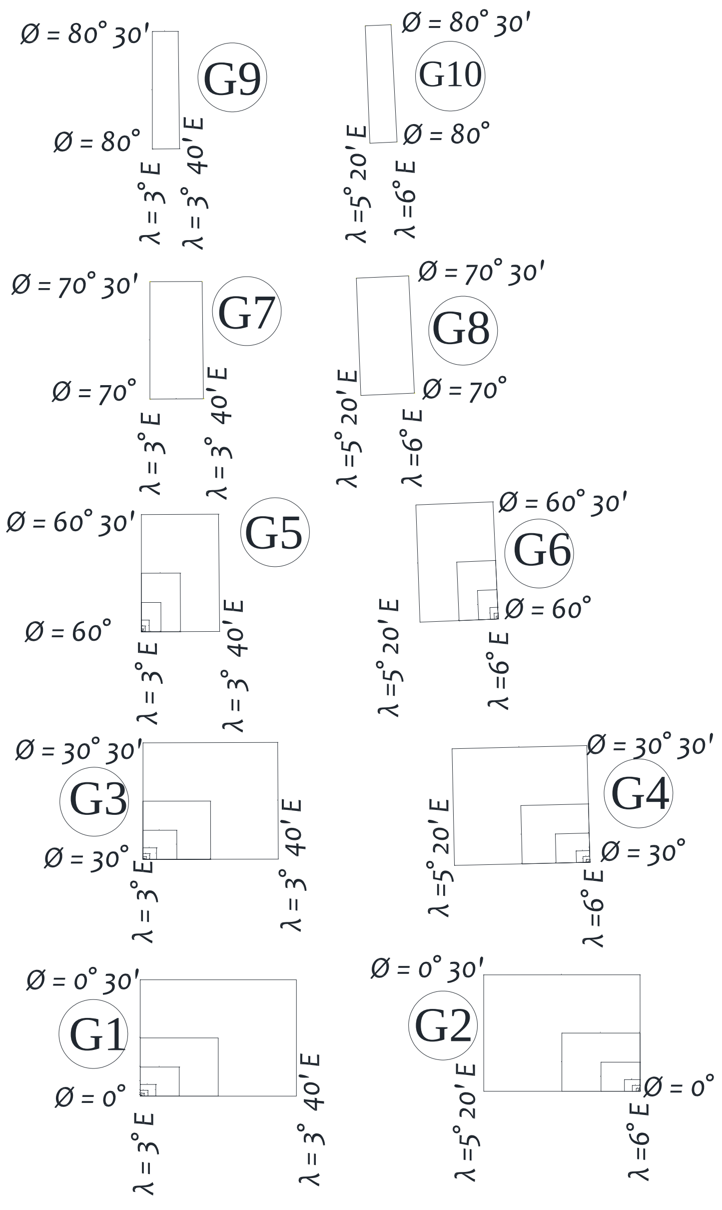


Figure (4-15): The distribution of Groups, in the global case.

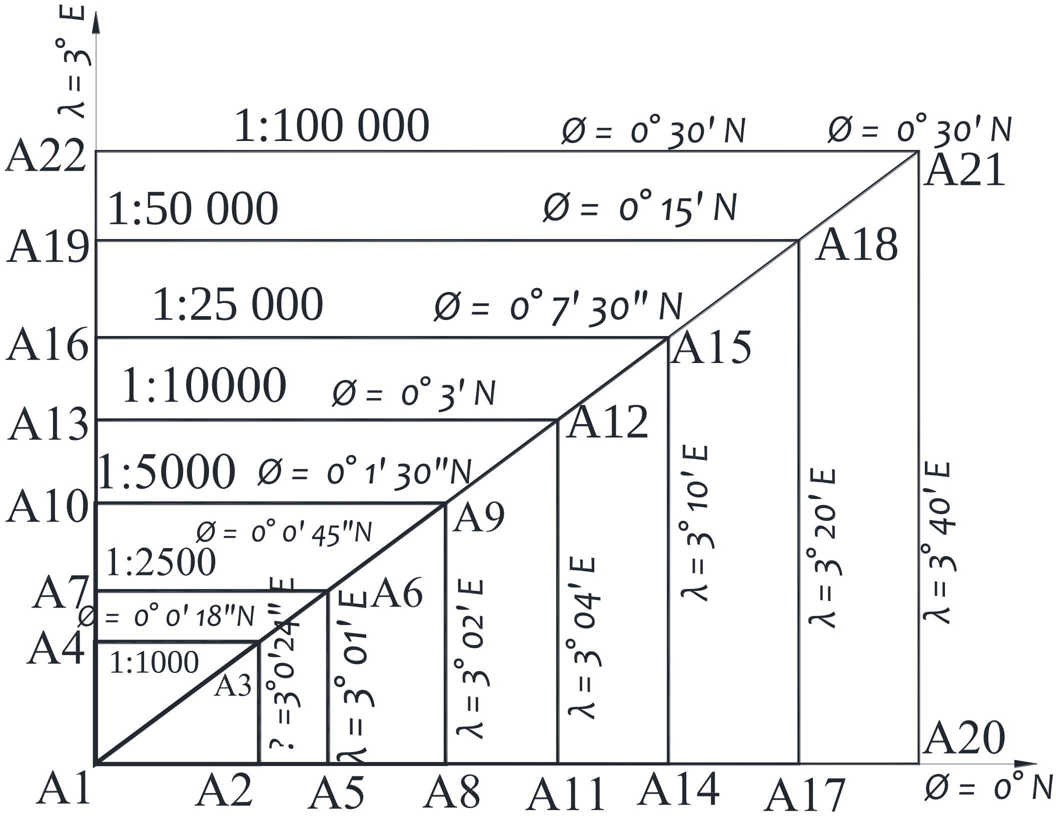


Figure (4-16): Group (1) of maps at central meridian of zone 31 in UTM at equator.

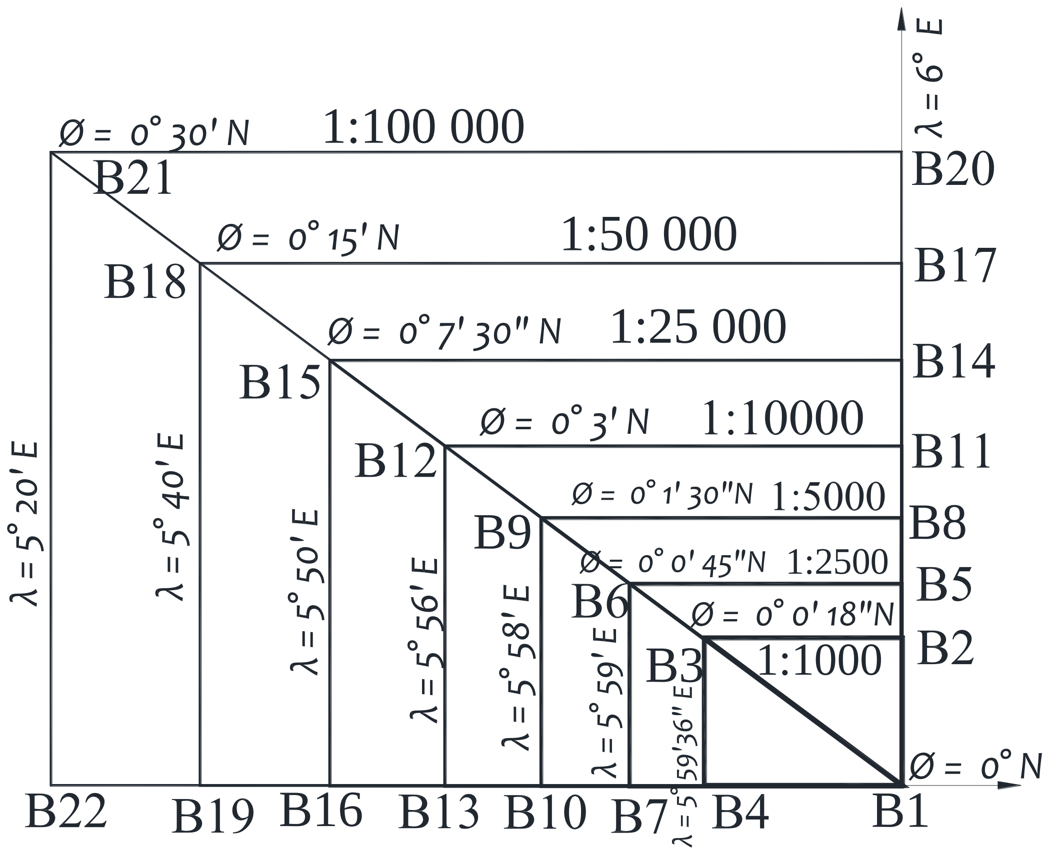


Figure (4-17): Group (2) of maps at edge of zone 31 in UTM at equator.

Chosen maps for the study have the following dimensions:

- 1:100000 as 40' x 30'with approximate dimensions 74200m x 55300 m at equator.
- 1:50000 as 20' x15'with approximate dimensions37100 m x 27650m at equator.
- 1:25000 as 10' x 7' 30"with approximate dimensions18550m x 13825 m at equator.
- 1:10000 as 4' x 3'with approximate dimensions 7420m x 5530m at equator.
- 1:5000 as 2' x1' 30"with approximate dimensions3710m x 2765 m at equator.
- 1:2500 as 1' x 45"with approximate dimensions1855 m x 1382 m at equator.
- 1:1000 as 24" x 18"with approximate dimensions 742m x 553 m at equator.

Also

- 1:100000 as 40' x 30' with approximate dimensions 64300m x 55400 m at latitude 30°.
- 1: 50000 as 20' x15'with approximate dimensions 32150 m x 27700 m at latitude 30°.
- 1: 25000 as 10' x7' 30"with approximate dimensions 16075 m x13850 m at latitude 30°.
- 1: 10000 as 4' x 3'with approximate dimensions 6430 m x 5540 m at latitude 30°.
- 1: 5000 as 2' x 1' 30"with approximate dimensions 3215 m x 2770 m at latitude 30°.
- 1: 2500 as 1' x 45"with approximate dimensions 1608 m x 1385 m at latitude 30°.
- 1: 1000 as 24" x 18"with approximate dimensions 643 m x 554 m at latitude 30°.

Also

- 1:100000 as 40' x 30' with approximate dimensions 37200 m x 55700 m at latitude 60°.
- 1:50000 as 20' x15' with approximate dimensions 18600 m x 27850 m at latitude 60°.
- 1:25000 as 10' x 7' 30"with approximate dimensions 9300 m x 13925 m at latitude 60°.
- 1:10000 as 4' x 3' with approximate dimensions 3720 m x 5570 m at latitude 60°.
- 1:5000 as 2' x 1' 30"with approximate dimensions 1860 m x 2785 m at latitude 60°.
- 1:2500 as 1' x 45" with approximate dimensions 930 m x 1392 m at latitude 60°.
- 1:1000 as 24" x 18"with approximate dimensions 372 m x 557 m at latitude 60°.

Group (1) and Group (2) At Equator, Maps are usually represented in map sheet of approximately 75 cm x 56 cm.

Table (4-5) includes the geodetic coordinates of the corner points of the studied maps related to WGS84 and the corresponding projected values (UTM) at different scales for Groups (1) and (2) at equator. Table (4-6) shows the data at latitude 30°N as Groups (3) and (4). Table (4-7) shows the data at latitude 60°N as Groups (5) and (6). Table (4-8) shows the data of Groups (7) and (8) at latitude 70°N & Groups (9) and (10) at latitude 80° N.

Table (4-5). Coordinates of map corners (WGS84 projected on UTM)
Group (1) & (2) at the equator.

	Point	Latitude	Longitude		East(m)	North(m)
Group1 at the central meridian of zone 31 , (Ø= 0º N , λ=3ºE)	A1	0° 00'00"	3° 00'00"		500000.000	0.000
	A2	0° 00'00"	3° 00'24"		500741.833	0.000
	A3	0° 00'18"	3° 00'24"		500741.833	552.650
	A4	0° 00'18"	3° 00'00"		500000.000	552.650
	A5	0° 00'00"	3° 01'00"		501854.583	0.000
	A6	0° 00'45"	3° 01'00"		501854.583	1381.626
	A7	0° 00'45"	3° 00'00"		500000.000	1381.626
	A8	0° 00'00"	3° 02'00"		503709.166	0.000
	A9	0° 01'30"	3° 02'00"		503709.165	2763.252
	A10	0° 01'30"	3° 00'00"		500000.000	2763.251
	A11	0° 00'00"	3° 04'00"		507418.333	0.000
	A12	0° 03'00"	3° 04'00"		507418.330	5526.506
	A13	0° 03'00"	3° 00'00"		500000.000	5526.502
	A14	0° 00'00"	3° 10'00"		518545.853	0.000
	A15	0° 07'30"	3° 10'00"		518545.810	13816.315
	A16	0° 07'30"	3° 00'00"		500000.000	13816.256
	A17	0° 00'00"	3° 20'00"		537091.865	0.000
	A18	0° 1500"	3° 20'00"		537091.514	27632.984
	A19	0° 15'00"	3° 00'00"		500000.000	27632.513
	A20	0° 00'00"	3° 40'00"		574184.994	0.000
	A21	0° 30'00"	3° 40'00"		574182.188	55268.803
	A22	0° 30'00"	3° 00'00"		500000.000	55265.037
Group2 at the edge of zone 31 (Ø= 0º N , λ=6ºE)	B1	0° 00'00"	6° 00'00"		833978.557	0.000
	B2	0° 00'18"	6° 00'00"		833978.556	553.414
	B3	0° 00'18"	5° 59'36"		833235.700	553.410
	B4	0° 00'00"	5° 59'36"		833235.701	0.000
	B5	0° 00'45"	6° 00'00"		833978.549	1383.534
	B6	0° 00'45"	5° 59'00"		832121.418	1383.513
	B7	0° 00'00"	5° 59'00"		832121.426	0.000
	B8	0° 01'30"	6° 00'00"		833978.525	2767.069
	B9	0° 01'30"	5° 58'00"		830264.292	2766.984
	B10	0° 00'00"	5° 58'00"		830264.324	0.000
	B11	0° 03'00"	6° 00'00"		833978.430	5534.138
	B12	0° 03'00"	5° 56'00"		826550.079	5533.802
	B13	0° 00'00"	5° 56'00"		826550.203	0.000
	B14	0° 07'30"	6° 00'00"		833977.766	13835.345
	B15	0° 07'30"	5° 50'00"		815407.758	13833.280
	B16	0° 00'00"	5° 50'00"		815408.505	0.000
	B17	0° 15'00"	6° 00'00"		833975.393	27670.690
	B18	0° 15'00"	5° 4000"		796838.334	27662.670
	B19	0° 00'00"	5° 4000"		796841.145	0.000
	B20	0° 30'00"	6° 00'00"		833965.902	55341.388
	B21	0° 30'00"	5° 20'00"		759704.028	55311.204
	B22	0° 00'00"	5° 20'00"		759713.862	0.000

Table (4-6). Coordinates of map corners (WGS84 projected on UTM)
Group (3) & (4) at latitude 30º N.

	Point	Latitude	Longitude		East(m)	North(m)
Group3 at the central meridian of zone 31, (Ø= 30º N , λ=3ºE)	C1	30° 00'00"	3° 00'00"		500000.000	3318785.353
	C2	30° 00'00"	3° 00'24"		500642.985	3318785.371
	C3	30° 00'18"	3° 00'24"		500642.952	3319339.412
	C4	30° 00'18"	3° 00'00"		500000.000	3319339.393
	C5	30° 00'00"	3° 01'00"		501607.461	3318785.470
	C6	30° 00'45"	3° 01'00"		501607.260	3320170.572
	C7	30° 00'45"	3° 00'00"		500000.000	3320170.455
	C8	30° 00'00"	3° 02'00"		503214.923	3318785.820
	C9	30° 01'30"	3° 02'00"		503214.117	3321556.028
	C10	30° 01'30"	3° 00'00"		500000.000	3321555.560
	C11	30° 00'00"	3° 04'00"		506429.846	3318787.223
	C12	30° 03'00"	3° 04'00"		506426.621	3324327.651
	C13	30° 03'00"	3° 00'00"		500000.000	3324325.779
	C14	30° 00'00"	3° 10'00"		516074.626	3318797.043
	C15	30° 07'30"	3° 10'00"		516054.442	3332648.216
	C16	30° 07'30"	3° 00'00"		500000.000	3332636.497
	C17	30° 00'00"	3° 20'00"		532149.320	3318832.112
	C18	30° 15'00"	3° 20'00"		532068.431	3346534.899
	C19	30° 15'00"	3° 00'00"		500000.000	3346487.905
	C20	30° 00'00"	3° 40'00"		564299.188	3318972.397
	C21	30° 30'00"	3° 40'00"		563974.390	3374380.421
	C22	30° 30'00"	3° 00'00"		500000.000	3374191.516
Group 4 at the edge of zone 31 (Ø= 30º N , λ=6ºE)	D1	30° 00'00"	6° 00'00"		789409.653	3322575.905
	D2	30° 00'18"	6° 00'00"		789395.124	3323130° 328
	D3	30° 00'18"	5° 59'36"		788751.729	3323113.484
	D4	30° 00'00"	5° 59'36"		788766.226	3322559.063
	D5	30° 00'45"	6° 00'00"		789373.326	3323961.963
	D6	30° 00'45"	5° 59'00"		787764.963	3323919.919
	D7	30° 00'00"	5° 59'00"		787801.088	3322533.871
	D8	30° 01'30"	6° 00'00"		789336.985	3325348.023
	D9	30° 01'30"	5° 58'00"		786120.677	3325264.148
	D10	30° 00'00"	5° 58'00"		786192.535	3322492.072
	D11	30° 03'00"	6° 00'00"		789264.261	3328120.150
	D12	30° 03'00"	5° 56'00"		782833.315	3327953.256
	D13	30° 00'00"	5° 56'00"		782975.466	3322409.179
	D14	30° 07'30"	6° 00'00"		789045.760	3336436.576
	D15	30° 07'30"	5° 50'00"		772980.929	3336025.779
	D16	30° 00'00"	5° 50'00"		773324.546	3322166.137
	D17	30° 15'00"	6° 00'00"		788680.491	3350297.439
	D18	30° 15'00"	5° 40'00"		756592.677	3349497.400
	D19	30° 00'00"	5° 40'00"		757240.603	3321779.849
	D20	30° 30'00"	6° 00'00"		787945.832	3378019.742
	D21	30° 30'00"	5° 20'00"		723938.130	3376506.627
	D22	30° 00'00"	5° 20'00"		725075.936	3321077.671

Table (4-7). Coordinates of map corners (WGS84 projected on UTM)
Group (5) & (6) at latitude 60º N.

	Point	Latitude	Longitude		East(m)	North(m)
Group 5 at the central meridian of zone 31, (Ø= 60º N , λ=3ºE)	E1	60° 00'00"	3° 00'00"		500000.000	6651411.191
	E2	60° 00'00"	3° 00'24"		500371.851	6651411.211
	E3	60° 00'18"	3° 00'24"		500371.795	6651968.049
	E4	60° 00'18"	3° 00'00"		500000.000	6651968.030
	E5	60° 00'00"	3° 01'00"		500929.628	6651411.309
	E6	60° 00'45"	3° 01'00"		500929.277	6652803.406
	E7	60° 00'45"	3° 00'00"		500000.000	6652803.289
	E8	60° 00'00"	3° 02'00"		501859.256	6651411.660
	E9	60° 01'30"	3° 02'00"		501857.853	6654195.858
	E10	60° 01'30"	3° 00'00"		500000.000	6654195.389
	E11	60° 00'00"	3° 0'400"		503718.512	6651413.065
	E12	60° 03'00"	3° 0'400"		503712.899	6656981.470
	E13	60° 03'00"	3° 00'00"		500000.000	6656979.598
	E14	60° 00'00"	3° 10'00"		509296.274	6651422.901
	E15	60° 07'30"	3° 10'00"		509261.182	6665343.969
	E16	60° 07'30"	3° 00'00"		500000.000	6665332.290
	E17	60° 00'00"	3° 20'00"		518592.508	6651458.030
	E18	60° 15'00"	3° 20'00"		518452.052	6679300.252
	E19	60° 15'00"	3° 00'00"		500000.000	6679253.652
	E20	60° 00'00"	3° 40'00"		537184.702	6651598.543
	E21	60° 30'00"	3° 40'00"		536622.160	6707282.609
	E22	60° 30'00"	3° 00'00"		500000.000	6707097.169
Group 6 at the edge of zone 31 (Ø= 60º N , λ=6ºE)	F1	60° 00'00"	6° 00'00"		667294.821	6655205.484
	F2	60° 00'18"	6° 00'00"		667269.565	6655761.941
	F3	60° 00'18"	5° 59'36"		666898.024	6655745.096
	F4	60° 00'00"	5° 59'36"		666923.224	6655188.641
	F5	60° 00'45"	6° 00'00"		667231.678	6656596.627
	F6	60° 00'45"	5° 59'00"		666303.035	6656554.592
	F7	60° 00'00"	5°59'00"		666365.827	6655163.441
	F8	60° 01'30"	6° 00'00"		667168.527	6657987.774
	F9	60° 01'30"	5° 58'00"		665311.935	6657903.956
	F10	60° 00'00"	5° 58'00"		665436.826	6655121.627
	F11	60° 03'00"	6° 00'00"		667042.200	6660770.068
	F12	60° 03'00"	5° 56'00"		663331.798	6660603.457
	F13	60° 00'00"	5° 56'00"		663578.802	6655038.705
	F14	60° 07'30"	6° 00'00"		666663.029	6669117.002
	F15	60° 07'30"	5° 50'00"		657407.888	6668708.119
	F16	60° 00'00"	5° 50'00"		658004.566	6654795.565
	F17	60° 15'00"	6° 00'00"		666030.435	6683028.712
	F18	60° 15'00"	5° 40'00"		647589.797	6682236.347
	F19	60° 00'00"	5° 40'00"		648713.642	6654409.079
	F20	60° 30'00"	6° 00'00"		664762.857	6710852.708
	F21	60° 30'00"	5° 20'00"		628160.809	6709368.945
	F22	60° 00'00"	5° 20'00"		630129.944	6653706.400

Table (4-8). Coordinates of 1:100000 map corners (WGS84 projected on UTM) Group (7), (8), (9) & (10) at latitudes 70ºN and 80º N.

	Poin	Latitude	Longitude		East(m)	North(m)
Group 7 Ø= 70º N, λ=3º E	K1	70° 00'00"	3° 00'00"		500000.000	7765873.1
	K2	70° 00'00"	3° 40'00"		525447.071	7766012.2
	K3	70° 30'00"	3° 40'00"		524836.442	7821769.6
	K4	70° 30'00"	3° 00'00"		500000.000	7821633.4
Group8 Ø= 70º N, λ=6º E	H1	70° 00'00"	5° 20'00"		589047.430	7767577.2
	H2	70° 00'00"	6° 00'00"		614473.715	7768690.1
	H3	70° 30'00"	6° 00'00"		611726.260	7824391.3
	H4	70° 30'00"	5° 20'00"		586910.399	7823301.8
Group9 Ø= 80º N, λ=3º E	I1	80° 00'00"	3° 00'00"		500000.000	8881585.8
	I2	80° 00'00"	3° 40'00"		512923.545	8881659.8
	I3	80° 30'00"	3° 40'00"		512283.577	8937464.7
	I4	80° 30'00"	3° 00'00"		500000.000	8937394.2
Group10 Ø= 80º N, λ=6º E	J1	80° 00'00"	5° 20'00"		545221.617	8882492.7
	J2	80° 00'00"	6° 00'00"		558132.215	8883084.9
	J3	80° 30'00"	6° 00'00"		555253.396	8938821.3
	J4	80° 30'00"	5° 20'00"		542982.201	8938257.6

- Tables (4-9) and table (4-10) include the geodetic and projected data in different used scales of Group (1) close to central meridian and Group (2) near zone edge at equator, the difference in geodetic distance and map distances calculated.

- Table (4-11) and table (4-12) include geodetic and projected data in different used scales of Group (3) close to central meridian and Group (4) near the zone edge at latitude 30º N, the distribution of points matches the distribution in Group (1) and Group (2)

- Table (4-13) and table (4-14) include geodetic and projected data in different used scales in Group (5) close to central meridian and Group (6) near the zone edge at latitude 60º N, the distribution of points matches the distribution in Group (1) and Group (2)

- Table (4-15) includes geodetic and projected data in scale 1: 100000 Group (7) close to central meridian and Group (8) near the zone edge at latitude 70º N. The table also includes geodetic and projected data in scale 1: 100000 Group (9) close to central meridian and Group (10) near the zone edge at latitude 80º N, for all groups 1 to 10 see figure (4-15).

Table (4-9). Geodetic and projected data in different used scales in Group (1) at equator.

Map	From Pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis.(m)		Diff. bet. Dis. (m)
1:1000	A1	A2	90°00'00"	742.130		90°00'00"	741.833		0.297
	A2	A3	0°00'00"	552.871		0°00'00"	552.650		0.221
	A3	A4	270°00'00	742.130		270°00'00	741.833		0.297
	A4	A1	180°00'00	552.871		180°00'00	552.650		0.221
	A1	A3	53°18'53"	925.432		53°18'53"	925.061		0.371
1:2500	A1	A5	90°00'00"	1855.325		90°00'00"	1854.583		0.742
	A5	A6	0°00'00"	1382.178		0°00'00"	1381.626		0.552
	A6	A7	270°00'00	1855.325		270°00'00	1854.583		0.742
	A7	A1	180°00'00	1382.178		180°00'00	1381.626		0.552
	A1	A6	53°18'53"	2313.579		53°18'53"	2312.654		0.925
1:5000	A1	A8	90°00'00"	3710.650		90°00'00"	3709.166		1.484
	A8	A9	0°00'00"	2764.357		0°00'00"	2763.252		1.105
	A9	A1	270°00'00	3710.649		270°00'00	3709.165		1.484
	A10	A1	180°00'00	2764.357		180°00'00	2763.251		1.106
	A1	A9	53°18'53"	4627.158		53°18'53"	4625.307		1.851
1:10000	A1	A1	90°00'00"	7421.299		90°00'00"	7418.333		2.966
	A11	A1	0°00'00"	5528.714		0°00'00"	5526.506		2.208
	A12	A1	270°00'00	7421.297		270°00'00	7418.330		2.967
	A13	A1	180°00'00	5528.714		180°00'00	5526.502		2.212
	A1	A1	53°18'53"	9254.315		53°18'53"	9250.615		3.700
1:25000	A1	A1	90°00'00"	18553.24		90°00'00"	18545.85		7.395
	A14	A1	0°00'00"	13821.78		359°59'59	13816.31		5.470
	A15	A1	270°00'01	18553.20		269°59'59	18545.81		7.395
	A16	A1	180°00'00	13821.78		180°00'00	13816.25		5.529
	A1	A1	53°18'52"	23135.77		53°18'52"	23126.55		9.222
1:50000	A1	A1	90°00'00"	37106.49		90°00'00"	37091.86		14.632
	A17	A1	0°00'00"	27643.57		359°59'57	27632.98		10.587
	A18	A1	270°00'03	37106.14		269°59'57	37091.51		14.632
	A19	A1	180°00'00	27643.57		180°00'00	27632.51		11.058
	A1	A1	53°18'52"	46271.48		53°18'51"	46253.24		18.246
1:100000	A1	A2	90°00'00"	74212.99		90°00'00"	74184.99		28.000
	A20	A2	0°00'00"	55287.15		359°59'50	55268.80		18.349
	A21	A2	270°00'10	74210.18		269°59'50	74182.18		27.999
	A22	A1	180°00'00	55287.15		180°00'00	55265.03		22.115
	A1	A2	53°18'48"	92542.41		53°18'45"	92507.50		34.916

Table (4 -10). Geodetic and projected data in different used scales in Group (2) at equator.

Map Scale	From Pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis.(m)		Diff. bet. Dis. (m)
	B1	B2	0°00'00"	552.871		0°00'00"	553.414		-0.543
	B2	B3	270°00'00"	742.130		269°59'59"	742.856		-0.726
1:1000	B3	B4	180°00'00"	552.871		180°00'00"	553.410		-0.539
	B4	B1	90°00'00"	742.130		90°00'00"	742.856		-0.726
	B1	B3	306°41'07"	925.432		306°41'07"	926.337		-0.905
	B1	B5	0°00'00"	1382.178		359°59'59"	1383.534		-1.356
	B5	B6	270°00'00"	1855.325		269°59'58"	1857.131		-1.806
1:2500	B6	B7	180°00'00"	1382.178		179°59'59"	1383.513		-1.335
	B7	B1	90°00'00"	1855.325		90°00'00"	1857.131		-1.806
	B1	B6	306°41'07"	2313.579		306°41'06"	2315.831		-2.252
	B1	B8	0°00'00"	2764.357		359°59'58"	2767.069		-2.712
	B8	B9	270°00'00"	3710.649		269°59'55"	3714.233		-3.584
1:5000	B9	B10	180°00'00"	2764.357		179°59'58"	2766.984		-2.627
	B10	B1	90°00'00"	3710.650		90°00'00"	3714.233		-3.583
	B1	B9	306°41'07"	4627.158		306°41'05"	4631.627		-4.469
	B1	B11	0°00'00"	5528.714		359°59'55"	5534.138		-5.424
	B11	B12	270°00'00"	7421.297		269°59'51"	7428.351		-7.054
1:10000	B12	B13	180°00'00"	5528.714		179°59'55"	5533.802		-5.088
	B13	B1	90°00'00"	7421.299		90°00'00"	7428.354		-7.055
	B1	B12	306°41'07"	9254.315		306°41'03"	9263.112		-8.797
	B1	B14	0°00'00"	13821.785		359°59'48"	13835.345		-13.560
	B14	B15	270°00'01"	18553.205		269°59'37"	18570.008		-16.803
1:25000	B15	B16	180°00'00"	13821.785		179°59'49"	13833.281		-11.496
	B16	B1	90°00'00"	18553.249		90°00'00"	18570.052		-16.803
	B1	B15	306°41'08"	23135.778		306°40'56"	23156.731		-20.953
	B1	B17	0°00'00"	27643.571		359°59'36"	27670.690		-27.119
	B17	B18	270°00'03"	37106.146		269°59'15"	37137.060		-30.914
1:50000	B18	B19	180°00'00"	27643.571		179°59'39"	27662.671		-19.100
	B19	B1	90°00'00"	37106.497		90°00'00"	37137.412		-30.915
	B1	B18	306°41'08"	46271.486		306°40'46"	46310.037		-38.551
	B1	B20	0°00'00"	55287.153		359°59'13"	55341.390		-54.237
	B20	B21	270°00'10"	74210.186		269°58'36"	74261.879		-51.693
1:100000	B21	B22	180°00'00"	55287.152		179°59'23"	55311.205		-24.053
	B22	B1	90°00'00"	74212.993		90°00'00"	74264.694		-51.701
	B1	B21	306°41'12"	92542.415		306°40'28"	92606.883		-64.468

Table (4-11). Geodetic and projected data in different used scales in Group (3) at latitude 30°N.

Map Scale	From Pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis.(m)		Diff. bet. Dis. (m)
1:1000	C1	C2	89°59'54"	643.242		89°59'54"	642.985		0.257
	C2	C3	0°00'00"	554.262		359°59'48"	554.041		0.221
	C3	C4	270°00'06"	643.210		269°59'54"	642.952		0.258
	C4	C1	180°00'00"	554.262		180°00'00"	554.041		0.221
	C1	C3	49°14'50"	849.086		49°14'50"	848.746		0.340
1:2500	C1	C5	89°59'45"	1608.105		89°59'45"	1607.461		0.644
	C5	C6	0°00'00"	1385.657		359°59'30"	1385.103		0.554
	C6	C7	270°00'15"	1607.903		269°59'45"	1607.260		0.643
	C7	C1	180°00'00"	1385.657		180°00'00"	1385.103		0.554
	C1	C6	49°14'37"	2122.668		49°14'37"	2121.819		0.849
1:5000	C1	C8	89°59'30"	3216.209		89°59'30"	3214.923		1.286
	C8	C9	0°00'00"	2771.316		359°59'00"	2770.208		1.108
	C9	C10	270°00'30"	3215.403		269°59'30"	3214.117		1.286
	C10	C1	180°00'00"	2771.316		180°00'00"	2770.208		1.108
	C1	C9	49°14'15"	4245.186		49°14'15"	4243.488		1.698
1:10000	C1	C11	89°59'00"	6432.419		89°59'00"	6429.847		2.572
	C11	C12	0°00'00"	5542.643		359°58'00"	5540.429		2.214
	C12	C13	270°01'00"	6429.192		269°59'00"	6426.621		2.571
	C13	C1	180°00'00"	5542.643		180°00'00"	5540.426		2.217
	C1	C12	49°13'32"	8489.767		49°13'32"	8486.373		3.394
1:25000	C1	C14	89°57'30"	16081.045		89°57'30"	16074.630		6.415
	C14	C15	0°00'00"	13856.687		359°54'59"	13851.188		5.499
	C15	C16	270°02'31"	16060.853		269°57'29"	16054.446		6.407
	C16	C1	180°00'00"	13856.687		180°00'00"	13851.144		5.543
	C1	C15	49°11'23"	21219.880		49°11'23"	21211.414		8.466
1:50000	C1	C17	89°55'00"	32162.082		89°55'00"	32149.354		12.728
	C17	C18	0°00'00"	27713.638		359°49'58"	27702.905		10.733
	C18	C19	270°05'02"	32081.162		269°54'58"	32068.465		12.697
	C19	C1	180°00'00"	27713.638		180°00'00"	27702.553		11.085
	C1	C18	49°07'47"	42424.591		49°07'47"	42407.801		16.790
1:100000	C1	C20	89°50'00"	64324.096		89°50'00"	64299.460		24.636
	C20	C21	0°00'00"	55428.335		359°39'51"	55408.976		19.359
	C21	C22	270°10'09"	63999.192		269°49'51"	63974.669		24.523
	C22	C1	180°00'00"	55428.335		180°00'00"	55406.164		22.171
	C1	C21	49°00'34"	84788.221		49°00'31"	84755.733		32.488

Table (4-12). Geodetic and projected data different used scales in Group (4) at latitude 30°N.

Map Scale	From Pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis.(m)		Diff. bet. Dis. (m)
	D1	D2	0°00'00"	554.262		358°29'56"	554.613		-0.351
	D2	D3	270°00'06"	643.210		268°30'01"	643.616		-0.406
1:1000	D3	D4	180°00'00"	554.262		178°30'08"	554.611		-0.349
	D4	D1	89°59'54"	643.242		88°30'02"	643.648		-0.406
	D1	D3	310°45'10"	849.085		309°15'06"	849.621		-0.536
	D1	D5	0°00'00"	1385.657		358°29'55"	1386.534		-0.877
	D5	D6	270°00'15"	1607.903		268°30'09"	1608.912		-1.009
1:2500	D6	D7	180°00'00"	1385.657		178°30'25"	1386.519		-0.862
	D7	D1	89°59'45"	1608.105		88°30'11"	1609.114		-1.009
	D1	D6	310°45'23"	2122.668		309°15'18"	2124.001		-1.333
	D1	D8	0°00'00"	2771.316		358°29'54"	2773.071		-1.755
	D8	D9	270°00'30"	3215.403		268°30'22"	3217.401		-1.998
1:5000	D9	D10	180°00'00"	2771.316		178°30'54"	2773.008		-1.692
	D10	D1	89°59'30"	3216.209		88°30'26"	3218.210		-2.001
	D1	D9	310°45'45"	4245.186		309°15'39"	4247.825		-2.639
	D1	D11	0°00'00"	5542.643		358°29'52"	5546.151		-3.508
	D11	D12	270°01'00"	6429.192		268°30'48"	6433.111		-3.919
1:10000	D12	D13	180°00'00"	5542.643		178°31'52"	5545.899		-3.256
	D13	D1	89°59'00"	6432.419		88°30'56"	6436.347		-3.928
	D1	D12	310°46'28"	8489.767		309°16'20"	8494.948		-5.181
	D1	D14	0°00'00"	13856.687		358°29'46"	13865.447		-8.760
	D14	D15	270°02'31"	16060.854		268°32'07"	16070.083		-9.229
1:25000	D15	D16	180°00'00"	13856.687		178°34'47"	13863.901		-7.214
	D16	D1	89°57'30"	16081.045		88°32'27"	16090.326		-9.281
	D1	D15	310°48'37"	21219.880		309°18'23"	21232.101		-12.221
	D1	D17	0°00'00"	27713.638		358°29'36"	27731.122		-17.484
	D17	D18	270°05'02"	32081.162		268°34'18"	32097.786		-16.624
1:50000	D18	D19	180°00'00"	27713.638		178°39'39"	27725.123		-11.485
	D19	D1	89°55'00"	32162.082		88°34'57"	32178.899		-16.817
	D1	D18	310°52'13"	42424.591		309°21'50"	42446.682		-22.091
	D1	D20	0°00'00"	55428.335		358°29'15"	55463.158		-34.823
	D20	D21	270°10'09"	63999.191		268°38'45"	64025.585		-26.394
1:100000	D21	D22	180°00'00"	55428.335		178°49'27"	55440.633		-12.298
	D22	D1	89°50'00"	64324.096		88°39'57"	64351.161		-27.065
	D1	D21	310°59'26"	84788.221		309°28'45"	84823.601		-35.380

Table (4-13). Geodetic and projected data in different used scales in Group (5) at latitude 60°N.

Map Scale	From Pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis. (m)		Diff. bet. Dis. (m)
1:1000	E1	E2	89°59'49"	372.000		89°59'49"	371.851		0.149
	E2	E3	0°00'00"	557.061		359°59'39"	556.838		0.223
	E3	E4	270°00'10"	371.944		269°59'50"	371.795		0.149
	E4	E1	180°00'00"	557.062		180°00'00"	556.839		0.223
	E1	E3	33°43'47"	669.836		33°43'47"	669.568		0.268
1:2500	E1	E5	89°59'34"	930.000		89°59'34"	929.628		0.372
	E5	E6	0°00'00"	1392.654		359°59'08"	1392.097		0.557
	E6	E7	270°00'26"	929.649		269°59'34"	929.277		0.372
	E7	E1	180°00'00"	1392.655		180°00'00"	1392.098		0.557
	E1	E6	33°43'21"	1674.533		33°43'21"	1673.863		0.670
1:5000	E1	E8	89°59'08"	1860.000		89°59'08"	1859.256		0.744
	E8	E9	0°00'00"	2785.312		359°58'16"	2784.198		1.114
	E9	E10	270°00'52"	1858.597		269°59'08"	1857.853		0.744
	E10	E1	180°00'00"	2785.312		180°00'00"	2784.198		1.114
	E1	E9	33°42'36"	3348.874		33°42'36"	3347.534		1.340
1:10000	E1	E11	89°58'16"	3720.000		89°58'16"	3718.512		1.488
	E11	E12	0°00'00"	5570.635		359°56'32"	5568.408		2.227
	E12	E13	270°01'44"	3714.385		269°58'16"	3712.900		1.485
	E13	E1	180°00'00"	5570.636		180°00'00"	5568.407		2.229
	E1	E12	33°41'08"	6696.977		33°41'08"	6694.298		2.679
1:25000	E1	E14	89°55'40"	9299.998		89°55'40"	9296.281		3.717
	E14	E15	0°00'00"	13926.669		359°51'20"	13921.113		5.556
	E15	E16	270°04'20"	9264.892		269°55'40"	9261.189		3.703
	E16	E1	180°00'00"	13926.669		180°00'00"	13921.099		5.570
	E1	E15	33°36'44"	16736.656		33°36'44"	16729.967		6.689
1:50000	E1	E17	89°51'20"	18599.981		89°51'20"	18592.567		7.414
	E17	E18	0°00'00"	27853.601		359°42'39"	27842.577		11.024
	E18	E19	270°08'41"	18459.469		269°51'19"	18452.111		7.358
	E19	E1	180°00'00"	27853.602		180°00'00"	27842.461		11.141
	E1	E18	33°29'23"	33453.998		33°29'22"	33440.663		13.335
1:100000	E1	E20	89°42'41"	37199.844		89°42'41"	37185.174		14.670
	E20	E21	0°00'00"	55708.261		359°25'16"	55686.907		21.354
	E21	E22	270°17'24"	36637.084		269°42'36"	36622.630		14.454
	E22	E1	180°00'00"	55708.261		180°00'00"	55685.978		22.283
	E1	E21	33°14'39"	66830.543		33°14'37"	66804.176		26.367

Table (4-14). Geodetic and projected data in different used scales in Group (6), at latitude 60°N.

Map Scale	From Pt	To Pt	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	map dis.(m)		Diff. bet. Dis. (m)
1:1000	F1	F2	0°00'00"	557.062		357°24'05"	557.030		0.032
	F2	F3	270°00'10"	371.944		267°24'15"	371.922		0.022
	F3	F4	180°00'00"	557.058		177°24'25"	557.026		0.032
	F4	F1	89°59'51"	372.000		87°24'17"	371.979		0.021
	F1	F3	326°16'13"	669.836		323°40'18"	669.798		0.038
1:2500	F1	F5	0°00'00"	1392.655		357°24'04"	1392.575		0.080
	F5	F6	270°00'26"	929.649		267°24'30"	929.594		0.055
	F6	F7	180°00'00"	1392.652		177°24'56"	1392.567		0.085
	F7	F1	89°59'35"	930.000		87°24'31"	929.945		0.055
	F1	F6	326°16'39"	1674.533		323°40'44"	1674.434		0.099
1:5000	F1	F8	0°00'00"	2785.314		357°24'04"	2785.154		0.160
	F8	F9	270°00'52"	1858.597		267°24'54"	1858.483		0.114
	F9	F10	180°00'00"	2785.311		177°25'48"	2785.131		0.180
	F10	F1	89°59'08"	1860.000		87°24'57"	1859.887		0.113
	F1	F9	326°17'24"	3348.874		323°41'27"	3348.669		0.205
1:10000	F1	F11	0°00'00"	5570.636		357°24'02"	5570.315		0.321
	F11	F12	270°01'44"	3714.385		267°25'44"	3714.142		0.243
	F12	F13	180°00'00"	5570.635		177°27'30"	5570.231		0.404
	F13	F1	89°58'16"	3720.000		87°25'49"	3719.760		0.240
	F1	F12	326°18'52"	6696.977		323°42'54"	6696.541		0.436
1:25000	F1	F14	0°00'00"	13926.669		357°23'59"	13925.857		0.812
	F14	F15	270°04'20"	9264.892		267°28'13"	9264.168		0.724
	F15	F16	180°00'00"	13926.669		177°32'39"	13925.343		1.326
	F16	F1	89°55'40"	9299.998		87°28'25"	9299.294		0.704
	F1	F15	326°23'16"	16736.656		323°47'15"	16735.369		1.287
1:50000	F1	F17	0°00'00"	27853.602		357°23'53"	27851.942		1.660
	F17	F18	270°08'41"	18459.469		267°32'23"	18457.654		1.815
	F18	F19	180°00'00"	27853.602		177°41'14"	27849.953		3.649
	F19	F1	89°51'20"	18599.981		87°32'45"	18598.238		1.743
	F1	F18	326°30'37"	33453.998		323°54'31"	33450.793		3.205
1:100000	F1	F20	0°00'00"	55708.261		357°23'41"	55704.797		3.464
	F20	F21	270°17'24"	36637.084		267°40'43"	36632.110		4.974
	F21	F22	180°00'00"	55708.261		177°58'26"	55697.364		10.897
	F22	F1	89°42'41"	37199.844		87°41'25"	37195.098		4.746
	F1	F21	326°45'21"	66830.543		324°09'05"	66821.788		8.755

Table (4-15). Geodetic and projected data in 1:100000 map scales in Groups (7, 8, 9, and 10) at latitude 70°N& 80°N.

Map Scale	From pt.	To Pt.	Geodetic Azimuth	Geodetic Dis. (m)		Map Bearing	Map dis.(m)		Diff. bet. Dis. (m)
Group (7) 1:100000	K1	K2	89°41'12"	25457.567		89°41'12"	25447.452		10.115
	K2	K3	0°00'00"	55782.582		359°22'21"	55760.700		21.882
	K3	K4	270°18'51"	24846.692		269°41'09"	24836.816		9.876
	K4	K1	180°00'00"	55782.582		180°00'00"	55760.269		22.313
Group (8) 1:100000	H1	H2	89°41'12"	25457.567		87°29'38"	25450.626		6.941
	H2	H3	0°00'00"	55782.582		357°10'34"	55768.998		13.584
	H3	H4	270°18'51"	24846.692		267°29'10"	24839.767		6.925
	H4	H1	180°00'00"	55782.582		177°48'14"	55765.551		17.031
Group (9) 1:100000	I1	I2	89°40'18"	12928.920		89°40'18"	12923.757		5.163
	I2	I3	0°00'00"	55830.799		359°20'35"	55808.575		22.224
	I3	I4	270°19'44"	12288.687		269°40'16"	12283.779		4.908
	I4	I1	180°00'00"	55830.799		180°00'00"	55808.467		22.332
Group (10) 1:100000	J1	J2	89°40'18"	12928.920		87°22'26"	12924.172		4.748
	J2	J3	0°00'00"	55830.799		357°02'36"	55810.659		20.140
	J3	J4	270°19'44"	12288.687		267°22'11"	12284.136		4.551
	J4	J1	180°00'00"	55830.799		177°42'01"	55809.794		21.005

Considering the data and results in pervious tables and concerning the deference between geodetic and map distances; the differences seem significant as absolute values but they are not noticeable as drawn in the map. It means one cannot notice a difference between geodetic and plan metric maps for the same area.

In map scale 1:1000 at equator, distortion value of 37 cm at G1 & 90 cm at G2 in 925.432 m is obtained. This is a big value especially when precise EDM is used in measuring distances in the field. The user does not know about distortion and the surveyor himself should bay attention while dealing with projected map and the scale factor while using Total Station in the field. This problem can vanish by using geodetic Total Station in the field and the proposed geodetic map. In the 1:1000 map itself, 37 & 90 cm differences in 925m will appear as (37 & 90 cm/1000m) which is not noticeable.

Distortion is variable in map from point to another; to resolve this issue practically we take an average value of distortion in limited region. The problem is more complex in case of international and intercontinental projects such international roads and petroleum pipelines. Again, the problem can

vanish by using the proposed geodetic mapping system especially in the presence of WGS84 as global geodetic coordinate system and GNSS as global observation tools.

For every map scale and concerning G1 maps which are adjacent to the central meridian of the used zone at equator, the maximum difference between the ellipsoidal and the corresponding distances are shown in the table (4-16) and table 3 beside their values as will appear in the map. The next table shows these results:

Table (4-16). Max differences between ellipsoidal and map distances for G1 maps adjacent to the central meridian of the zone and at Equator.

Map Scale	1:1000	1:2500	1:5000	1:10000	1:25000	1:50000	1:100000
Max diff (m) (ellipsoidal dis-map dis)	0.37	0.92	1.85	3.70	9.22	18.25	34.92
Max diff drawn in the map (mm)	0.37	0.37	0.37	0.37	0.37	0.36	0.33

Table (4-17). Max differences between ellipsoidal and map distances for G2 maps adjacent to the edge of the zone and at Equator.

Map Scale	1:1000	1:2500	1:5000	1:10000	1:25000	1:50000	1:100000
Max diff (m) (ellipsoidal dis-map dis)	0.90	2.25	4.46	8.80	20.95	38.55	64.47
Max diff drawn in the map (mm)	0.90	0.90	0.89	0.88	0.84	0.77	0.64

Again, in all scales in group G2 and other groups the differences, drawn in the map, are not noticeable. This means again that the form of the proposed geodetic map will not differ from its corresponding projected one when print the two maps when both of them are put in coincidence, table (4-17).

4.4. AREA CALCULATION ON PROJECTED MAP AND GEODETIC DATUM

Map scales 1: 100000, 1:10000 and 1:5000 of groups G1 & G2 are chosen in area calculation on the map and on the geodetic datum, (G1 near central meridian of zone 31 and G2 near edge of zone 31, also G1 and G2 at the equator, figure (4-15).

The map area is divided into two triangles by its diagonal, the coordinates of map corners will be taken from table (4-5). The coordinates are related to WGS84 projected using UTM. From the map coordinates, map area (projected area) can be computed by applying equation (3-6) directly and it can also be computed from map distances by applying equation (3-4). **The correction of the projected distance should be applied to obtain the correct distances and areas.**

Through the ellipsoid distances and by using the equations (3-119) or (3-120) we can calculate the area of spherical triangle through the area of the corresponding plane triangle. It should be remarked here that the corresponding projected distances in the two maps at G1 (negative distortion) and G2

(positive distortion) are different while they are the same in the corresponding ellipsoidal case. This assures the need for the proposed geodetic map.

4.4.1. Area Calculation on map 1:100,000

The considered area in the map 1:100,000 in group G1 is determined by A1-A20-A21-A22 points with dimensions 30'($\Delta\phi$) x 40'($\Delta\lambda$). The area is divided in to two triangles A1-A20-A21 and A1-A21-A22. Also, the considered area in the map 1:100000 in group G2 is determined by B1-B20-B21-B22 points with dimension 30'($\Delta\phi$) x 40'($\Delta\lambda$). The area is divided into two triangles B1-B20-B21 and B1-B21-B22.

a) Area on projected map, map 1:100,000

The area of map 1: 100,000 is computed in G1 and G2; distortion in area is not the same from projected triangle sides; that is appear in table (4-18) at G1and table (4-19) at G2, the map side distances need corrections to near the value of the geodetic distances by using equations (2-7) the corrected areas will be obtained by useing corrected distances in the tables(4-18a) at G1and table (4-19a) at G2,

Table (4-18). Projected map area as two triangles (from projected distances) 1:100,000 map in G1.

Map study	2 triangles	Map distances (sides of triangle) (m)			Projected area (m^2)
1:100 000 A1A20A21A22	ΔA1A20A21	74184.994	55268.803	92507.500	2,050,057,906.829
	ΔA1A21A22	74182.188	55265.037	92507.500	2,049,840,679.639
Area of projected map (Negative distortion)					4,099,898,586.468

Table (4-18a). Corrected area of projected map as two triangles **(from corrected distances by scale factor)** 1:100,000 map in G1.

Map study	2 triangles	Corrected Map distances (sides of triangle) by scale factor (m)			Corrected area (m^2)
1:100 000 A1A20A21A22	ΔA1A20A21	74212.99383	55287.15186	92542.41588	2,051,512,529.73
	ΔA1A21A22	74210.18711	55287.15161	92542.41588	2,051,434,931.50
Corrected Area of projected map					**4,102,947,461.23**

Table (4-19). Projected map area as two triangles (from projected distances) 1:100,000 map in G2.

Map study	2 triangles	Map distances(sides of triangle) (m)			Projected area (m^2)
1:100 000 B1B20B21B22	ΔB1B20B21	74261.879	55341.390	92606.883	2,054,877,771.460
	ΔB1B21B22	74264.694	55311.205	92606.883	2,053,834,824.588
Area of projected map (positive distortion)					4,108,712,596.048

Table (4-19a). Corrected area of projected map as two triangles **(from corrected distances by scale factor)** 1:100,000 map in G2.

Map study	2 triangles	Corrected Map distances (sides of triangle) by scale factor (m)			Corrected area (m^2)
1:100 000 B1B20B21B22	ΔB1B20B21	74210.18658	55287.15274	92542.41586	2,051,434,958.79
	ΔB1B21B22	74212.99324	55287.15275	92542.41586	2,051,512,546.06
				Corrected Area of projected map	**4,102,947,504.85**

Geometrically, the areas in G1 and G2 in map 1: 100,000 are the same from geometric of ellipsoid. By comparison, the corrected values in tables(4-18a) & (4-19a) are more closed than the projected value in tables (4-18) & (4-19).

b) Area on geodetic datum, map 1:100,000

In table (4-23); the area computed as two spherical or ellipsoidal triangles using equations (3-119) or (3-120) through the area of the corresponding plane triangle. we get the matched geodetic distances in map 1:100,000 in groups (G1) & (G2) are equals, that mean the area is the same too.

Table (4-20). Area as two ellipsoidal triangles by using geodetic distances; map 1:100,000 in groups (G1) & (G2).

Area as two triangles	Ellipsoidal distances (sides of triangle) (m)			Average of Gaussian mean radii (m)	F ellipsoidal area (precise) (m^2)
ΔA1 A20 A21	74212.9938621946	55287.1520026738	92542.4160049207	6356753.394	2051548769.470
ΔA1 A21 A22	74210.1869428477	55287.1520026738	92542.4160049207	6356754.475	2051471173.036
				F Area (Ellipsoidal Area)	**4103019942.506**

Finally,

For dimension map 1: 100 000 = 30'(Δϕ) x 40'(Δλ)

a) Corrected map area from corrected distances by using scale factor equations (2-7)

Corrected Area of projected map at G1 = 4,102,947,461.23m^2

Corrected Area of projected map at G2 = 4,102,947,504.85m^2

b) Area as two ellipsoidal triangles by using geodetic distances = 4,103,019,942.506 m^2

We can chick by using equation (3-107)

Area as trapezoid on sphere by double integration method. = 4,102,974,041.049 m^2

The value computed by double integration is less than the ellipsoidal area by using geodetic distances. It is less because that area limited between two parallels and two meridians,

meridians as a great circle, parallels on line at $\phi = 0$ as great circle and another $\phi = 0° 30'$ not great circle. The distortions in projected map is clear in negative and positive distortion tables (4-18) & (4-19). the corrected values in tables(4-18a) & (4-19a) are more closed. The area on geodetic datum (Area as two ellipsoidal triangles by using geodetic distances) is the same in G1 and G2, that is logic form geometric of ellipsoid (the map as trapezoid 30'($\Delta\phi$) x 40'($\Delta\lambda$)), this area is greater than corrected area because it is curved area, see sec 3.2.5c.

4.4.2. Calculation on map 1:10,000

the considered area in the map 1:10000 in group G1 is determined by A1-A11-A12-A13 points with dimensions 3'($\Delta\phi$) x 4'($\Delta\lambda$). The area is divided in to two triangles A1-A11-A12 and A1-A12-A13. Also, the considered area in the map 1:10000 in group G2 is determined by B1-B11-B12-B13 points with dimension 3'($\Delta\phi$) x 4'($\Delta\lambda$). The area is divided into two triangles B1-B11-B12 and B1-B12-B13.

a) Area on projected map, map 1:10000

The area of map 1: 10000 is computed in G1 and G2; distortion in area is not the same from projected triangle sides not the same in value and sign negative and positive; that is clear in table (4-21) at G1 and table (4-22) at G2.

Table (4-21). Projected map area as two triangles (from projected distances) 1:10,000 map in G1.

Map study	2 triangles	Map distances (sides of triangle) (m)			Projected area (m^2)
1:10 000	ΔA1A11A12	7418.333	5526.506	9250.615	20,498,730.917
A1A11A12A13	ΔA1A12A13	7418.330	5526.502	9250.615	20,498,707.791
				Area of projected map (Negative distortion)	40,997,438.708

Table (4-21a). Corrected area of projected map as two triangles **(from corrected distances by scale factor)** 1:10,000 map in G1.

Map study	2 triangles	Corrected Map distances (sides of triangle) by scale factor (m)			Corrected area (m^2)
1:10 000	ΔA1A11A12	7421.29983	5528.71372	9254.31520	20,515,121.11
A1A11A12A13	ΔA1A12A13	7421.29683	5528.71349	9254.31520	20,515,111.94
				Corrected Area of projected map	**41,030,233.05**

Table (4-22). Projected map area as two triangles (from projected distances) 1:10,000 map in G2.

Map study	2 triangles	Map distances (sides of triangle) (m)			Projected area(m^2)
1:10 000	ΔB1B11B12	7428.351	5534.138	9263.112	20,554,759.768
B1B11B12B13	ΔB1B12B13	7428.354	5533.802	9263.112	20,553,520.106
		Area of projected map (positive distortion)			41,108,279.874

Table (4-22a). Corrected area of projected map as two triangles **(from corrected distances by scale factor)** 1:10,000 map in G2.

Map study	2 triangles	Corrected Map distances (sides of triangle) by scale factor (m)			Corrected area(m^2)
1:10 000	ΔB1B11B12	7421.29665	5528.71404	9254.31558	20,515,113.517
B1B11B12B13	ΔB1B12B13	7421.29964	5528.71400	9254.31558	20,515,121.581
		Corrected Area of projected map			**41,030,235.098**

Geometrically, the areas in G1 and G2 in map 1: 10,000 are the same. By comparison, the corrected values in tables(4-21a) & (4-22a) are more closed than the projected value in tables (4-21) & (4-22).

b) Area on geodetic datum, map 1:10,000

In table (4-23) the computed area of trapezoid as two spherical triangles using equation (3-120) through the area of the corresponding plane triangle; map 1:10000 in groups (G1) & (G2). Again, from figures (4-34) & (4-35) we get the matched geodetic distances in map 1:100000 in groups (G1) & (G2) are equals, that mean the area is the same.

Table (4-23). Area of trapezoid as two ellipsoidal triangles by using geodetic distances; map 1:10,000 in groups (G1) & (G2).

Area as two triangles	Ellipsoidal distances (sides of triangle)			Average of Gaussian mean radii (m)	F area ellipsoidal (precise) (m^2)
ΔA1A11A12	7421.2993862195	5528.7138050949	9254.3150492781	6356752.325	20515123.8075
ΔA1A12A13	7421.2965793151	5528.7138050949	9254.3150492781	6356752.336	20515116.0482
				F Area (Ellipsoid area)	**41,030,239.856**

Finally

For dimensions of the map 1: 10000 3'(Δϕ) x 4'(Δλ)

a) Corrected Area of projected map at G1 = 41,030,233.05m^2

Corrected Area of projected map at G2 = 41,030,235.098m^2

b) Area as two ellipsoidal triangles by using geodetic distances **= 41,030,239.856 m^2**

We can chick by using equation (3-107)

Area as trapezoid on sphere by double integration method. = 41,030,237.454 m^2

Also, the value computed by double integration is less than the ellipsoidal area by using geodetic distances. the corrected values in tables(4-21a) & (4-22a) are more closed. The ellipsoidal area is greater than corrected area because the ellipsoidal area is curved area.

4.4.3. Calculation on map 1:5000

The considered area in the map 1:5000 in group G1 is determined by A1-A8-A9-A10 points with dimensions 1' 30" (Δϕ) x 2'(Δλ). The area is divided in to two triangles A1-A8-A9 and A1-A9-A10. Also; the considered area in the map 1:5000 in group G2 is determined by B1-B8-B9-B10 points with dimensions 1' 30" (Δϕ) x 2'(Δλ). The area is divided into two triangles B1-B8-B92 and B1-B9-B10.

a) Area on projected map, map 1:5000

The area of map 1: 5000 is computed in G1 and G2; distortion in area is not the same from projected triangle sides; that is clear in table (4 - 24) and table (4 - 25).

Table (4-24). Projected map area as two triangles (from projected distances) 1:5,000 map in G1.

Map study	2 triangles	Map distances (sides of triangle) (m)			Projected area(m^2)
1:5 000	ΔA1A8A9	3709.166	2763.252	4625.307	5,124,678.889
A1A8A9A10	ΔA1A9A10	3709.165	2763.251	4625.307	5,124,677.733
				Area of projected map (negative distortion)	10,249,356.623

Table (4-24a). Corrected area of projected map as two triangles **(from Corrected Distances by scale factor)** 1:5,000 map in G1.

Map study	2 triangles	Corrected Map distances (sides of triangle) (m)			Corrected area(m^2)
1:5 000	ΔA1A8A9	3710.65005	2764.35727	4627.15779	5,128,781.224
A1A8A9A10	ΔA1A9A10	3710.64905	2764.356743	4627.15779	5,128,778.859
				Corrected Area of projected map	**10,257,560.083**

Table (4-25). Projected map area as two triangles (from projected distances) 1:5,000 map in G2.

Map study	2 triangles	Map distances sides of triangle (m)			Projected area(m^2)
1:5 000	ΔB1B8B9	3714.233	2767.069	4631.627	5,138,769.358
B1B8B9B10	ΔB1B9B10	3714.233	2766.984	4631.627	5,138,612.799
		Area of projected map (positive distortion)			10,277,382.157

Table (4-25a). Corrected area of projected map as two triangles **(from corrected distances by scale factor)** 1:5,000 map in G2.

Map study	2 triangles	Corrected Map distances (sides of triangle) (m)			Corrected area(m^2)
1:5 000	ΔB1B8B9	3710.64945	2764.35702	4627.15793	5,128,779.931
B1B8B9B10	ΔB1B9B10	3710.64945	2764.35648	4627.15793	5,128,778.931
		Corrected Area of projected map			**10,257,558.861**

Also, the projected areas in G1 and G2 in map 1: 5,000 is the same geometrically. the corrected values in tables(4-24a) & (4-25a) are more closed than the projected value in tables (4-24) & (4-25).

b) Area on geodetic datum, map 1: 5000

In table (4-33) the computation area of trapezoid as two spherical triangles using equations (3-78) & (3-79) through the area of the corresponding plane triangle; map 1:5000 in groups (G1) & (G2)

Table (4-26). Area as two ellipsoidal triangles by using geodetic distances; map 1:5,000 in groups (G1) & (G2).

Area as two triangles	Ellipsoidal distances (sides of triangle)			Average of Gaussian mean radii (m)	F area ellipsoidal (precise) (m^2)
ΔA1A8A9	3710.6496931098	2764.3568972626	4627.1578028489	6356752.317	5128780.263
ΔA1A9A10	3710.6493422467	2764.3568972626	4627.1578028489	6356752.320	5128779.777
				F Area (Ellipsoid area)	**10257560.040**

Finally

For dimension map 1: 5 000 1' 30 (Δϕ) x 2' (Δλ)

Corrected Area of projected map at G1 = 10,257,560.083m^2
Corrected Area of projected map at G2 = 10,257,558.861m^2

b) Area as two ellipsoidal triangles by using geodetic distances = **10,257,560.040** m^2

We can chick by using equation (3-107),

(Area as trapezoid on sphere by double integration method) = 10,257,559.759 m^2

Again, the distortions in projected map are clear in negative and positive distortions; In our Program we use for calculation area on datum by equation (3-120) in complete form.

In table (4-27) the projected areas in different scale between group (1) & group (2), in the global case are bounded by the same $\Delta\phi$, $\Delta\lambda$ on datum. The areas from geometric of ellipsoid are equal in the same equivalent map scale group (1) and group (2) but the distortion makes this difference, the corrected area is listed in Table (4-27a).

Table (4-27). Projected Areas in some different scales between group (1) & group (2), in the global case.

Map Scale	Projected area at equator		Area difference.
	G1 (mt^2)	G2(mt^2)	Area G2 - Area G1(mt^2)
1:100,000	4,099,898,604.52	4,108,712,592.54	8,813,988.03
1:10,000	40,997,438.27	41,108,278.08	110,839.81
1:5,000	10,249,356.64	10,277,382.21	28,025.57

Table (4-27a). Corrected projected Areas in some different scales between group (1) & group (2), in the global case.

Map Scale	Corrected projected area at equator		Area difference.
	G1 (mt^2)	G2(mt^2)	Area G2 - Area G1(mt^2)
1:100,000	4102947461.2323	4102947504.8549	43.6227
1:10,000	41030233.0469	41030235.0975	2.0506
1:5,000	10257560.0833	10257558.8611	1.2222

In table (4-28) areas on datum in different scale between group (1) & group (2), in the global case. Geometrically, the two corresponding areas (G1) and (G1) in the same scale are equal on the ellipsoid, it is computed by The Automatic Real Map software.

Table (4-28). Output Areas from The Automatic Real Map Program in different scale between group (1) & group (2), in the global case.

Map Scale	Area on datum at equator		Area difference.
	G1 (mt^2)	G2(mt^2)	Area G2 - Area G1(mt^2)
1:100,000	**4,103,019,942.5064**	**4,103,019,942.5054**	0.001
1: 50,000	**1,025,755,751.5389**	**1,025,755,751.5383**	0.000
1:25,000	**256,438,985.7104**	**256,438,985.7102**	0.000
1:10,000	**41,030,239.8558**	**4,1030,239.8557**	0.000
1:5,000	**10,257,560.0404**	**1,0257,560.0404**	0.000
1:2,500	**2,564,390.0149**	**2,564,390.0149**	0.000
1:1,000	**410,302.4026**	**410,302.4026**	0.000

4.5. GEODETIC TOTAL STATION(GTS)

Total station (TS) or any traditional surveying tools are observed on the earth's surface, and are not distorted observation. TS computes on plane geometry but it is observed on curved surface, in Geodetic Total Station (GTS), the control points start from geodetic coordinates and geodetic computation, the geodetic computations could be explained as the following:

1. The geodetic work will be done for the control points traditionally or by using GPS to produce a geodetic coordinate (φ, λ, h).
2. GTS will be occupied a control point and the geodetic values of that point $(\varphi, \lambda, h)_1$, will be entered to the GTS, the values of ξ and η (deflection of the vertical components) and geoidal undulation (N), at the base Point are given to the GTS or by interpolating their values from a geoid model file which is attached to the GTS. The calculations are done by using equations which will be applied to obtain the geodetic coordinates (φ, λ), The value of ellipsoidal height (h) will be calculated from interpolation as in the geoid model by knowing the value of geoidal undulation of some points of known geodetic.

   ellipsoidal height (h)= Orthometric height(H) + Geoidal undulation (N) (5-7)

3. GTS will be oriented at a second point (back sight point) and its geodetic coordinates $(\varphi, \lambda, h)_2$ will be entered to the GTS.
4. The orientation program in the GTS will be computed the geodetic distance (s_{12}) and geodetic azimuth (α_{12}) by using the inverse geodetic problem.

After the orientation steps the user of GTS have to make his choices between the following:

I. Measuring mode: to get a coordinate of a new surveyed point.

II. Stake out mode: to find the location of a certain point of known coordinates.

Before applying any mode, the observations will be reduced to the reference ellipsoid, the reduction for spatial distances, horizontal directions, horizontal angles and vertical angles will be done by geoid data (ξ, η and N) if it is available. unless the geoid data is available, we can use the observation without reductions like pressing in traditional TS, [Mahmoud, S. M., 2004].

4.6. MAP WITHOUT PROJECTION

Surveying engineering nowadays could be generally divided into modern (satellite based) and tradition ways. In modern (satellite based) way, the required geodetic coordinates are obtained

directly related to the geodetic datum. In the surveying traditional way, the required geodetic coordinates cannot be directly observed. They are obtained by computing them from taken traditional observations.

The traditional observations are distances, vertical angles, and horizontal angles. Those observations are taken related to the direction of actual gravity, while the geodetic computations will be carried out on the surface of the adopted reference ellipsoid (direction of normal gravity). Thus, fictitious observations related to the direction of the normal to the ellipsoid should be obtained from the taken observations. It is therefore convenient to reduce the taken observations to the used reference ellipsoid. The new observations after reduction can then be used to calculate the geodetic coordinates (ϕ, λ).

Map projection is used to transform the obtained geodetic coordinates into plan (map) coordinates. In map projection process, distortion in distance, azimuth, area, or shape must happen. It is difficult to the user and not convenient to the specialist to deal with this distortion.

we can tell the map projection is divide to calculation for transform the obtained geodetic coordinates into plan (map) coordinates and operation of drawn map soft or hard copies. In sec 4.2. & 4.3 we approved there is no difference in hard copy in all surveying map scales to 1: 100000 is not noticeable, see table (4-16) and (4-17).

In soft copy the distortion is noticeable from scale 1 :1000 to up, 1 km effect by decimeters. if we represent the map directly by difference latitude and longitude with respect to choose map base point without projection the map can then be produced as soft copy and we can print hard copy too.

In the past, computing and drawing the maps were manually done. Nowadays, computations and map production are automatically done by using electronic computers. Therefore, it is the time now to draw the map using the geodetic coordinates directly and avoiding the noisy distortion. The proposed map will be computerized soft copy one and could be plotted whenever needed.

Parallels and meridians will be the background of the proposed map. Points will be represented by their geodetic coordinates (ϕ, λ) through choose base map point in south west corner, any place in map or out of map. The needed surveying elements, (distances, azimuths, and areas), will be obtained by computing them using subroutine programs. Those subroutines will be part of the proposed electronic map. Just push button (hot keys) to obtain the needed element.

In the same datum, any point on the earth has unique geodetic value of coordinates latitude and longitude (ϕ, λ). In projection systems like UTM (universal) and ETM (national); the same point lying at the border between two zones like longitude 33^{o}E in ETM (between red and blue

zones) and also longitude12^{o}E in UTM (between zones 32 and 33) has two different pairs of coordinates. Pair of (E, N) from the first zone and another different pair (E, N) from the second adjacent zone will be obtained. The same values of (E, N) are repeating in the sixty zones of UTM. In large projects like petroleum pipe lines and international roads, when the project is located in two zones, a problem happens. One project should belong to one coordinate system but the projection makes it in two different zones or systems of coordinates. The followed solution is to relate the whole project to one zone or system of coordinates despite the resulting great value of distortion.

Surveying elements (distances, azimuths, areas) from the proposed geodetic maps do not involve scale distortion. The shape of the features in the proposed map will not differ from the corresponding features in the projected map. For example, in table (3-4), the lineN1-N19 of 60000m ellipsoidal distance has 60019.879 projected distance in the map of 1:100000. The difference between the two values in the map is approximately (20/100000 m) i.e. 0.2mm which cannot even measure by a ruler.

The proposed automatic real map is digital map presented by Parallels and Meridians directly; these maps depend on digital form in basic entities point, line and arc in tables form by latitude and longitude coordinates. Calculation of distances, azimuths, and areas will be done using the appropriate geodetic equations by hot keys ad joint to the map; these points are known in geodetic datum like WGS84 and EGD30. The map could be plotted when a hard copy is needed.

4.6.1. STEPS OF AUTOMATIC REAL MAP PRODUCTION

In the case that the computations will be done on the adopted reference ellipsoid in two-dimension calculations. Hence, the results will be point coordinates in the geodetic 2D form direct geodetic problem.

$$\phi_2, \lambda_2, \alpha_{21} = f\,(\phi_1, \lambda_1, \alpha_{12}, S_{12}) \qquad (4\text{-}1)$$

Also, in the case that the computations will be done in three-dimension calculations, the local horizon system coordinates (u, v, w) will be first obtained as:

$$u, v, w = f\,(s_{12}, \alpha_{12}, z_{12}) \qquad (4\text{-}2)$$

or

$$u = s \sin z \cos \alpha,$$
$$v = S \sin z \sin \alpha, \qquad (4\text{-}3)$$
$$w = s \cos z$$

where α is the geodetic azimuth

z is geodetic zenith distances

s is spatial distances for the observed line

We have two local systems one with respect to plumb line, its name is astronomic local horizon system and the other with normal, its name is geodetic local horizon system. (u, v, w) is local system tied to the occupied station. we need geoid model to get the geodetic local horizon system. The problem of geoid data is still found when used of projected coordinates (E, N) and it is ignored as well if the geoid data is not available.

The relation between the local Cartesian system and Geodetic Cartesian system takes the form:

$$\begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix} = \begin{vmatrix} -\sin\phi\cos\lambda & -\sin\lambda\cos\phi & \cos\lambda \\ -\sin\phi\sin\lambda & \cos\lambda\cos\phi & \sin\lambda \\ \cos\phi & 0 & \sin\phi \end{vmatrix} * \begin{vmatrix} u \\ v \\ w \end{vmatrix} \qquad (4\text{-}4)$$

[Nassar, 1994 and Shaker, 1982]

And

$$\begin{aligned} X_2 &= X_1 + \Delta X_{12} \\ Y_2 &= Y_1 + \Delta Y_{12} \\ Z_2 &= Z_1 + \Delta Z_{12} \end{aligned} \qquad (4\text{-}5)$$

Now curve-linear coordinates could be computed from the obtained rectangular coordinates:

$(\phi, \lambda, h) = f(X, Y, Z, a, b)$ Call sec 3.2.4

Also, we can obtain the geodetic coordinates from GPS and proposal geodetic total station (GTS)

After obtaining the geodetic coordinates (Ø, λ) for all the project points, the map can be drowned and stored in its digital form, it can be also plotted when needed.

The two axes of the map are chosen at the south-west corner of the map or at any points in the map. Then the difference of latitude and longitude between the concerned point and the corner of the map is defined. The shape of the proposed map does not differ from the corresponding map drawn by projected coordinates (E, N). This is explained in sections 4.2 and 4.3.

All the above-mentioned equations used in computations are programmed and ad joint to the map as an essential part of it. Any needed information can be obtained from the proposed automatic real map using hot keys (push button). The required information will be obtained directly from the geodetic coordinates and the projection distortion will be totally avoided.

4.7. The DESCRIPTION AND FACILITIES OF THE DESIGNED PROGRAM

The program for producing Automatic Real Map is created by Visual basic for application & third-party component, this is available in some programs like AutoCAD and Microsoft office. In AutoCAD or Civil3D, to draw the map using latitude and longitude is possible;

1. The map is recorded as points in Microsoft excel tables in which form latitude and longitude.
2. The map data can be imported from total station and GPS as points, lines, polylines and arcs

which are connecting between these points.

3. The points are recorded by actual latitudes and longitudes.
4. Base point (map corner or any point) is specified to calculate the differences in latitude and longitude between that base point and all other points.
5. Latitude and longitude differences are computed in meter units using suitable geodetic equations.
6. Then all points are represented and connected to each other by lines and polygons if needed.
7. The line between any two points can be drawn and then selected and using certain program keys to get its azimuth and distance.
8. All properties of any line (geodetic distance, azimuth, rectangular and geodetic coordinates for its two terminal points, difference in latitude and longitude, difference in rectangular coordinates also spatial distance) can be obtained once pushing the specified key.
9. Any point can be selected and using point properties key, point properties (geodetic and rectangle coordinates, orthometric and ellipsoidal heights) can be obtained if ζ (Meridional component of the deflection of the vertical), η (Prime vertical component of the deflection of the vertical) and N (Prime vertical component of the deflection of the vertical) are available and stored in the program.
10. A polyline between 3 points can be drawn as triangle; then it is selected by specified key to compute the ellipsoidal area and also the geodetic circumference.
11. The closed polyline between several points can be drawn and then selected. The enclosed ellipsoidal area can be computed using the specified area key; also the geodetic circumference can be obtained.
12. The user can add new point to the map by;
    - Free hand by click insertion
    - Rectangular coordinates X, Y, Z.
    - Geodetic Latitude and geodetic longitude
    - Geodetic distance and geodetic azimuth from chosen point
    - Spatial distance and geodetic azimuth from chosen point
    - Latitude and longitude differences from chosen point

5. MAP INDEX

Map index is a type of finding aid that allows users to find a series of maps covering their regions of interest along with the name or number of the linked map sheet. A map index provides geographical spatial representation on either a computer screen or a paper sheet. In this way, a map acts as a kind of gazetteer, with the position represented within a grid overlaying the maps surface.

Information is being searched by geodetic or map coordinates, rather than the metadata for any country. Additionally, in various authorities and institutions, maps are cataloged individually or as series, resulting in several levels of specificity. A map index provides detailed information about a series of sheets. It is a graphic key that provides information about coverage, availability, and any additional information available, [htyhtps://en.wikipedia.org/wiki/Index_map].

5.1 MAP INDEX IN EGYPT

5.1.1 Quadrant System

This system used in the first cadaster (1898) and based upon the geographical coordinates with respect to special origin. Egypt was divided into 4 quadrants formed by the intersection of the longitude 31°E and latitude 30°N. The 4 quadrants are North East [N.E.], North West [N.W.], South East [S.E.] and South West [S.W.].

In such a system an area 1° x 1° is divided into 20 maps of scale 1: 50,000 whose dimensions are 15' longitude x 12' latitude. The 1:50,000 map , in turn , covers an area of 25 maps to scale 1:10,000 whose dimensions are 3' longitude x 2' 24'' latitude (15'/5 and 12' /5). The 1:10,000 map is divided into 16 maps of scale 1:2,500 of dimensions 45'' longitude x 36'' latitude (3'/4 and 2' 24''/4).

The numbering of the 1:50,000 map has been cancelled and concentration was given to both the 1:10,000 and 1:2,500 maps. Numbering for the 1:10,000 maps starts from the origin giving the numerals 1, 2, 3 ….both eastwards and westwards of longitude 31° E and to the north and to the south of latitude 30° N, figure (5-1). The designation for the 1:10,000 maps comprises of two numerals followed by the quadrant in which such map lies. The first numerals refers to the horizontal coordinates of the south-western corner of the map 1:10,000 and the second letter denoted the vertical coordinate of the same corner, figure (5-1), [Behairy, A. M., 1987, p105 to p111].

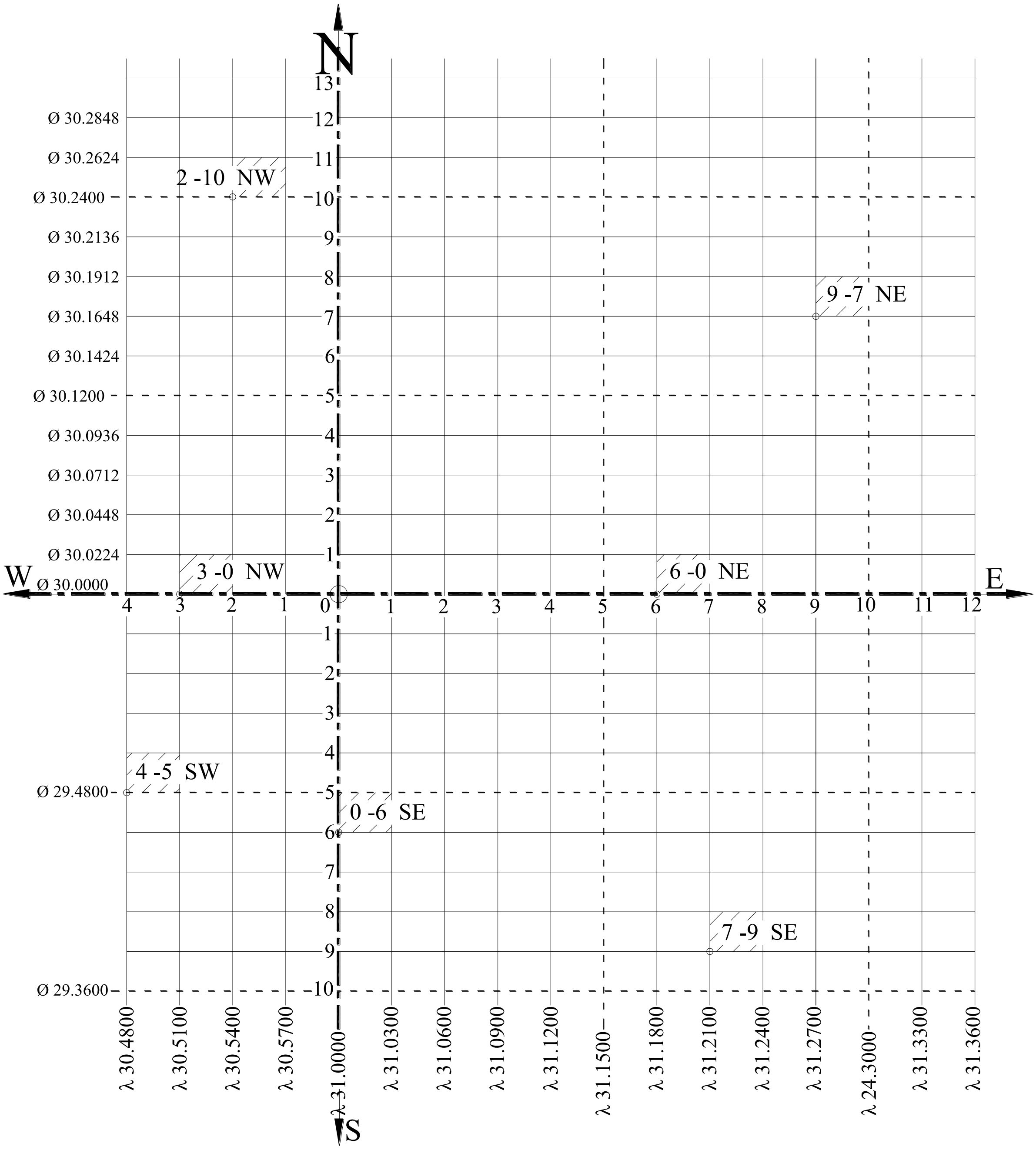


Figure (5 -1): Map index of the 1:10,000 in quadrant system.

As has been mentioned that, the map 1:10,000 is divided into 16 maps of scales 1: 2500 figure (5 - 2). The designation of the 1:2,500 maps is given as the number (from 1 to 16) of the map preceded by the two numerals of the 1:10,000 map and then followed by its quadrant. For example if we have the 1:10,000 designation is 54-18 NE and that the 2500 map is number 7 in the 1: 10,000 map then its designation will be 54-18-7 N E.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Figure (5-2): sub divisions of 1: 10,000 map (16 maps of 1:2500).

5.1.2 Kilometric System

This system is used in recent maps. In such a system each point is defined by the rectangular coordinates (x, y), x is the vertical coordinate and y is the horizontal coordinate, related to a chosen origin. In order to get Egypt in one quadrant to overcome any ambiguity, the origin has been chosen as the point of intersection of longitude 25˚E and latitude 22˚N, table (5-1) represents the different scales used in this system.

Table (5-1). Specifications of different scales which are used in Kilometric system.

Scale	Map dimensions	Dim. of the covered area	Map index in fraction form		Kind of map
			Numerator	Denominator	
1:100,000	60 cm x 40 cm	60 km x 40 km	Tens of kilometers	Tens of kilometers	Topographic
1:25,000	60 cm x 40 cm	15 km x 10 km	Tens of kilometers	Kilometers	Topographic
1:10,000	60 cm x 40 cm	6 km x 4 km	Kilometers	Kilometers	Agricultural
1:5,000	60 cm x 40 cm	3 km x 2 km	Kilometers	Kilometers	Agricultural
1:2500	60 cm x 40 cm	1.5 km x 1 km	Kilometers	Kilometers	Agricultural
1:1000	60 cm x 40 cm	0.6 km x 0 .4 km	Kilometers	Kilometers	Cities

The map in this system is represented by the coordinates of the south western corner of the map in the form of a fraction in which its numerator is the vertical x coordinate and denominator is the horizontal y coordinate.

a) 1 : 100,000 maps

E.g. 16/24 Egypt

This means that the south western corner is at a 160 km and 240 km distances from both the latitude 22° N and the longitude 25° E respectively. It is the convention to omit one zero from both the numerator and denominator, figure (5-3), [Behairy, 1987, p111 to p114].

20/18	**20/24**	**20/30**
16/18	**16/24**	**16/30**
12/18	**12/24**	**12/30**

Figure (5-3): Map index in the 1:100,000 by kilometric system.

b) 1: 25,000 maps

E.g. 80/ 300 Cairo

It is the same as mentioned for the 1:100,000 but here only one zero from the numerator is cancelled, figure (5-4).

81/285	**81/300**	**81/315**
80/285	**80/300**	**80/315**
79/285	**79/300**	**79/315**

Figure (5-4): Map index in the 1:25,000 by kilometric system.

c) 1: 2500 maps

E.g. 818/ 625.5, figure (5-5).

819/624	**819/625.5**	**819/627**
818/624	**818/625.5**	**818/627**
817/624	**817/625.5**	**817/627**

Figure (5-5): Map index in the 1:2500 by kilometric system.

d) 1:1000 maps

E.g. 47/ 48.6, figure (5-6).

47.4/48	**47.4/48.6**	**47.4/49.2**
47/48	**47/48.6**	**47/49.2**
46.6/48	**46.6/48.6**	**46.6/49.2**

Figure (5-6): Map index in the 1:1000 by kilometric system.

5.1.3 The Millionth Map (application in Egypt)

The bases for a reference of maps of different scales in Egypt are provided by the National Million Map 1: 1,000,000. Such maps use unified symbols, conventional signs and colors across the world.

The contour interval agreed upon is 100 m. each country is free to choose its own local scales other than the 1: 1,000,000.

The sheets of this map are arranged as follows. The surface of the earth is divided into bands (rows) with parallels spaced at intervals 4° of latitude from the equator and into columns with meridians drawn at intervals of 6° longitude. Thus 60 columns are obtained. The bands are lettered A, B, C… from the Equator to the north and south poles. The columns are numbered with numerals 1, 2, 3…anti-clockwise and starting the 180°west meridian, figure (5-7).

The location of any point on the surface of the earth can be determined by its geographical coordinates; longitude (λ) and latitude (Ø). The longitude is either east or west of the longitude zero (prime longitude) which passes by Greenwich (England), and the latitude is either to the north or the south of the latitude zero which is the Equator.

To obtain the reference of maps on larger scales, it is usual to divide each sheet of the 1:1,000,000 map. Each part then has its own reference. What follows is a system of subdivision for the NH36 and this system is adopted in Egypt, [Behairy, 1987 p84 to p91].

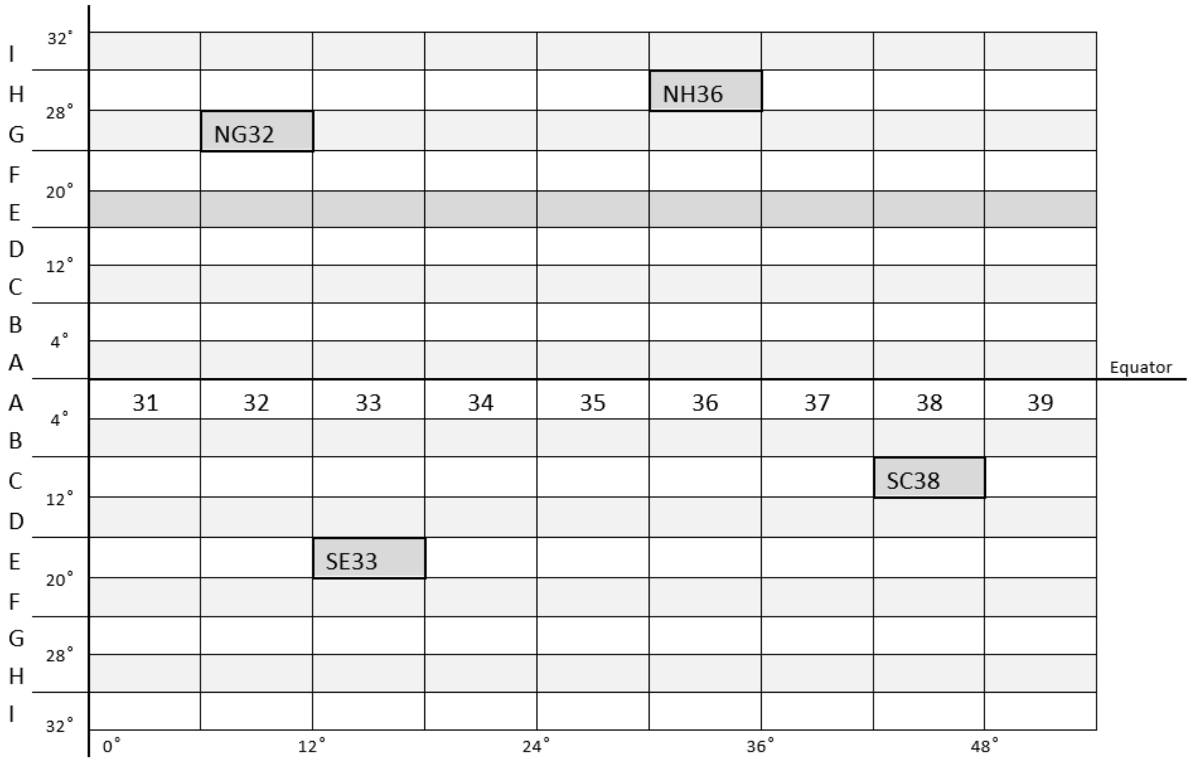


Figure (5-7): International millionth map system.

- Millionth map is divided to 4 maps of scale 1: 500000 NW, NE, SW ,SE with dimensions 3°x 2°; the corner name to millionth map name is added as in figure (5-8),
- Also Millionth map is divided to 16 maps of scale 1: 250 000 A, B, C… P with dimensions 1° 30' x 1°; the name as number from A to P is added to millionth map name as in figure (5-9) .
- To get 1:100 000 map, 1:250 000 map is divided to 6 maps from 1 to 6 with dimensions 30'x30'; the number 1 to 6 is added to 1:250 000 map name as in figure (5-10).
- To get 1:50 000 map, 1:100 000 map is divided to 4 maps a, b, c, d with dimensions 15'x15'; one from those letters is added to 1:100 000 map name as on figure (5-11).
- To get 1:25 000 map, 1: 50 000 map is divided to 4 maps 1, 2, 3, 4 with dimensions 7.5' x 7.5'; one from those numbers is added to 1:50 000 map name as in figure (5-12), table (5-2) illustrates different dimensions and scales in millionth map system, [Maghraby S., 2008].

NW	NE
SW	SE

NH36 NE 1:500 000 map

3°x 2°

Figure (5-8): Map index 1:500 000 based on millionth map.

M	N	O	P
I	J	K	L
E	F	G	H
A	B	C	D

NH36 - J 1:250 000 map

1° 30' x 1°

Figure (5-9): Map index 1:250 000 based on millionth map.

4	5	6
1	2	3

NH36 - J1 1:100 000 map

30' x 30'

Figure (5-10) : Map index 1:100 000 based on 1: 250 000 map.

C	D
A	B

NH36 - J1A 1:50 000 map

15' x15'

Figure (5-11): Map index 1:50 000 based on 1: 100 000 map.

3	4
1	2

NH36 - J1a4 1:25 000 map
7' 30" x 7' 30"

Figure (5-12): Map index 1:25 000 based on 1: 50 000 map.

Table (5-2). different dimensions and scales in millionth map system.

Ex.	($\Delta\lambda$)	($\Delta\varnothing$)	Scale
NH36	6°	4°	1: 1000 000
NH36 NE	3°	2°	1: 500 000
NH36 -J	1° 30'	1°	1: 250 000
NH36 -J1	30'	30'	1: 100 000
NH36 -J1a	15'	15'	1: 50 000
NH36 -J1a4	7' 30"	7' 30"	1:25 000

5.1.4 Ortho photo system

It is based on 1:50 000 in millionth map index with dimensions 15'x15'; see figure (5-11); it is divided to 6 divisions 1 to 6 number with 7' 30" x 5'; each division is subdivided to 4 maps a, b, c, d with 3' 45" x 2' 30" as scale map 1:10000, [Maghraby S., 2008].

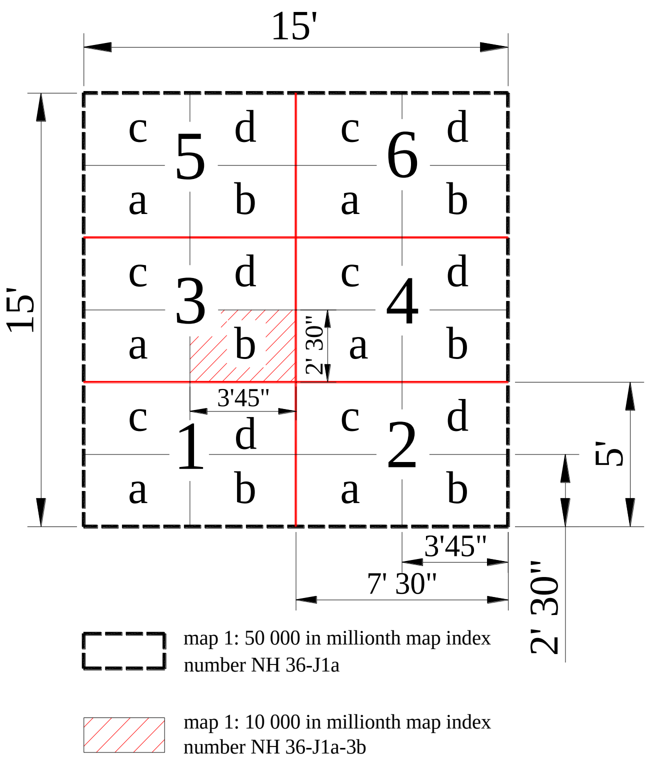


Figure (5-13): Map index in ortho-photo system.

5.2 Geographic system

This system is simple, quick to use, has consistent world-wide application. It is based on the millionth map but each country arbitrarily chooses its own scales other than the 1:1000 000 as to fulfill its own purposes.

(a) The geographic system divides the globe into quadrangle of dimension 15° longitude by 15° latitude. The equator is divided into 24 divisions (360° /15°) and the latitude is divided into 12 divisions (180/15). This code divides the earth's surface into 24 x 12 = 288 as 15° quadrangles, each of which is arranged by two letters, The first letter points to longitude and the second to latitude. The notations start from the south-western corner of the arrangement with A to Z inclusive without I and O eastwards from A to M inclusive without I northwards from the South Pole to north pole for latitude, figure (5-14)

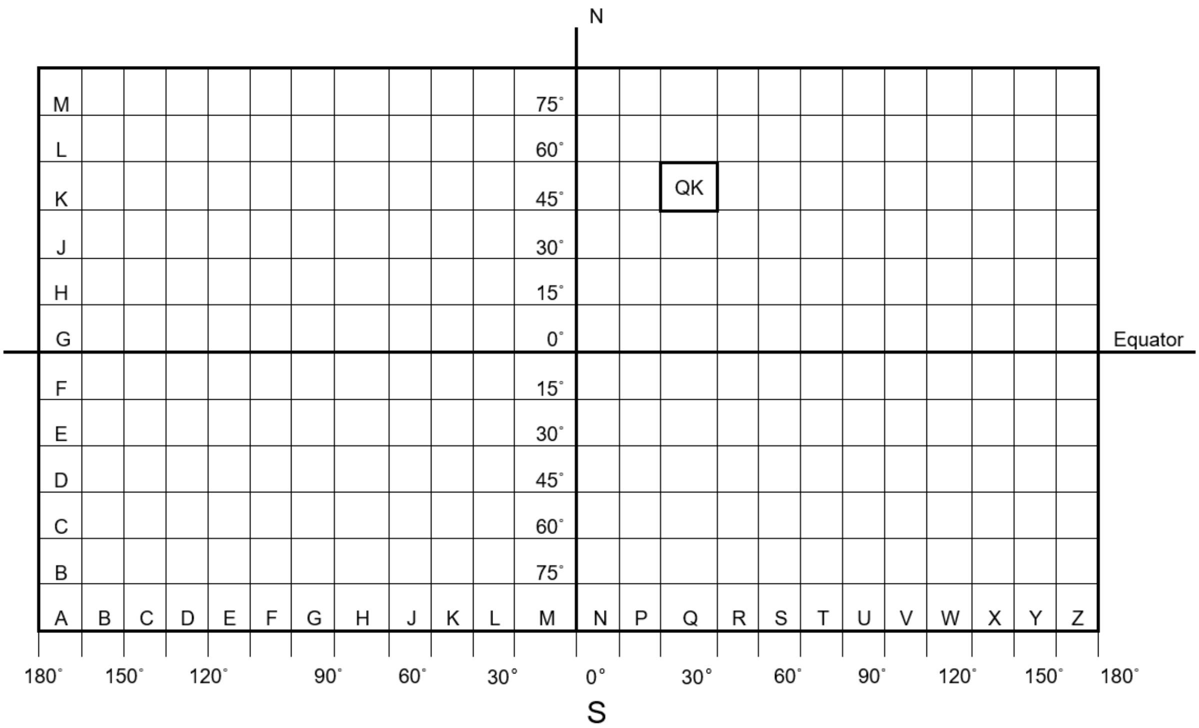


Figure (5-14): Geographical system arrangement.

(b) Each 15°x 15°quadrangle is subdivided into 15 one-degree zones of longitude, eastwards from the western side of the quadrangle geographic map. This one-degree units being named from A to Q inclusive without I and O. Each 15° x 15°quadrangle is also subdivided into 15 one-degree bands of latitude northwards from the southern side of the quadrangle; also, this bands being lettered from (A to Q) inclusive without I and O. A 1° x 1° quadrangle geographic map anywhere on the globe may now be identified by 4 letters; the first 2 letters being the reference of 15° x 15°quadrangle,

the third letter being that of the one-degree longitude zone and the forth letter of the one-degree latitude; figure (5-15)

(c) Each 1° x 1° quadrangle is divided into 6 x 6 = 36 divisions each division is 10' x 10' (figure 5-16.) the south western corner is the start of the numbering which increases eastward for longitude and northward for latitude. The direction of numbering is used wherever the 1° x 1° quadrangle map is unique position, i.e. it does not disturbance even though the position may be west of the Greenwich meridian or the south hemisphere. A unique reference defining the location of a map can now be given by using 4 letters and 4 numerals.

The name starts by defining the 15°x 15° quadrangle. In figure (5-14) we choose (QK) quadrangle; it is bounded by the quadrangle included between 30° E & 45 °E and 45°N & 60°N. Then regarding the 1°x 1° quadrangle (FG); we find the position is included between 35°E & 36° E and 51°N & 52 °N as shown in figure (5-15). We then locate its final position on the 10'x 10' quadrangle, figure (5-16) and the four numerals are preceded by four letters. Thus the map designation is (QK FG 25' 15'), [Behairy, 1987 p99 to p 105]

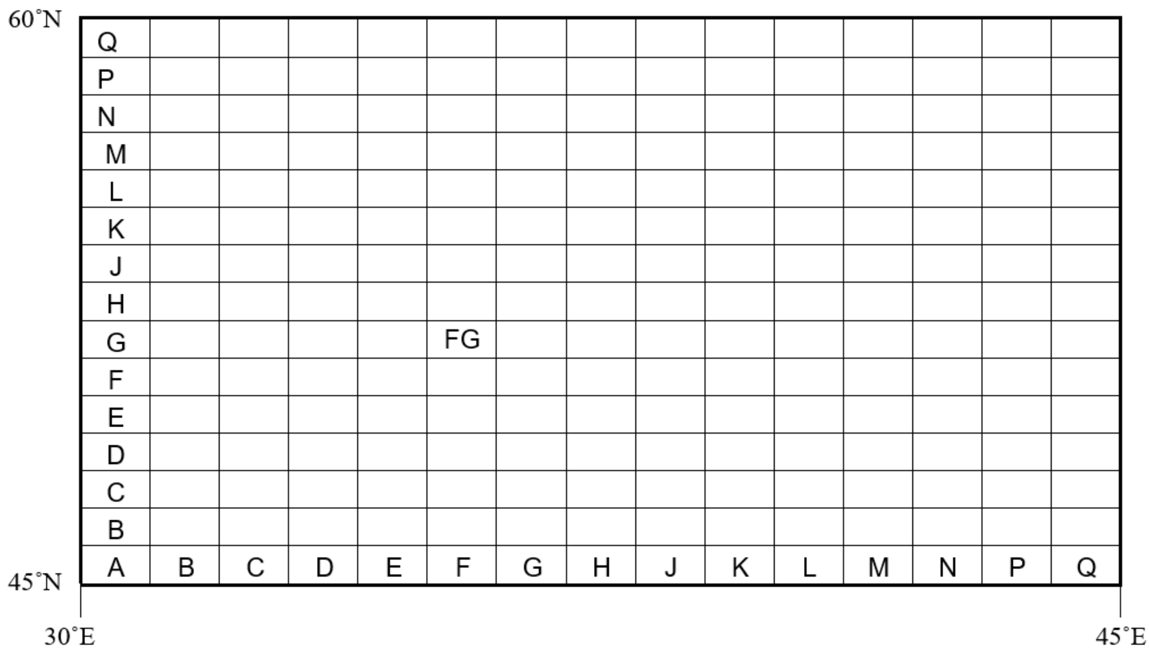


Figure (5-15): Subdivisions of 15° x 15° quadrangle into 1° x 1° quadrangles.

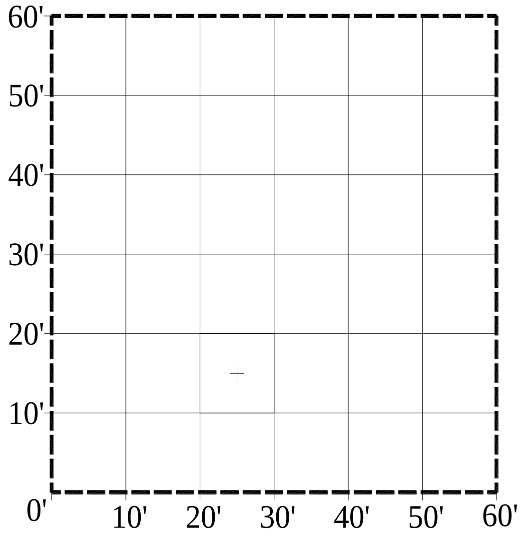


Figure (5-16): Subdivisions of 1˚ x 1˚ quadrangle into 10' x 10' quadrangles.

5.3 PROPOSED MAP INDEXES

In order to draw a map, some factors should be regarded:

- The accepted paper size for dealing and trading
- The dimensions of the mapped area
- The required drawing scale

Recalling that, the projected map does not represent the reality because of the well known distortion. Every country, in the old system of projection, has its own system beside that most of the countries are divided into different zones. Data (projected coordinates) from different countries or inside the same country but in different zones cannot be used (collected) together. The same conclusion can be drawn on the Universal Transverse Mercator (UTM).

Nowadays, universal surveying field tools like satellite positioning missions (GNSS), satellite imagery, and satellite gravity missions are widely used. The produced coordinates and coordinates based services are related to a worldwide geodetic datum like WGS84. So, the field tools of collecting data became global and the reference geodetic datums became global too but the mapping system not yet.

This research has two main objectives, the first one is proposing a real geodetic map in an electronic computerized copy and the second objective is introducing a universal map index. The first objective is explained previously in separate chapter. In this part of the research the second objective will be investigated and proposed. So, in this research, a map index is introduced in addition to the proposed geodetic real map to work together as universal mapping system.

The proposed universal mapping system, and unlike the old system, will enable:

- Collecting the maps of one country together
- Collecting the maps from different neighboring countries together
- Using surveying (geodetic) data wherever on the globe in one system without transformation
- Computing distances, azimuths, and areas between any points on the globe without distortion
- The map scale will not affect the accuracy of the extracted elements from the map (distance, azimuth, and area). They will be calculated from the geodetic coordinates with their observed accuracies.

5.3.1 Universal Map Index Proposal

Firstly, a map of smallest surveying scale was defined i.e. 1:100,000 scale map. This map could geodetically either cover 30'(ΔØ) x 40'(Δλ) for the whole globe. The map 1:100,000 covering dimensions of 30'(ΔØ) x 40'(Δλ) could be chosen. By applying the inverse geodetic problem, the corresponding metric values at the equator are 55288 m x 74214 m (by apply inverse geodetic problem) that needs printing paper dimensions of about 55 cm x 75 cm. Also, the corresponding values at latitude 30˚ are 55429 m x 64325 m that needs printing paper dimensions of about 55 cm x 65 cm. and the corresponding values at latitude 60˚ are 55709 m x 37200 m that needs printing paper dimensions of about 56 cm x 37 cm.

The convergence of meridians near the poles causes a problem that one of the dimensions of the map where 30′ or 40' as (Δλ) becomes metrically small (covers small distance). This may restrict the proposal to some high latitude as it will come later.

In table (5-3), starting from 1:100,000 map which covers 30'(ΔØ) x 40'(Δλ), map dimensions at equator, latitude 30˚ and latitude 60˚ in different surveying scales 1:100,000 - 1:50,000 - 1:25,000 - 1:10,000 - 1:5000 - 1:2500 - 1:1000 and 1:500 are illustrated.

The dimensions 30'(ΔØ) x 40'(Δλ) could be chosen because it covers the whole world without over lab. For (ΔØ), each 2 maps make complete one degree and for (Δλ) each 3 maps make complete two degrees that means the whole globe can be covered in one system. This proposed system is named GMIS30/40 (Global Map Index System 30/40).

Table (5-3). geodetic and metric map dimensions in different scales based on 1: 100,000 map as 30' x 40'.

Scale	Geodetic Dim.	Dim. at latitude 0° (m)			Dim. at latitude 30° (m)			Dim. at latitude 60° (m)	
		ΔØ (m)	Δλ(m)		ΔØ(m)	Δλ(m)		ΔØ(m)	Δλ(m)
1:100000	30' x 40'	55,288	74,214		55,429	64,325		55,709	37,200
1: 50 000	15' x 20'	27,644	37,107		27,714	32,162		27,854	18,600
1:25 000	7' 30"x 10'	13,822	18,553		13,857	16,081		13,927	9300
1:10 000	3' x 4'	5529	7421		5543	6432		5571	3720
1:5 000	1'30" x 2'	2764	3711		2771	3216		2785	1860
1:2 500	45" x 1'	1382	1855		1386	1608		1393	930
1:1 000	18" x 24"	553	742		554	643		557	372
1:5 00	9" x 12"	276	371		277	322		279	186

Maps need to be numbered to indicate its location, scale, and dimensions. Numbering maps in one simple system includes all scales and verifies what have been mentioned is not an easy task. In the proposed real geodetic map in this thesis, the map has one, not repeated, place on the globe. So simple fraction is introduced as proposed index to the proposed real geodetic map. The Numerator contains the start latitude to the end latitude of the map and the Denominator contains the start longitude to the end longitude of the map. The latitude and longitude differences in Numerator and Denominator indicate the map scale. This format is very simple and it indicates the place, dimensions, and map scale. Figure (5-17) illustrates the proposed map index for the scale 1: 100000 map which covers 30' x 40'. In figure (5-17), the numerator starts at latitude 30°30' 00" **(which wrote as 30.3000, the degree, mints and second signs were erased, the decimal point is putted between degrees and mints)** ends at latitude 31°00' 00" and Denominator starts at longitude 24°40' 00" and ends at longitude 25°20' 00". The latitude difference 30′ and the longitude difference 40′ indicate the map scale, i.e. 1:100000;

$$\frac{\text{Ø } 30.3000/31.0000 \text{ E}}{\lambda\ 24.4000/25.2000 \text{ N}}$$

The map index of the scale 1: 100,000 is described in figure (5-17) in geodetic dimensions 30' x 40'.
The map index of the scale 1: 50,000 is described in figure (5-18) in geodetic dimensions 15' x 20'.
The map index of the scale 1: 25,000 is described in figure (5-19) in geodetic dimensions 7' 30" x 10'.
The map index of the scale 1: 10,000 is described in figure (5-20) in geodetic dimensions 3' x 4'.
The map index of the scale 1: 5000 is described in figure (5-21) in geodetic dimensions 1' 30" x 2'.
The map index of the scale 1: 2500 is described in figure (5-22) in geodetic dimensions 45" x 1'.
The map index of the scale 1: 1 000 is described in figure (5-23) in geodetic dimensions 18" x 24".
The map index of the scale1: 500 is described in figure (5-24) in geodetic dimensions 9" x 12".

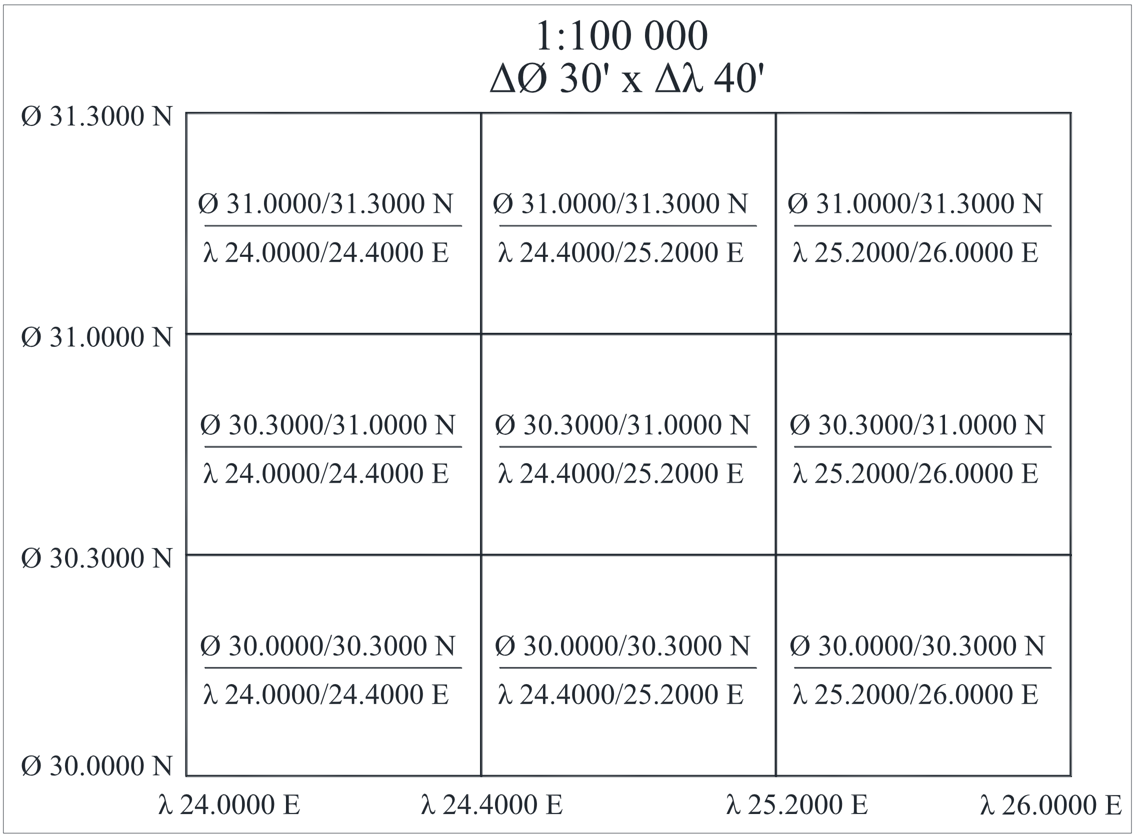


Figure (5-17): Neighboring maps in map index of the 1:100,000 with dimensions 30' x 40'.

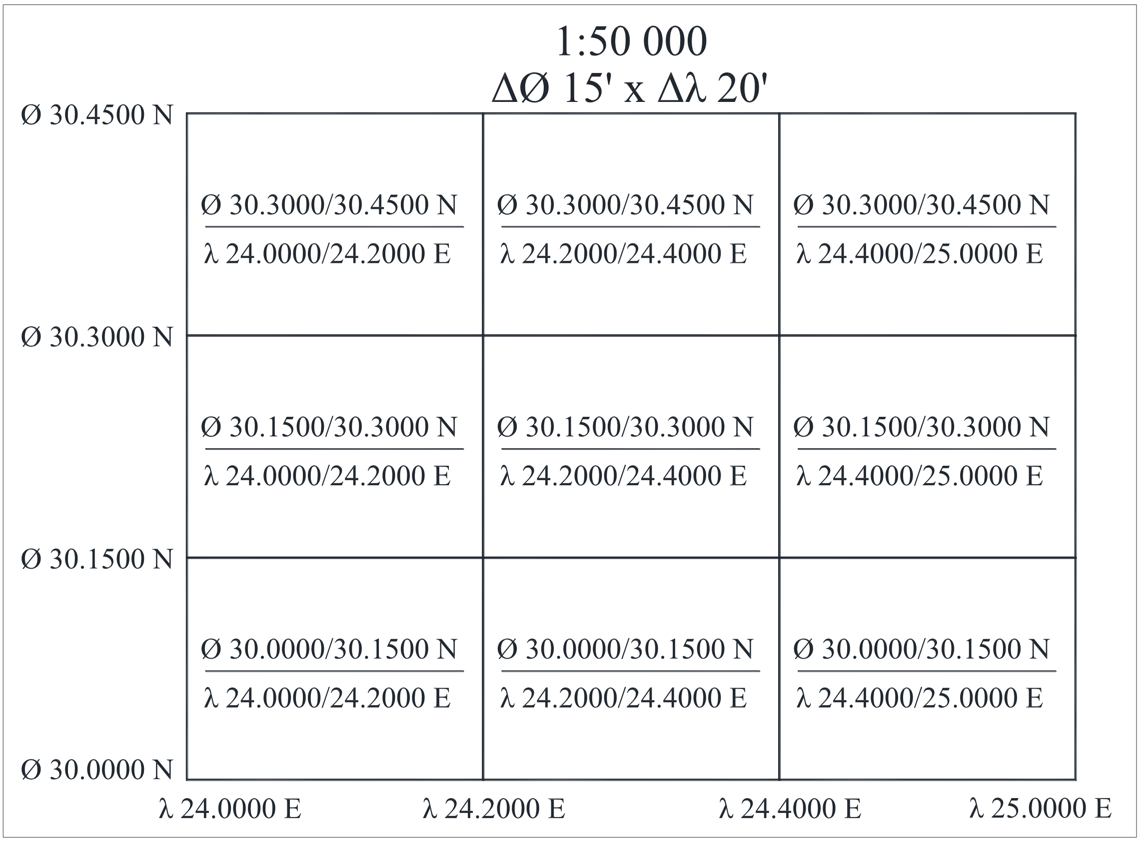


Figure (5-18): Neighboring maps in map index of the 1:50,000 with dimensions 15' x 20'.

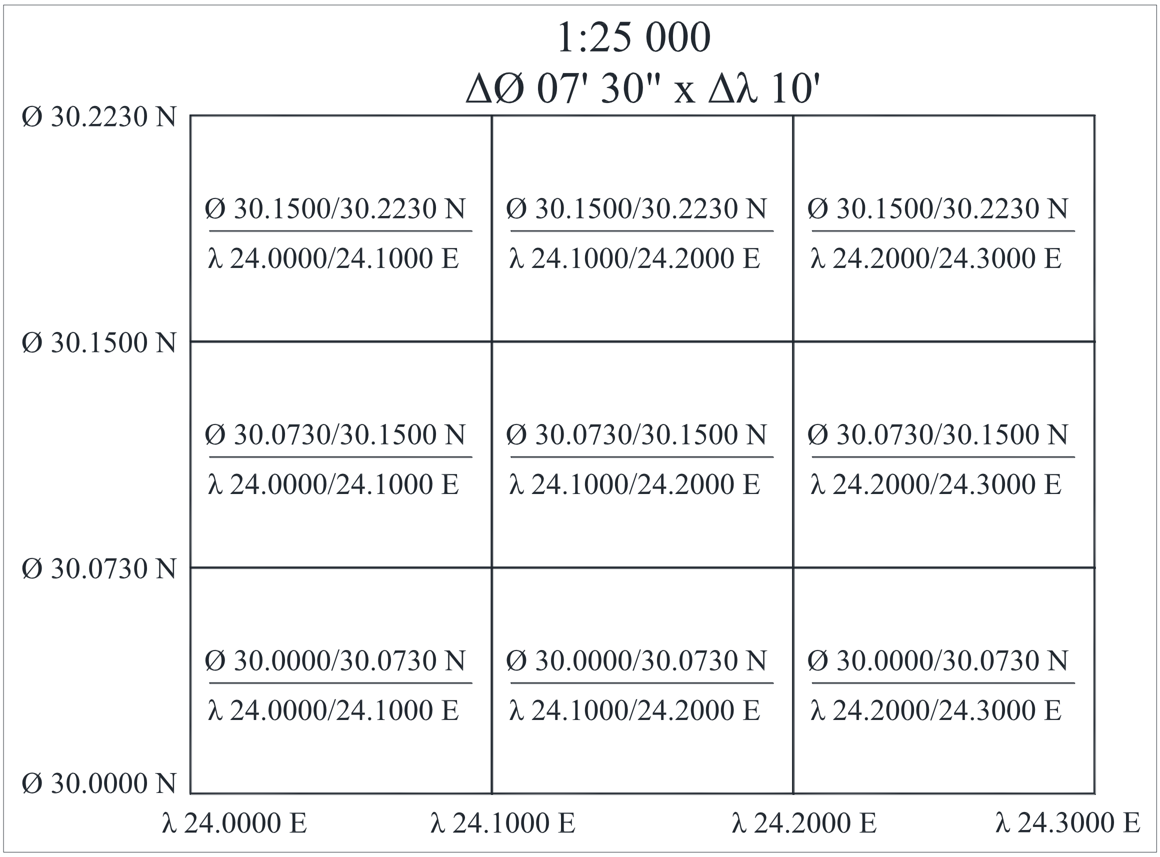


Figure (5-19): Neighboring maps in map index of the 1:25,000 with dimensions 7'30″ x 10'.

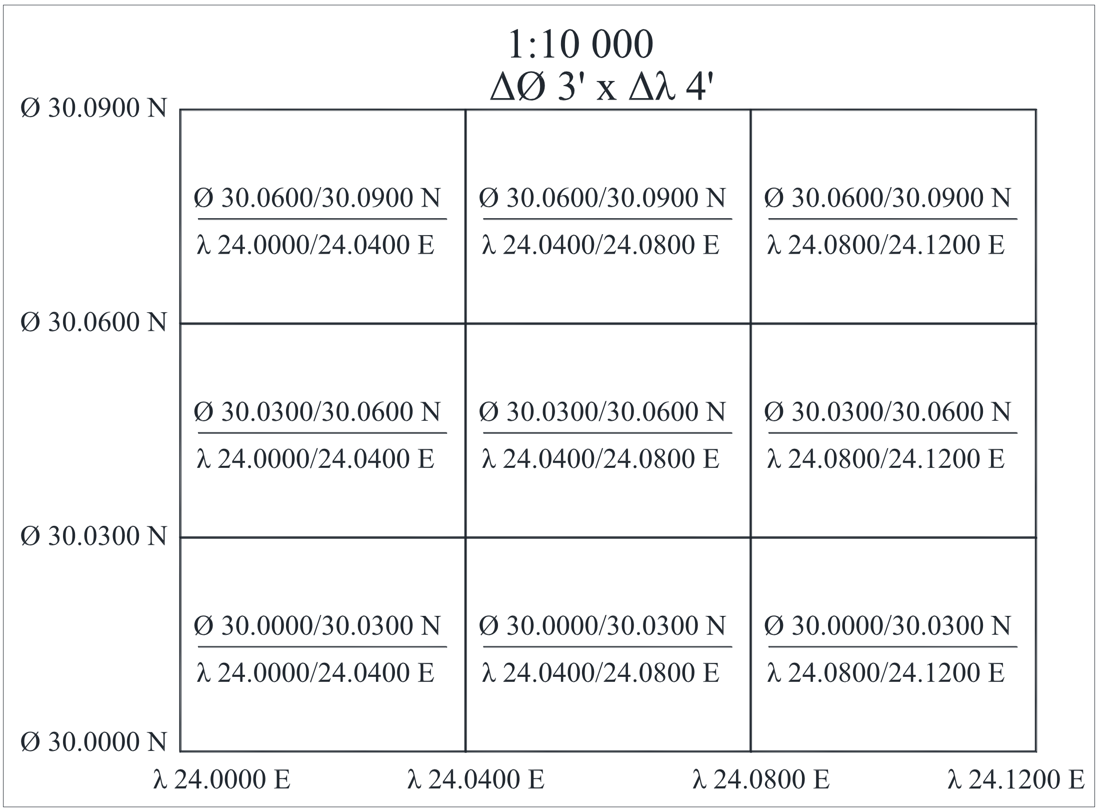


Figure (5-20): Neighboring maps in map index of the 1:10,000 with dimensions 3' x 4'.

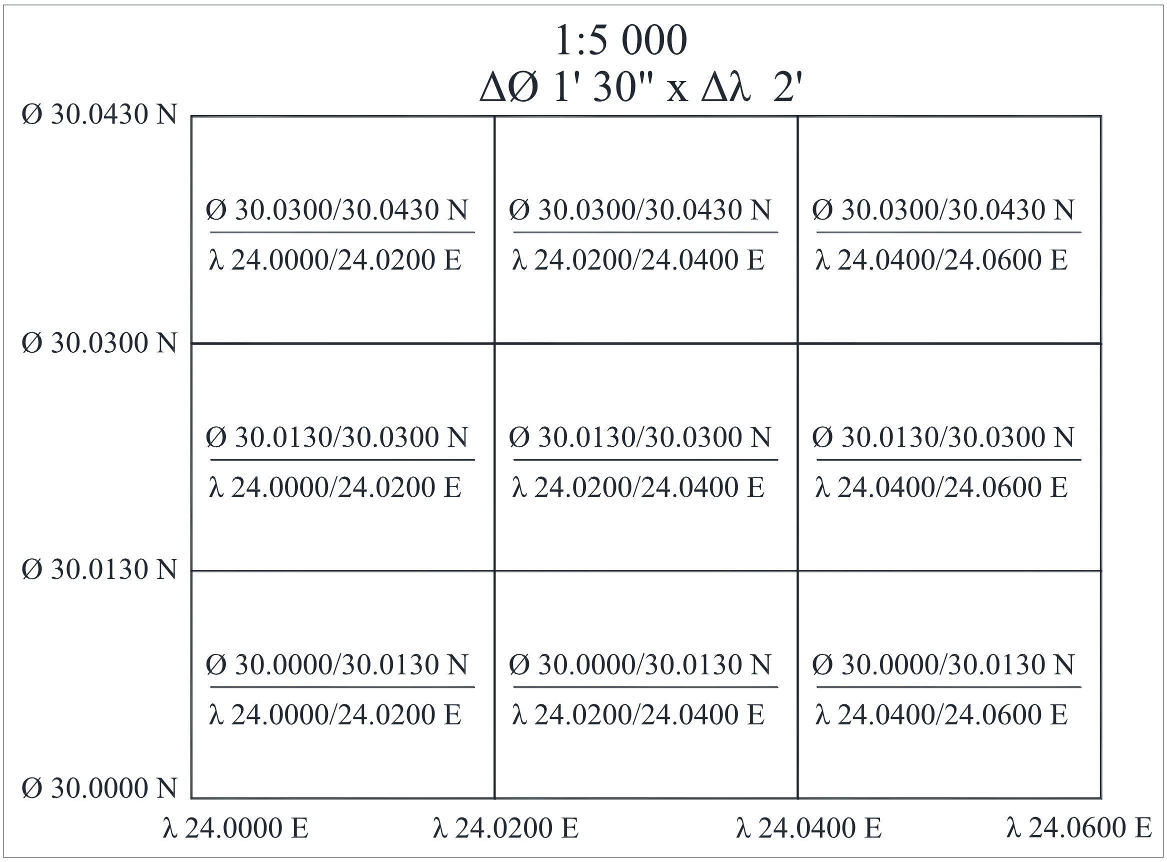


Figure (5-21): Neighboring maps in map index of the 1:5000 with dimensions 1'30''' x 2'.

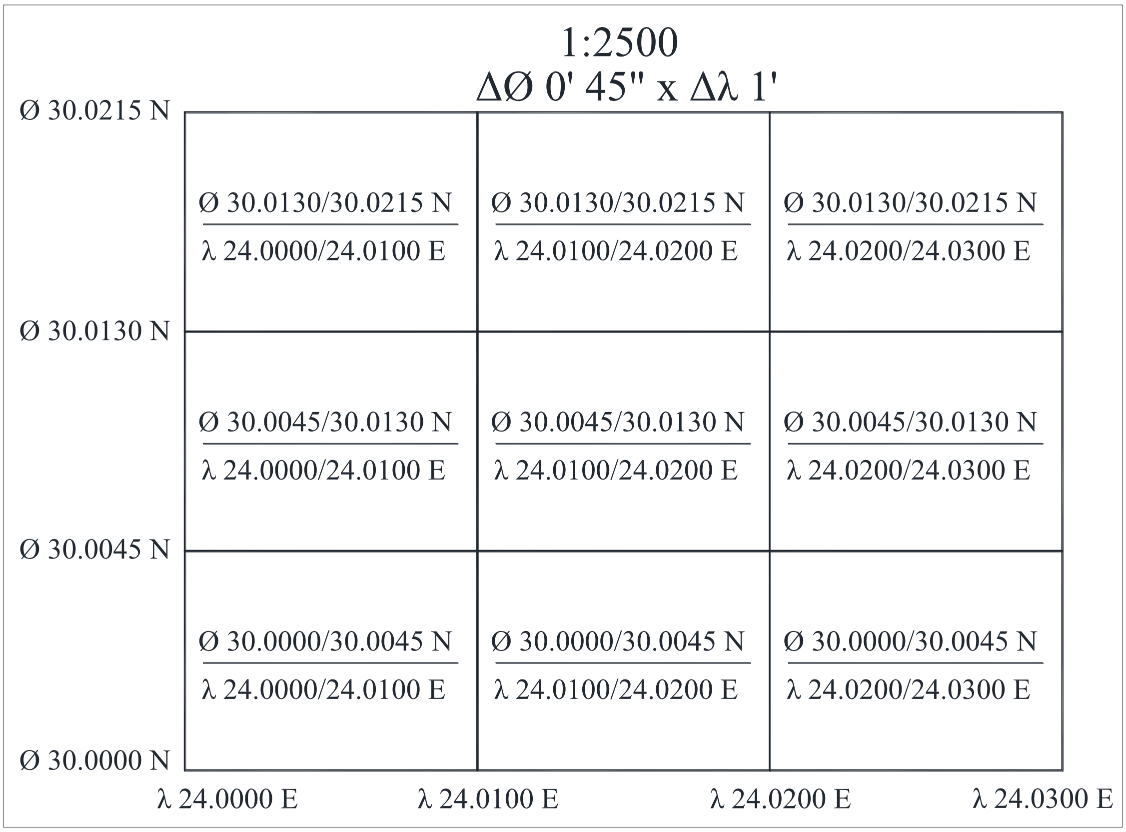


Figure (5-22): Neighboring maps in map index of the 1:2500 with dimensions 45'' x 1'.

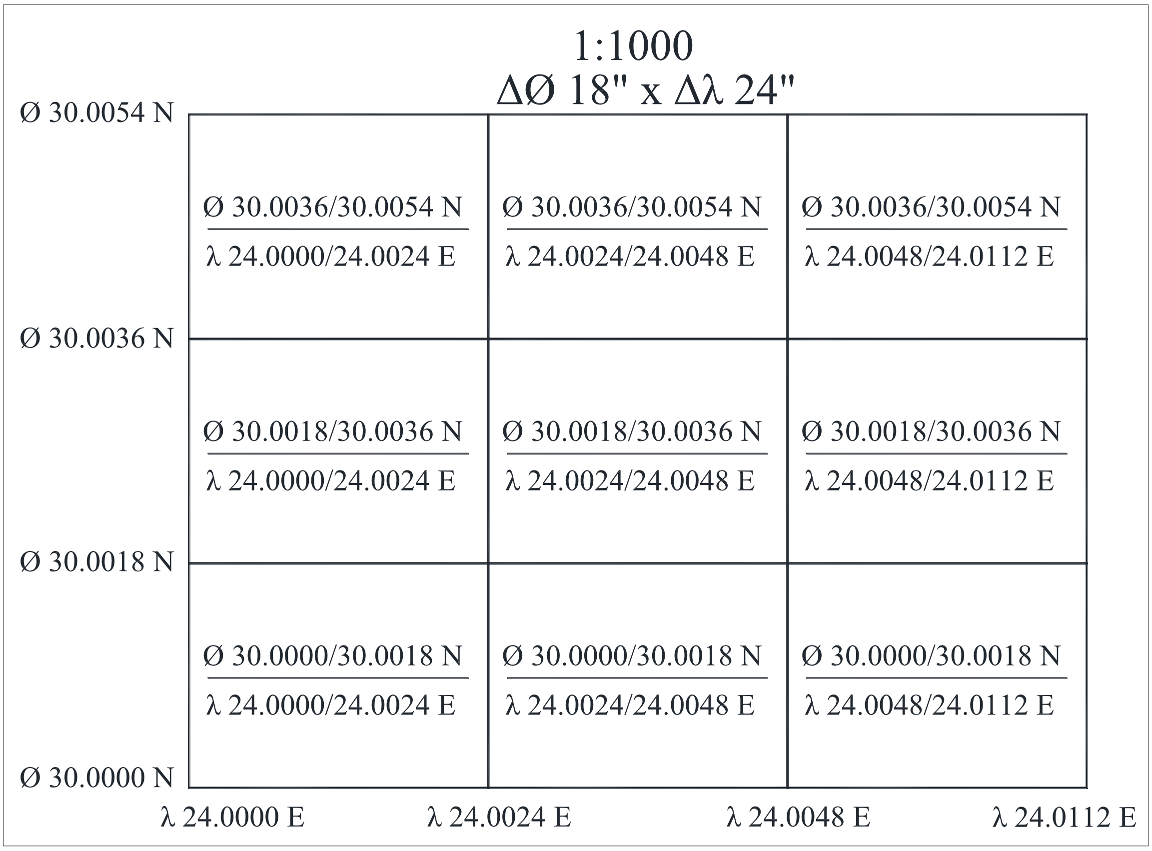


Figure (5-23): Neighboring maps in map index of the 1:1000 with dimensions 18" x 24".

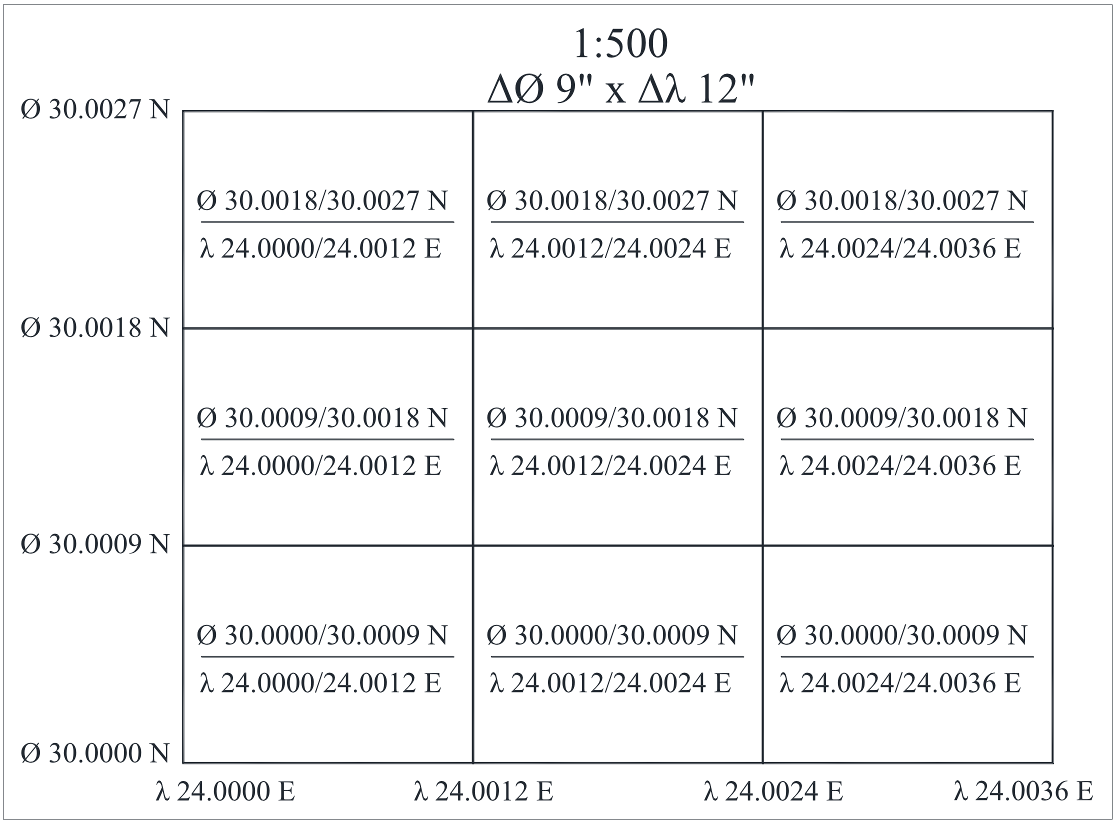


Figure (5-24): Neighboring maps in map index of the 1:500 with dimensions 9" x 12".

5.4 Effect of Convergence of Meridians on Longitude Difference

It is well known that the meridians are converging as they come nearer to the poles till they intersect at the poles. Thus the metric value of (Δλ) is getting shorter towards the poles. So the question will be about the shape of the map near the poles. In table (5-4) the change in (Δλ) as 40' is registered starting from the equator to the pole every 5º **which was calculated by inverse geodetic problem from the automatic real map software.** The calculations are done at the Egyptian Datum (EGD) and the used Ellipsoid is Helmert 1906. The change in (ΔØ) while getting nearer to the pole is small and is not noticeable in the map, see table (5-3).

Table (5-4). the effect of convergence of meridians on the longitude differences (Δλ) =40' at different latitudes

Distances at	(Δλ) =40' (Length)mt.		Distances at	(Δλ) =40' (Length)mt.
latitude 0˚	74213.727		latitude 70˚	25457.808
latitude 5˚	73933.198		latitude 75˚	19268.084
latitude 10˚	73093.618		latitude 80˚	12929.041
latitude 15˚	71701.005		latitude 81˚	11647.610
latitude 20˚	69765.363		latitude 82˚	10362.562
latitude 25˚	67300.651		latitude 83˚	9074.295
latitude 30˚	64324.724		latitude 84˚	7783.210
latitude 35˚	60859.258		latitude 85˚	6489.706
latitude 40˚	56929.656		latitude 86˚	5194.187
latitude 45˚	52564.915		latitude 87˚	3897.053
latitude 50˚	47797.469		latitude 88˚	2598.708
latitude 55˚	42662.999		latitude 89˚	1299.556
latitude 60˚	37200.198		latitude 90˚	0.000
latitude 65˚	31450.507			

In the case that the map of 1:100,000 scale which covers (30′ as ΔØ and 40′ as Δλ) is chosen, 40′ as Δλ covers 74213 m at the equator which equals about 75 cm in the map. This value gets lesser towards the pole. It equals 64324 m at latitude 30º which equals about 65 cm in the map. This value equals about 10 cm in the map at latitude 82º and about 6.5 cm at latitude 85º and about 1.3 cm at latitude 89º. So, the maps of scale 1:100,000 can be used until latitude 82º, see figure (5-25).

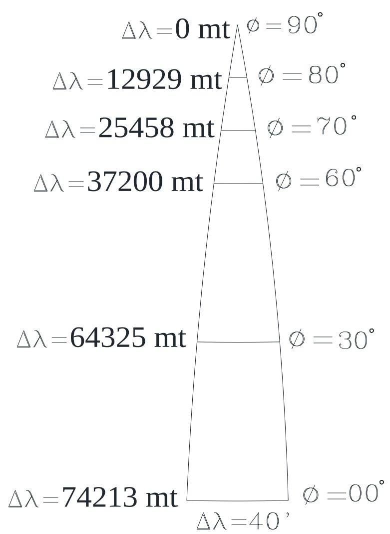


Figure (5-25): The effect of convergence of meridians on the Δλ = 40' at different latitudes.

6. SUMMARY, CONCLUSIONS, and RECOMMENDATIONS

INTRODUCTION

Surveying nowadays could be generally divided into modern (satellite based) and traditional ways. In modern way, the required geodetic coordinates are obtained directly related to the specified geodetic datum. In the surveying traditional way, the required geodetic coordinates cannot be directly observed. They are obtained by computing them from taken traditional observations.

The traditional observations are distances, vertical angles, and horizontal angles. Those observations are taken related to the direction of actual gravity, while the geodetic computations will be carried out on the surface of the reference ellipsoid. Thus fictitious observations related to the direction of the normal to the ellipsoid should be obtained from the taken observations. It is therefore convenient to reduce the taken observations to the used reference ellipsoid. The new observations after reduction can then be used to calculate the geodetic coordinates (ø, λ).

Map projection is used to transform the obtained geodetic coordinates into plan (map) coordinates. In map projection process, distortion in distance, azimuth, area, or shape must happen. It is difficult to the user and not convenient to the specialist to deal with this distortion.

In the past, the computations and drawing the maps were manually done. Nowadays, computations and map production are automatically done by using electronic computers. Therefore it is the time now to draw the map using the geodetic coordinates directly and to avoid the noisy distortion. The proposed map will be computerized soft copy one and will be plotted whenever needed.

Parallels and meridians will be the background of the proposed map. Points will be represented by their geodetic coordinates (ø, λ). The needed surveying elements, (distances, azimuths, and areas), will be obtained by computing them using ad joint functions. Those functions (computer programs) will be part of the proposed electronic map. Just push button (hot keys) to obtain the needed element.

In the same datum, any point on the earth has unique geodetic value of coordinates; latitude and longitude (ø, λ). In projection systems like UTM (universal) and ETM (national); the same point lying at the border between two zones like longitude 33ºE in ETM (between red and blue zones) and also longitude 12ºE in UTM (between zones 32 and 33) has two different pairs of coordinates. Pair of (E, N) from the first zone and another different pair (E, N) from the second

adjacent zone will be obtained. The same values of (E, N) are repeating in the sixty zones of UTM.

In large projects like petroleum pipe lines and international roads, when the project is located in two zones, a problem happens. One project should belong to one coordinate system but the projection makes it in two different zones or systems of coordinates. The followed solution is to relate the whole project to one zone or system of coordinates despite the resulting great value of distortion.

Distances from the proposed map do not involve scale distortion. The shape of the feature in the proposed map will not differ from the corresponding feature's shape in the projected map. Parallels and meridians will be straights in the proposed map with its all scales. For example, the line of 60,000m, as an ellipsoidal distance, has 60019.879m, as a projected distance, in the map of 1:100,000. The difference between the two values in the map is approximately (20/100000) m i.e. 0.2mm which cannot even measured by a ruler.

The proposed automatic real map is digital map presented by Parallels and Meridians and calculation of distances, azimuths, and areas will be done using the appropriate geodetic equations by hot keys ad joint to the map; these points are known in geodetic datum like WGS84. The map could be plotted whenever a hard copy is needed.

6.1 SUMMARY

6.1.1 Ellipsoidal Versus Plan Distances

The earth as a planet has a curved surface. In geodesy, that curved surface is geometrically represented by an ellipsoid or a sphere. This means that the geodetic computations are the default and it should be followed. In the surveying field and when small areas are considered, the plan surveying computations are followed. The area is considered small when the curvature of the earth does not appear, i.e. when the difference between the curved area and its plan surface is not significant compared to the required accuracy. When viewing an image of a small area in Google Earth, it looks like a flat area although the curvature of the earth exists.

The chord and curved distances between the same two points are computed with varying the distances from 1000 m till 100,000 m. The difference between chord and its arc distance for the same two points on the earth is very small in short lines. Difference between arc and its corresponding chord distance reached 1 mm at distance 10 km, 10 cm at distance 45 km, and 1 m at distance 100 km.

When using the smallest scale map 1:100,000 which covers 60 km * 40 km in one sheet while the differences of 22cm and 6.5cm at distances 60km and 40km respectively. Difference between Distances of 60,000.22m and 60,000m both drawn at scale 1:100,000 will not be noticeable to the user eye. Therefore using the geodetic coordinates directly in mapping will not show difference with mapping the same area using plan coordinates. I.e. differences between curves and straights will not appear on the map.

This part is computed and illustrated here to prove that the background of the proposed geodetic map (grid of latitudes and longitudes) will still be straights and not curves. The mapped features using ø, λ will not also differ in their form from their corresponding form in the projected map in all the surveying map scales.

6.1.2 Geodetic Versus Projected Maps in Different Surveying Scales

The computations on WGS84 (World Geodetic System 1984) and UTM (Universal Transverse Mercator) are done. The computations have been done once more on the Egyptian national mapping system. In zone number 31 of UTM, two main groups of maps are chosen for the study, one of these groups is at the central meridian of the zone and the other group is at the zone border. The differences in the distances and azimuths at the surface of the ellipsoid and the map are studied on various scales 1: 1000, 1:2500, 1:5000, 1: 10,000, 1: 25,000, 1: 50,000, and 1:100,000.

The computations are done in sub groups G1 & G2 at equator, G3 & G4 at latitude 30ºN, G5 & G6 at latitude 60º N, G7 & G8 at latitude 70º N, and G9 & G10 at 80º N. In UTM, zone width is 6 degrees.

The geodetic coordinates of the corner points of the studied maps related to WGS84 and the corresponding projected values (UTM) at different scales for G1 and G2 at equator are computed. These data are also prepared at latitude 30° N as Groups (3) & (4) and at latitude 60°N as Groups (5) & (6) and the data of Groups (7), (8), (9) and (10) at latitude 70°N, 80°N.

Considering the data and obtained results and concerning the deference between geodetic and map distances; the differences seem significant as absolute values but they are not noticeable as drawn in the map. It means one cannot notice a difference between geodetic and plan metric maps for the same area.

In map scale 1:1000 at equator, distortion value of 37 cm at G1 & 90 cm at G2 in 925.432 m are obtained. This is a big value especially when precise EDM is used in measuring distances in the field. The user does not know about distortion and the surveyor himself should bay

attention while dealing with projected map and the scale factor while using Total Station in the field. This problem can vanish by using Geodetic Total Station (GTS) in the field and the proposed geodetic map, see sec4.5 & sec4.6. In the 1:1000 map itself, 37 & 90 cm differences in 925m will appear as (37 & 90 cm/1000m) which is not noticeable.

Distortion is variable in map from point to another; to resolve this issue practically we take an average value of distortion in limited region. The problem is more complex in case of international and intercontinental projects such international roads and petroleum pipelines. Again, the problem can vanish by using the proposed geodetic mapping system especially in the presence of WGS84 as global geodetic coordinate system and GNSS as global observation tools.

6.1.3 Area Calculation on Projected Map and Geodetic Datum

a. Area on Projected Map

Map scales 1: 100 000, 1:10 000 and 1:5000 of groups G1 and G2 are chosen in area calculation on the map and on the geodetic datum. The whole area of the map is divided into two triangles by its diagonal. The coordinates are related to WGS84 projected using UTM; G1 in the middle of zone 31 and G2 in its adage. The areas computed from projected map by plane geometry. Both projected areas of G1 and G2 are not the same and the distortion in both cases is noticeable, to compute the corrected area we need first calculate the corrected distances by scale factor, then the corrected areas are more closed in the same scale in G1 and G2.

b. Area as Trapezoid by Double Integration on surface of the Sphere

The area of the map as a trapezoid on sphere by double integration through our simple proposed equation is computed. With respect to the area under discussion; that method of trapezoid on sphere cannot be compared with that of two spherical triangles in large distances because this form is surrounded by two longitudes and two latitudes, and a latitude does not represent a great circle to determine the border limit of spherical triangles. Also, the trapezoid shape is special case for computing area pounded by random points.

c. Area as two spherical / ellipsoidal triangles by using geodetic distances (the best method)

Through the ellipsoid distances and by using the proposed equations (3-78) or (3-79) we can calculate the area of spherical triangle through area of corresponding plane triangle. It should be remarked here that the corresponding projected distances in the two maps at G1 (negative distortion) and G2 (positive

distortion) are different while they are the same in the corresponding ellipsoidal case. This assures the need for the proposed geodetic map.

This method is the best, it is more stable than that of computing the area as spherical triangles by using spherical excess because it depends on P area as initial value (plane area computed from geodetic distances) and correct that by spherical factor in proposed equations (3-78) or (3-79) to get corresponding F area.

Sometimes the computed area of a spherical triangle using spherical excess is less than P area by great value, this value is not logic because the small change in value of spherical excess affects the area result. The area computed by spherical excess is very sensitive towards the value of ρ (206265) or (206264.8062) especially in large areas.

6.1.4 Map Index Proposal

This research has two main objectives, the first one is proposing a real geodetic map in an electronic computerized copy and the second objective is introducing a universal map index.

A map of smallest surveying scale was defined i.e. 1:100,000 scale map. This map can geodetically cover **30'(ΔØ) x 40'(Δλ)** for the whole globe. The corresponding metric values for map 30'(ΔØ) x 40'(Δλ) at the equator are 55288 m x 74214 m that needs printing paper dimensions of 56 cm x 75 cm. Also, the corresponding values at latitude 30˚ are 55429 m x 64325 m that needs printing paper dimensions of 56 cm x 65 cm. and the corresponding values at latitude 60˚ are 55709 m x 37200 m that needs printing paper dimensions of about 56 cm x 38 cm.

starting from 1:100,000 map which covers 30'(ΔØ) x 40'(Δλ), map dimensions at equator, latitude 30˚ and latitude 60˚ in different surveying scales 1:100,000 - 1:50,000 - 1:25,000 - 1:10,000 - 1:5000 - 1:2500 - 1:1000 and 1:500 are illustrated.

The map of dimensions 30'(ΔØ) x 40'(Δλ) could cover the whole world without over lab. For (ΔØ) each 2 maps make complete one degree and for (Δλ) each 3 maps make complete two degrees that means the whole globe can be covered in one system. This proposed system is named **GMIS30/40 (Global Map Index System 30/40).**

Maps need to be numbered to indicate its location, scale, and dimensions. Numbering maps in one simple system includes all scales and verifies what have been mentioned is not an easy task. In the proposed real geodetic map in this thesis, the map has one, not repeated, place on the globe. So simple fraction is introduced as proposed index to the proposed real geodetic map.

The Numerator contains the start latitude to the end latitude of the map and the Denominator contains the start longitude to the end longitude of the map. The latitude and longitude differences in Numerator and Denominator indicate the map scale. This format is very simple and it indicates the place, dimensions, and map scale.

This example illustrates the proposed map index for the scale 1: 100,000 map which covers 30' x 40'. The numerator starts at latitude 30˚00' 00" and ends at latitude 30˚30' 00" and Denominator starts at longitude 24˚20' 00" and ends at longitude 25˚00' 00". The latitude difference 30′ and the longitude difference 40′ indicate the map scale, i.e. 1:100000;

$$\frac{\text{Ø } 30.0000/30.3000 \text{ N}}{\lambda\ 24.2000/25.0000 \text{ E}}$$

Geodetic and metric map dimensions in different scales based on 1: 100,000 map as 30' x 40' mentioned in sec 5.3.1

6.1.5 Steps of Automatic Real Map Production

After obtaining the geodetic coordinates (Ø, λ) for each of the project points, the map could be drown and stored in its digital form, it could be also plotted when needed. The two axes of the map are chosen at the south-west corner of the map. Then the difference of latitude and longitude between the concerned point and the corner of the map is defined. All the equations used in computations are programmed and ad joint to the map as an essential part of it. Any needed information can be obtained from the proposed automatic real map using hot keys (push button). The required information will be obtained directly from the geodetic coordinates and the projection distortion will be totally avoided.

The computer program for producing Automatic Real Map is created by **Visual Basic for Applications** this is available in some programs like AutoCAD and Microsoft office. In AutoCAD, to draw the map using latitude and longitude is possible;

- The map is recorded as points and lines in Microsoft excel tables.
- The map data can be imported from total station and GPS as points, lines, polylines and arcs which are connecting between these points.
- The points are recorded by their actual latitudes and longitudes.
- Base point (map corner or any point) is specified to calculate the differences in latitude and longitude between that base point and all other points.

- Latitude and longitude differences are computed in meter units using suitable geodetic equations.
- Then all points are represented and connected to each other by lines and polygons if needed.
- The line between any two points can be drawn and then selected and using certain program keys to get its azimuth and distance.
- All properties of any line (geodetic distance, azimuth, rectangular and geodetic coordinates for its two terminal points, difference in latitude and longitude, difference in rectangular coordinates also spatial distance) can be obtained once pushing the specified key.
- Any point can be selected and using point properties key, point properties (geodetic and rectangle coordinates, orthometric and ellipsoidal heights) can be obtained if ζ, η, N are available and stored in the program.
- A polyline between 3 points can be drawn as triangle; then it is selected by specified key to compute the ellipsoidal area and also the geodetic circumference and spherical excess.
- The closed polyline between several points can be drawn and then selected. The enclosed ellipsoidal area can be computed using the specified area key; also the geodetic circumference can be obtained.
- The user can add new point to the map by;
    - Free hand by click insertion
    - Rectangular coordinates X, Y, Z.
    - Geodetic Latitude and geodetic longitude
    - Geodetic distance and geodetic azimuth from chosen point
    - Spatial distance and geodetic azimuth from chosen point
    - Latitude and longitude differences from chosen point

6.2 CONCLUSIONS

The projected map does not represent the reality because of the well-known distortion. Every country, in the old system of projection, has its own system beside that often every country is divided into different zones. Data (projected coordinates) from different countries or inside the

same country but in different zones cannot be used (collected) together. The same conclusion can be drawn on the Universal Transverse Mercator (UTM).

Nowadays, universal surveying field tools like satellite positioning missions (GNSS), satellite imagery, and satellite gravity missions are widely used. The produced coordinates and coordinates based services are related to a worldwide geodetic datum like WGS84. So, the field tools of collecting data became global and the reference geodetic datums became global too but the mapping system not yet.

This research is proposing a real geodetic map in an electronic computerized copy. The proposal is a universal mapping system, but the old system cannot be. This study will enable:

- Collecting the maps of one country together
- Collecting the maps from different neighboring countries together
- Using surveying (geodetic) data wherever on the globe in one system without transformation
- Computing distances, azimuths, and areas between any points on the globe without distortion
- The map scale will not affect the accuracy of the extracted elements from the map (distance, azimuth, and area). They will be calculated from the geodetic coordinates with their observed accuracies.

6.3 RECOMMENDATIONS

Based on the preceding summary and conclusions, the following recommendations can be suggested:

1. Matching the big jumps in computing devises and programming facilities and with advents of satellite missions, using the proposed real digital geodetic mapping system is strongly recommended.
2. Using the proposed Global Map Index System (GMIS) in mapping is recommended for its easy and simple indication to the map scale, map dimensions and location in one system in the world.

3. Using the suggested equation for computing the area of ellipsoidal triangles by using geodetic distances is recommended rather than of using the equation of computing the area of the Spherical Triangle through the spherical excess.

4. Increasing the map width above latitude 80º to be 2º or more is recommended because of the meridian convergence.

5. Using the geodetic computations beside the plan metric computations in Total Stations sets is recommended to match the other recent surveying tools GNSS and remote sensing to get rid of the noisy distortion.

6. Using large drawing software companies such as Autodesk, Esri and Microsoft our software is added to their software to produce real automatic map after coordination with authors.

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A. THE DESCRIPTION OF THE DESIGNED PROGRAM

The automatic program Real Map is created by Visual basic for Application (VBA), this is available in some programs like AutoCAD, Civil 3D and Microsoft office. In AutoCAD or Civil 3D, to draw the map using latitude and longitude is possible by additional authors program (automatic real map), The creation of new menu is appeared in AutoCAD or Civil 3D programs, figure (A-1):

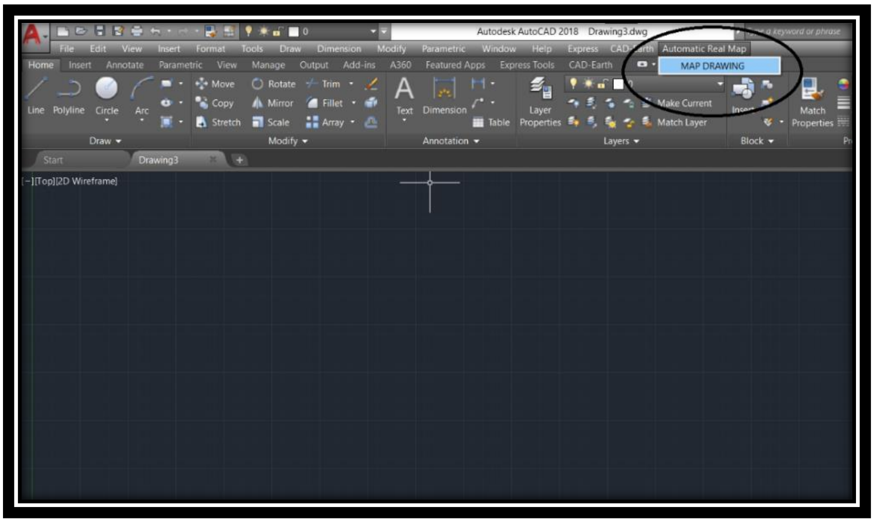

Figure (A-1): Automatic Real Map menu in AutoCAD program.

Once the menu is pushed (Automatic Real Map), its content button (map drawing) appear. The main form for the automatic real map is started, see figure (A-2), The description of the automatic real map program is divided into two parts (upper and lower):

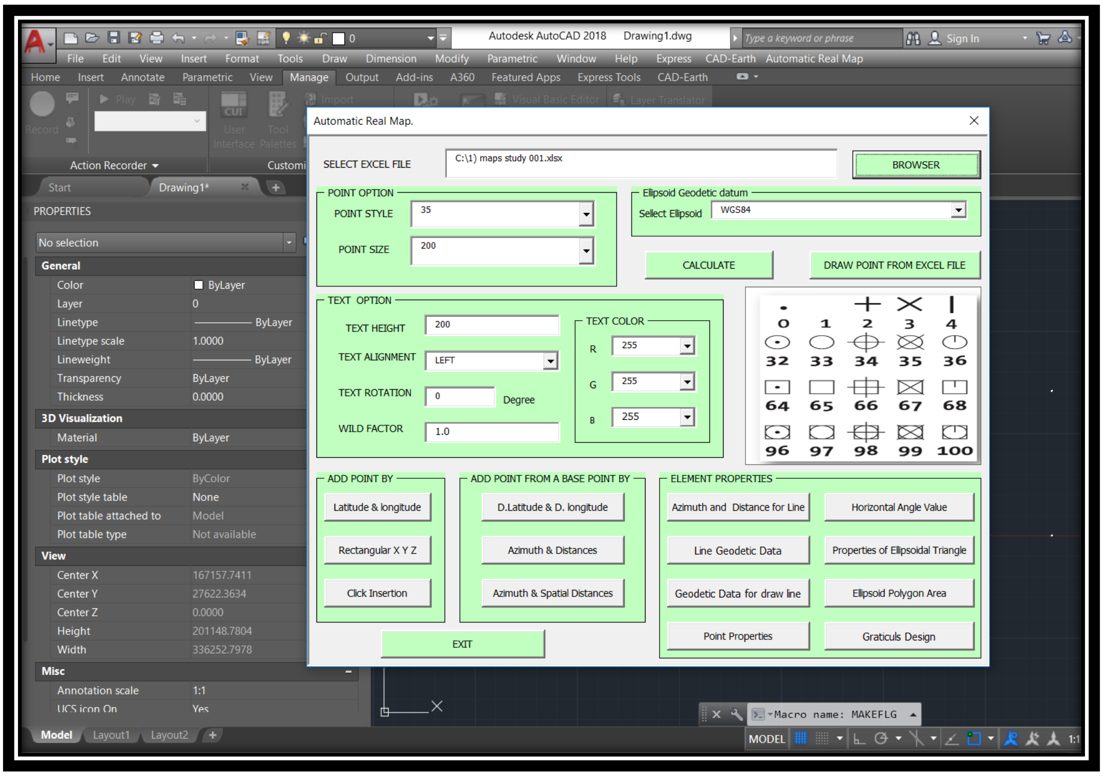

Figure (A-2): The main form of the automatic real map.

A.1. The upper part in the automatic real map main form:

- First, the datum is chosen from **(Ellipoid Geodetic Datum)** frame, the used default datum is WGS84.
- The data is imported from excel file as point name, geodetic coordinates (ϕ, λ, h) and geoidal data (N, ζ, η), if they are available, the geodetic data can be taken from geodetic total station (GTS) or Global Position System (GPS) as geodetic Coordinates or computed from the observation by 2D or 3D geodetic computation, see sec (4.5.1). **(BRAWSER)** button is used to choose the excel file data from the computer.
- Point style and point size are chosen from **(point option)** frame**.**
- The scale of data point (name and coordinates), style, rotation and colour are chosen from **(text option)** and **(text colour)** frames**.**
  - this program will choose the map base point which is specified to calculate the differences in latitude and longitude between this point and all other points in meter values by **(Calculate)** button. Latitude and longitude differences ($\Delta\phi$, $\Delta\lambda$) are computed in meters units using suitable geodetic equations. (differences between base point and every points)
  - The export data to AutoCAD program from excel file using **(Draw Point from Excel File)** button.
  - Then all points are represented and connected to each other by lines, arcs and polygons if they are needed.

A.2. The lower part in the automatic real map main form is divided into 3 frames:

- **The left frame is in lower part for addition points (ADD POINT BY)**

1. Add point by geodetic value (ϕ, λ, h) & (N, ζ, η). if it is available, use **(latitude & longitude)** button.
2. Add point by geodetic rectangular coordinates (X, Y, Z), use **(Rectangular X, Y, Z)** button.
3. Add point by click in screen at chosen point, use **(Click Insertion)** button.

- **The middle frame is in lower part for addition point from a base point (ADD POINT FROM A BASE POINT BY) frame**

4. Add point by selected point in AutoCAD, then the difference in geodetic values ($\Delta\phi$, $\Delta\lambda$, Δh) are inputted by keyboard, use **(D. Latitude D. Longitude)** button.

5. Add point by selected point in AutoCAD, then the geodetic azimuth and geodetic distance by keyboard, use **(Azimuth & Distances)** button.
6. Add point by selected point in AutoCAD and geodetic azimuth & spatial distances by keyboard, use **(Azimuth & Spatial Distances)** button.

The point data (name, latitude, longitude, and ellipsoidal height) is written in AutoCAD drawing as different layers.

- **The right frame in lower part is for (Element properties)**

7. To get the geodetic and back azimuths in addition to geodetic and spatial distances are by selecting line which is connected between two geodetic points, so use **(Azimuth and Distance for line)** button.
8. To get geodetic properties of selected line which is connected between two geodetic points, so use **(Line Geodetic Data)** button
9. Drawing line and getting geodetic data are achieved by using **(Geodetic Data for Draw Line)** button.
10. The user can select any database point to get all geodetic, astronomic and geoid data for that point by using **(Point Properties)** button.
11. To get the horizontal angle for selected polyline which is connected between 3 points, so use **(Horizontal Angle Value)** button.
12. To get area, circumference, internal angles sides length and spherical excess of ellipsoidal triangle should use **(Properties of Ellipsoidal Triangle)** button.
13. **(Ellipsoid Polygon Area)** button is used to get area and circumference of ellipsoidal closed polyline.
14. To draw the graticules for the map, use **(Graticules Design)** button.

A.3. Forms descriptions of the 14 buttons (hot keys) are:

1. In frame (**ADD POINT BY**), the user can use **(latitude & longitude)** button to add new point by keyboard input data, the required values of geodetic coordinates are (ϕ, λ, h) and geoid data are (ζ, η, N) if they are available, figure (A-3).

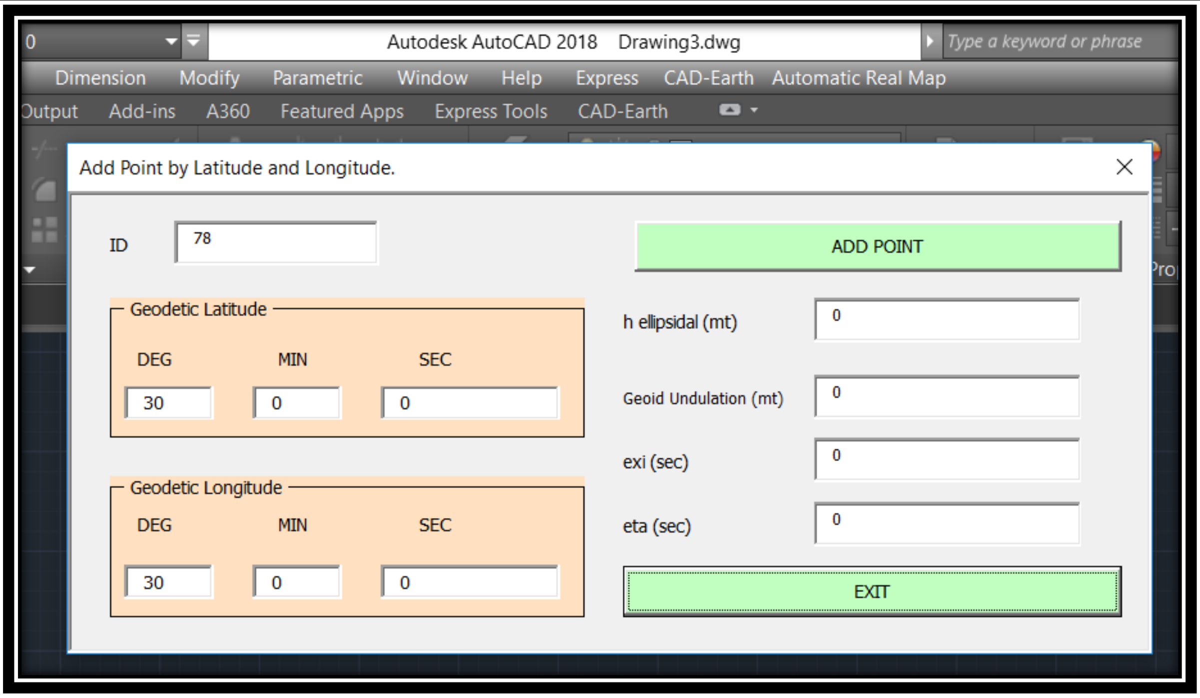

Figure (A-3): Draw point by geodetic coordinates (ϕ, λ, h).

2. In frame (**ADD POINT BY)**, the user can use (**Rectangular X, Y, Z)** button to add a new point to the map by using Geodetic Rectangular coordinates X, Y, Z and geoid undulation N, figure (A-4a). After all the values are inputted then push **(ADD)** button. The geodetic coordinate (ϕ, λ, h) will be computed and recorded in database and AutoCAD, the form will be changed, see figure(A-4b)

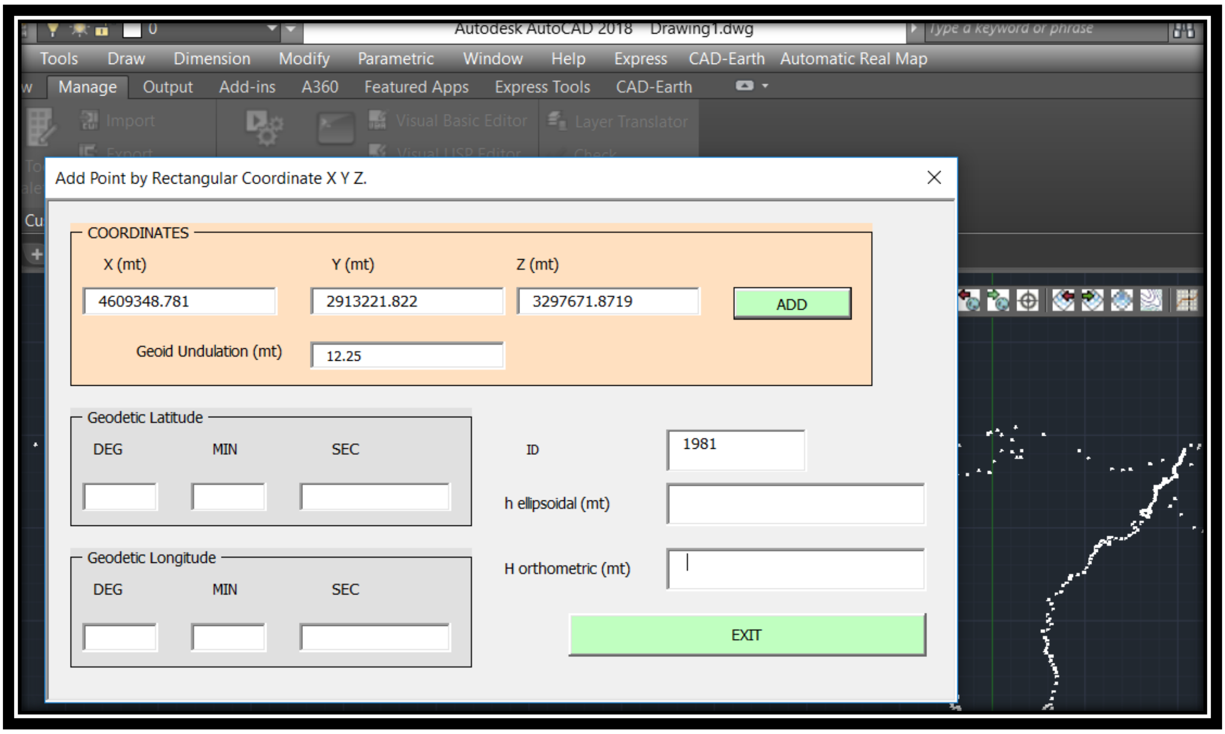

Figure (A-4a): Draw point by rectangular coordinates components (X, Y, Z), input data.

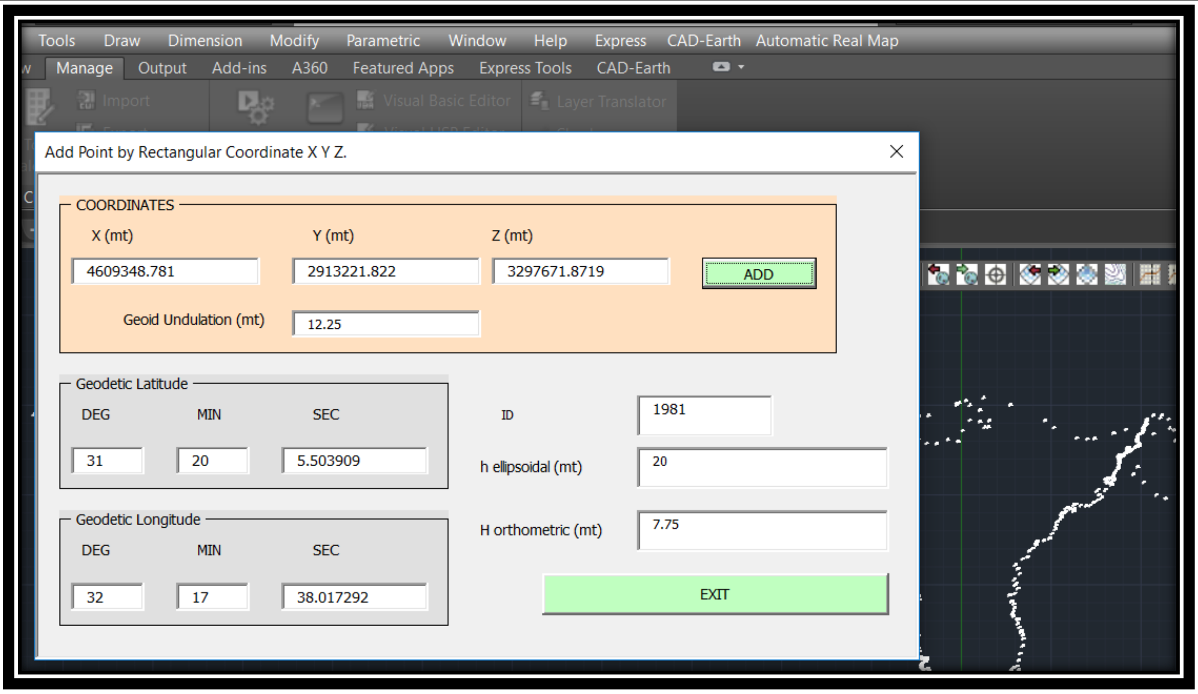

Figure (A-4b): Draw point by rectangular coordinates components (X, Y, Z), result.

3. In frame **(ADD POINT BY),** the user can use **(Click insertion)** button to add new point to the map, and can input geoid data if they are available before the order, the geodetic coordinate will be computed and recorded in database and AutoCAD, figure (A-5).

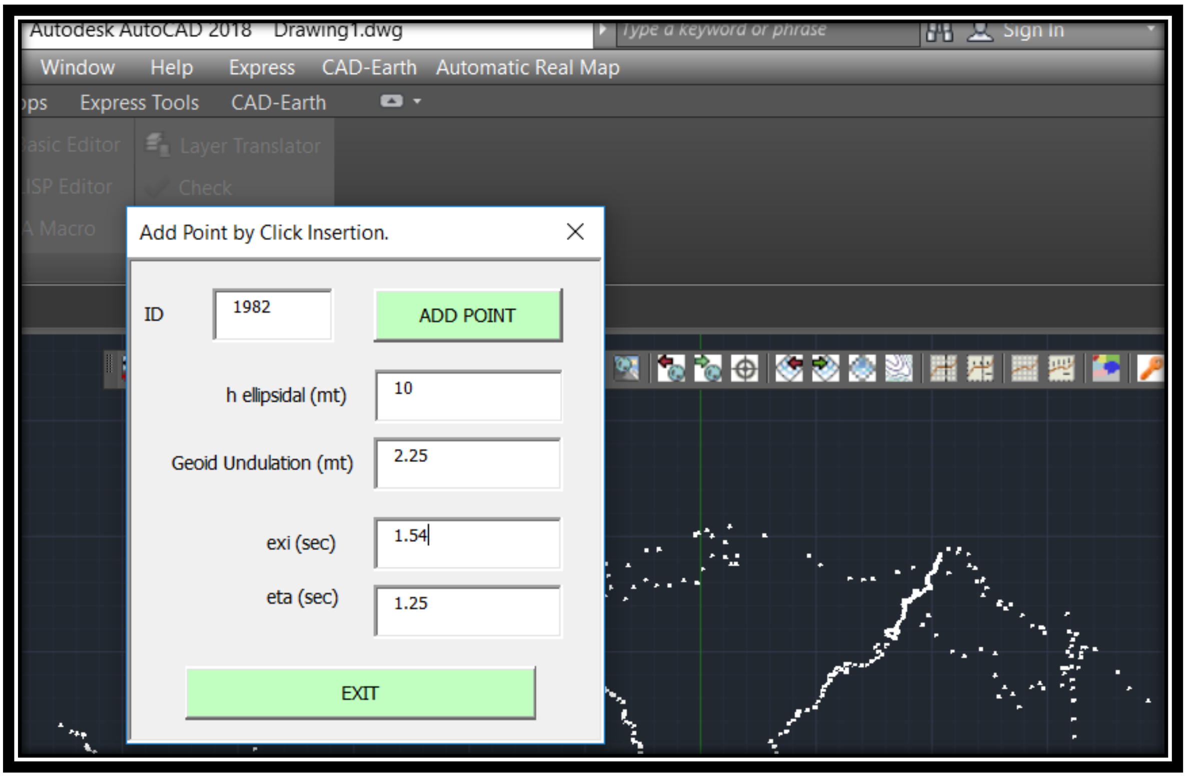

Figure (A-5): Draw point by click on screen.

4. In frame **(ADD POINT FROM BASE POINT BY)**, the user can use **(D. Latitude D. Longitude)** button to add new point to the map by geodetic coordinates differences (Δϕ, Δλ, Δh) which are related to chosen point from the map by click selection. Push **(add new point)** button in this form after selection and input coordinate differences. The geodetic coordinate (ϕ, λ, h) will be computed and recorded in database and AutoCAD, figure (A-6).

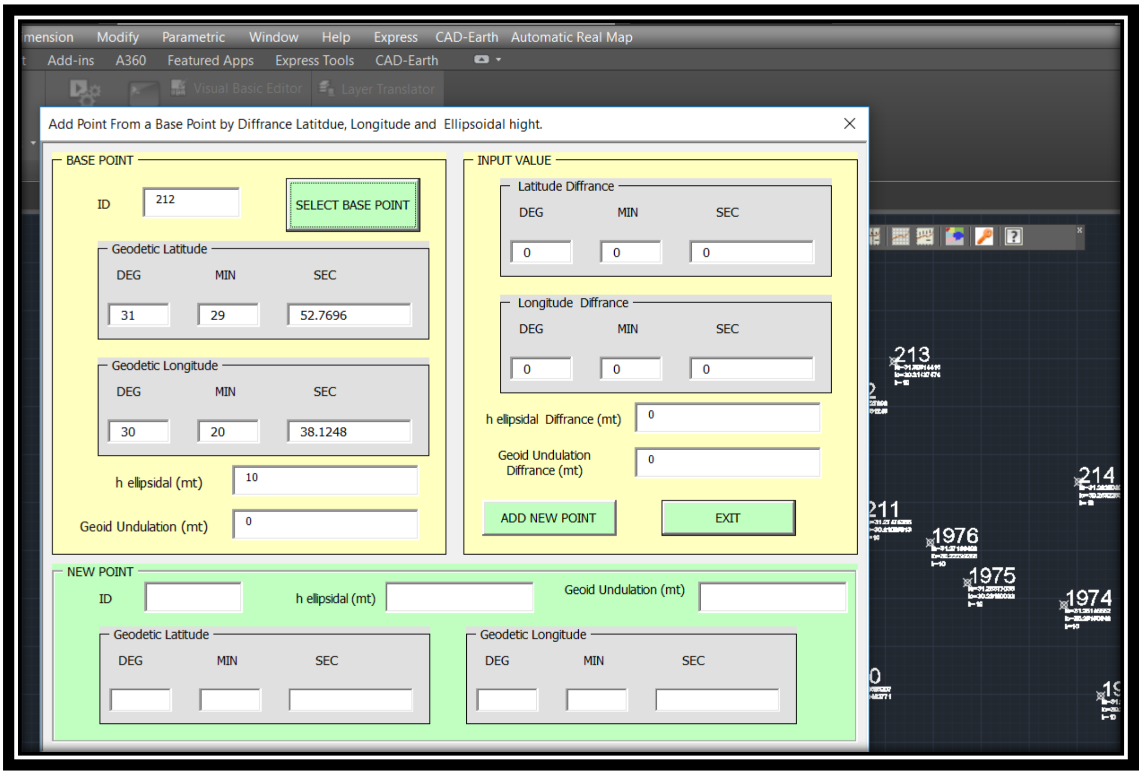

Figure (A-6): Draw point by different latitude, longitude and ellipsoidal height.

5. In frame **(ADD POINT FROM BASE POINT BY)**, the user can use **(Azimuth & Distances)** button to add a new point to the map by Geodetic distance and geodetic azimuth from chosen point, after selection and input geodetic distance and azimuth then push **(add new point)** button in this form. The geodetic coordinate (ϕ, λ, h) will be computed and recorded in database and AutoCAD. figure (A-7),

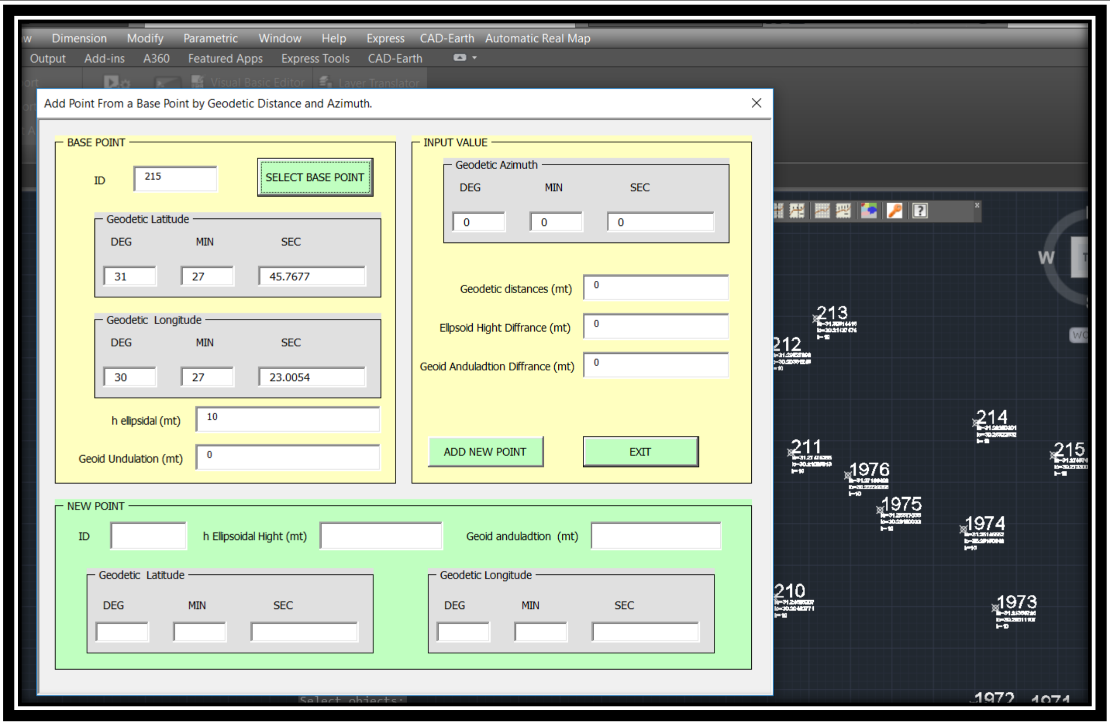

Figure (A-7): Draw point by geodetic distance and geodetic azimuth.

6. The geodetic distances aren't usually available for users,3 the spatial distances are observed by Electronic Distance Measurements (EDM). In frame **(ADD POINT FROM BASE POINT BY)**, The user can use **(Azimuth & Spatial Distances)** button to add new point to the map by special distance and geodetic azimuth from chosen point. push (add new point) button in this form after selection and input them, the geodetic coordinate (ϕ, λ, h) will be computed and recorded in database and AutoCAD. figure (A-8),

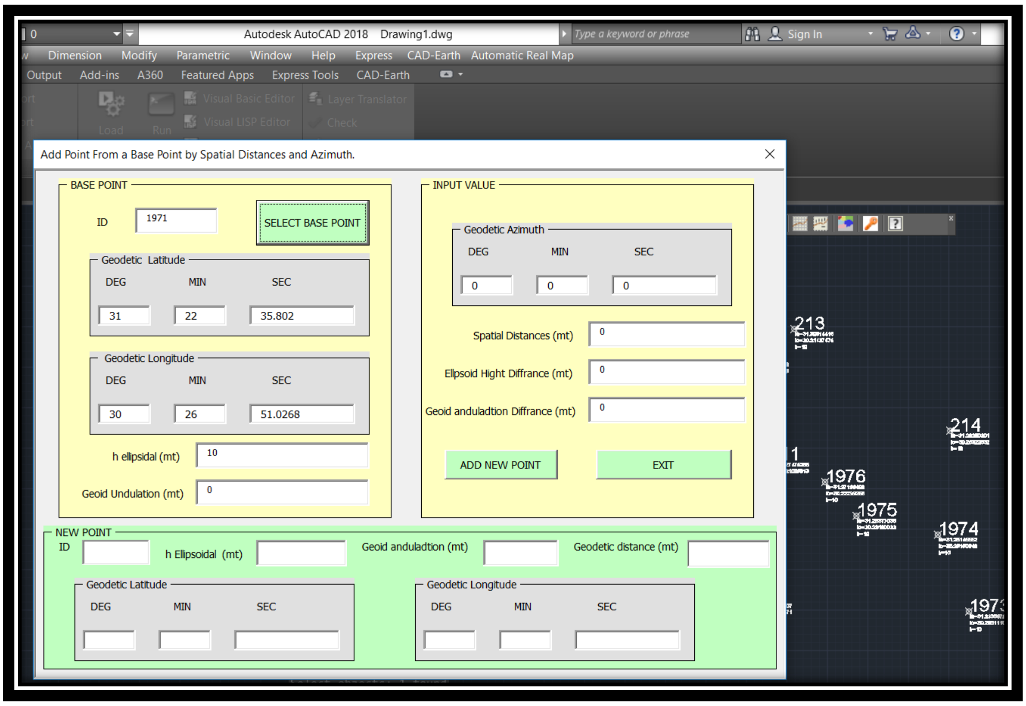

Figure (A-8): Draw point by spatial distance and geodetic azimuth.

7. In frame **(ELEMENT PROPERTIES)**, the user can use **(Point Properties)** button to query about geodetic properties of selected point. The point properties are (geodetic curvilinear, astronomic and geodetic rectangular coordinates, also orthometric and ellipsoidal heights) can be obtained if geoid data are available, figure (A-9).

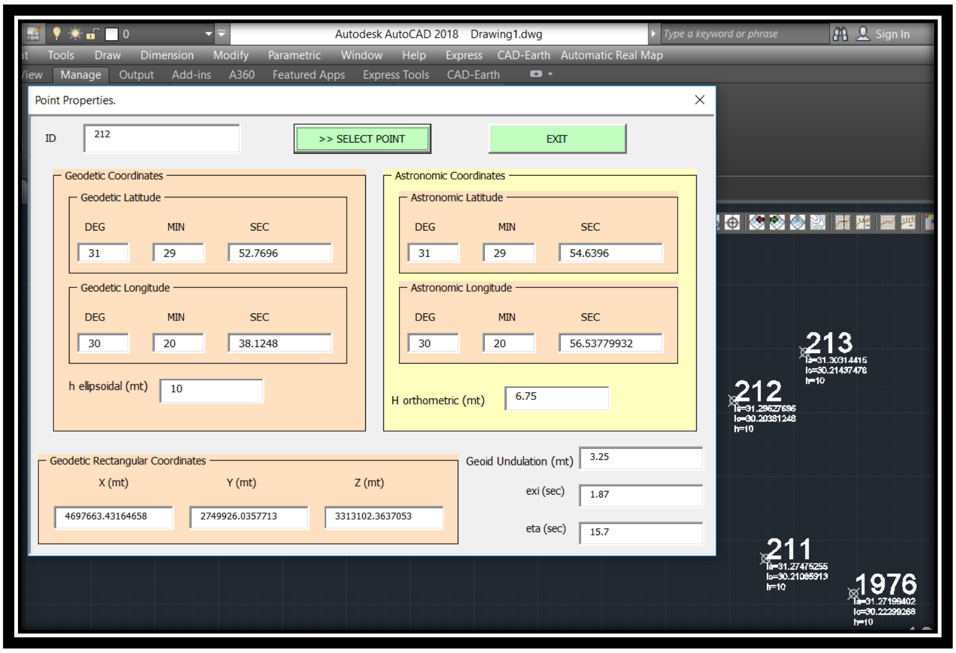

Figure (A-9): Geodetic properties of the data base point.

8. In frame **(ELEMENT PROPERTIES),** the user can use **(Azimuth and Distance for line)** button to query of selected line which linked between two geodetic database points. The output results are forward and back geodetic azimuth in addition geodetic and special distances of selected line, figure (A-10)

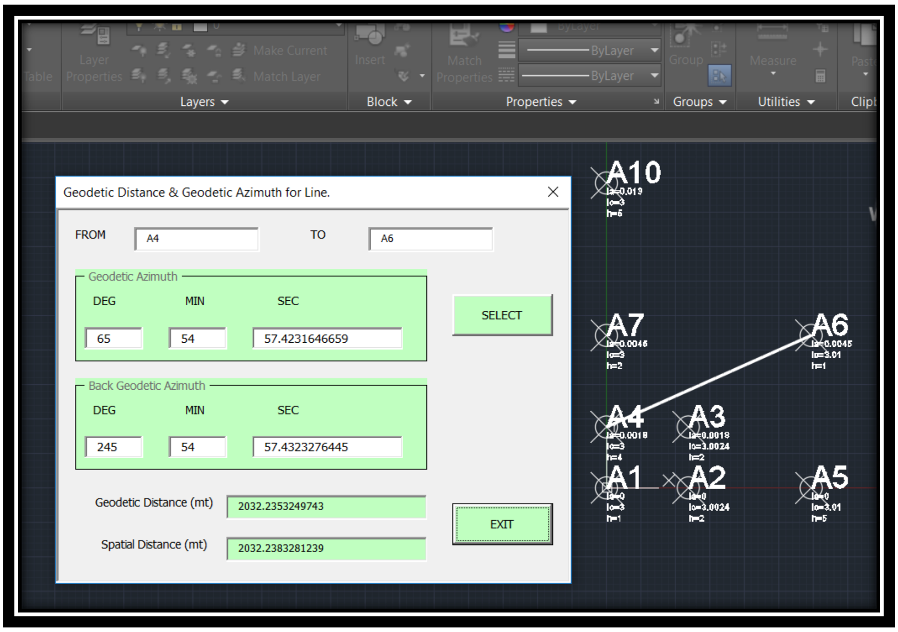

Figure (A-10): Geodetic azimuth and Geodetic & special distances of the selected line.

9. In frame **(ELEMENT PROPERTIES),** the user can use **(Line Geodetic Data)** button to query of selected line and its of all properties (geodetic distance, geodetic azimuth & geodetic back azimuth, geodetic rectangular coordinates and geodetic curvilinear coordinates of its two terminal points, latitude and longitude differences, rectangular coordinates differences also spatial distance) can be obtained once pushing the specified key, figure (A-11),

Figure (A-11): Geodetic data of the selected line.

10. (Geodetic Data for draw Line) button is used in frame **(ELEMENT PROPERTIES)** to query for drawing line and getting geodetic data .The geodetic properties of drawn line is obtained by click insertion only, the command don't need to exist database points, this program will recognize the geodetic coordinate points of the drawn line, then calculate all geodetic properties for this line, figure(A-12).

Figure (A-12): Geodetic data of the drawn line.

11. (Horizontal Angle Value) button is used in frame **(ELEMENT PROPERTIES** to compute and query the angle value which is connected between 3 point as polyline, the form shows points number and geodetic coordinates of these points. This angle is computed by the calculation of the difference between two azimuth of the two angle lines, in addition to the internal and external angles as degree and radian are shown, figure (A-13).

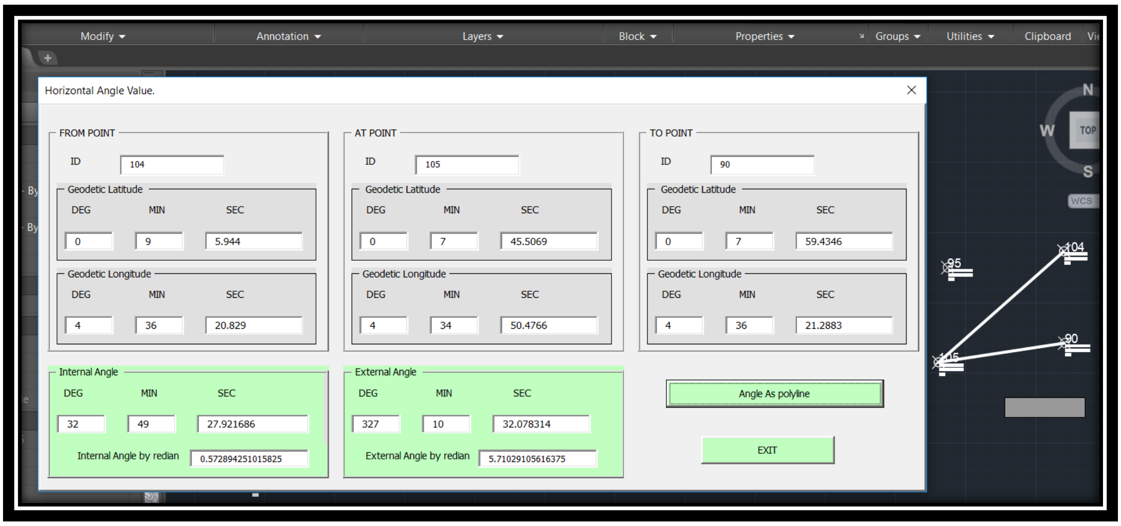

Figure (A-13): The horizontal angle of the selected angle as polyline.

12. (Properties of Ellipsoidal Triangle) button is used to get all geodetic properties of the ellipsoidal triangle as a closed polyline in frame **(ELEMENT PROPERTIES).** A closed polyline between three points can be drawn as triangle, then it is selected by this key to compute the ellipsoidal area and the geodetic circumference. Also, this from shows the points of triangle vertices name , coordinates, internal angles, ellipsoid distances of triangle sides, average of Gaussian Mean Radius at point triangle vertices and spherical excess, figure (A-14).

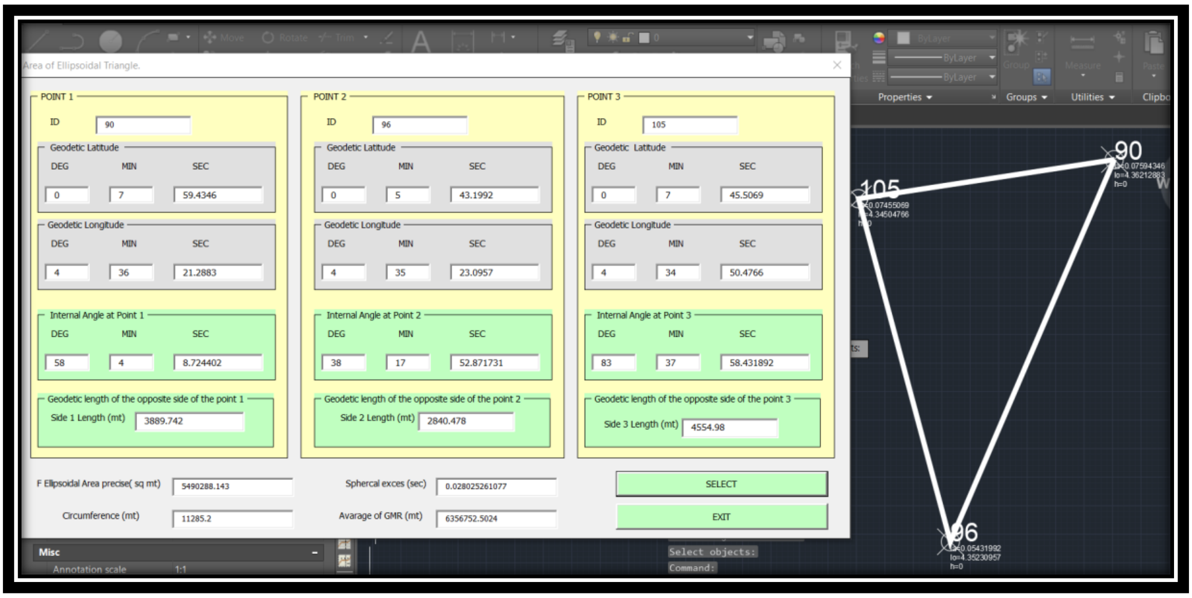

Figure (A-14): Geodetic data of the ellipsoidal Triangle selected as triangle polyline.

13. **(Ellipsoidal Polygon Area)** button is used in frame **(ELEMENT PROPERTIES)** to compute and query the ellipsoidal area and the geodetic circumference in the closed polyline between several points. The area divides into multi ellipsoidal triangle, the summation of the areas of ellipsoidal triangle will be collected, in addition to the summation of ellipsoidal distances of sides will be collected to calculate the circumference, figure (A-15).

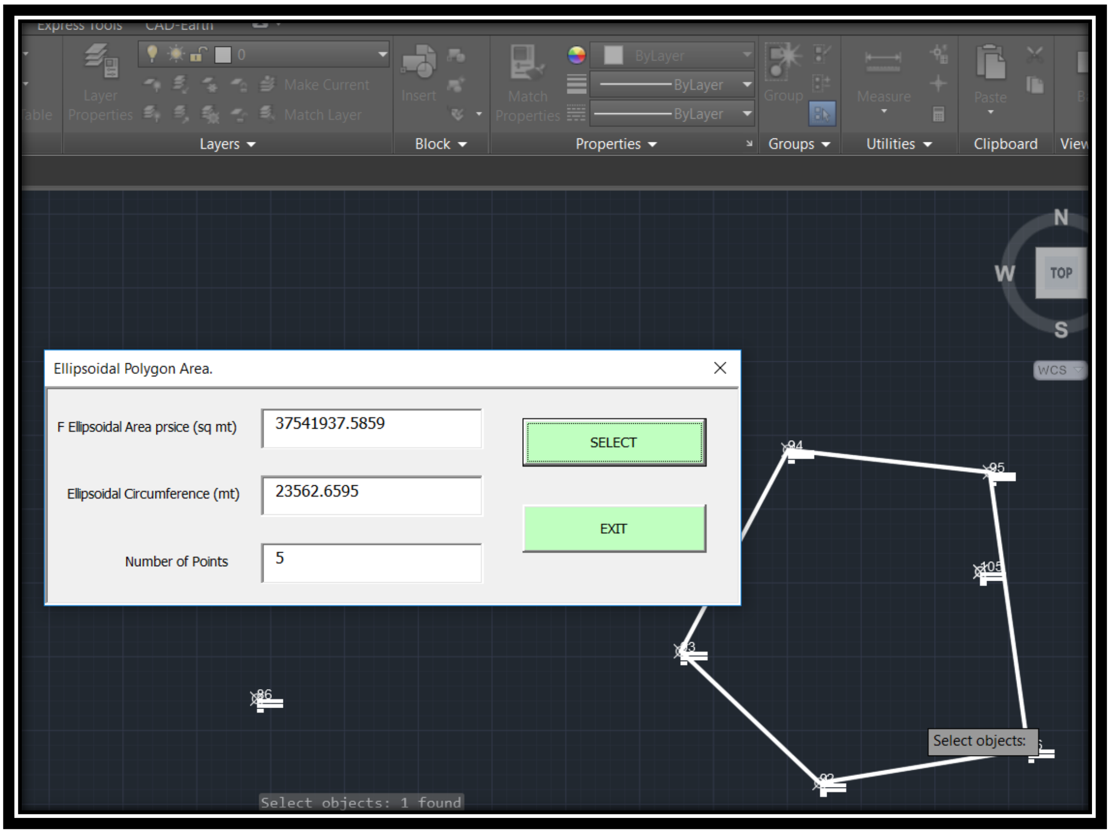

Figure (A-15): Ellipsoidal area of selected polygon as closed polyline.

14. In frame **(ELEMENT PROPERTIES),** the user can use **(Graticules Design)** button to draw the map graticules by choosing the dimensions and graticule limits, the program can compute the maximum and minimum of latitude and longitude. For cadastral maps design graticule may be 1" x 1" or 2" x 2" or any value, in topographic maps at small scales are chosen as 1' or 2' or 5' or more in geographic map could be designed as degrees.

- In figure (A-16), the output of the automatic real map program, geographic map of Egypt with graticules is 1 º x 1º.
- In figure (A-17), the output of the automatic real map program, the geographic map of Africa with graticules is 5 º x 5º.

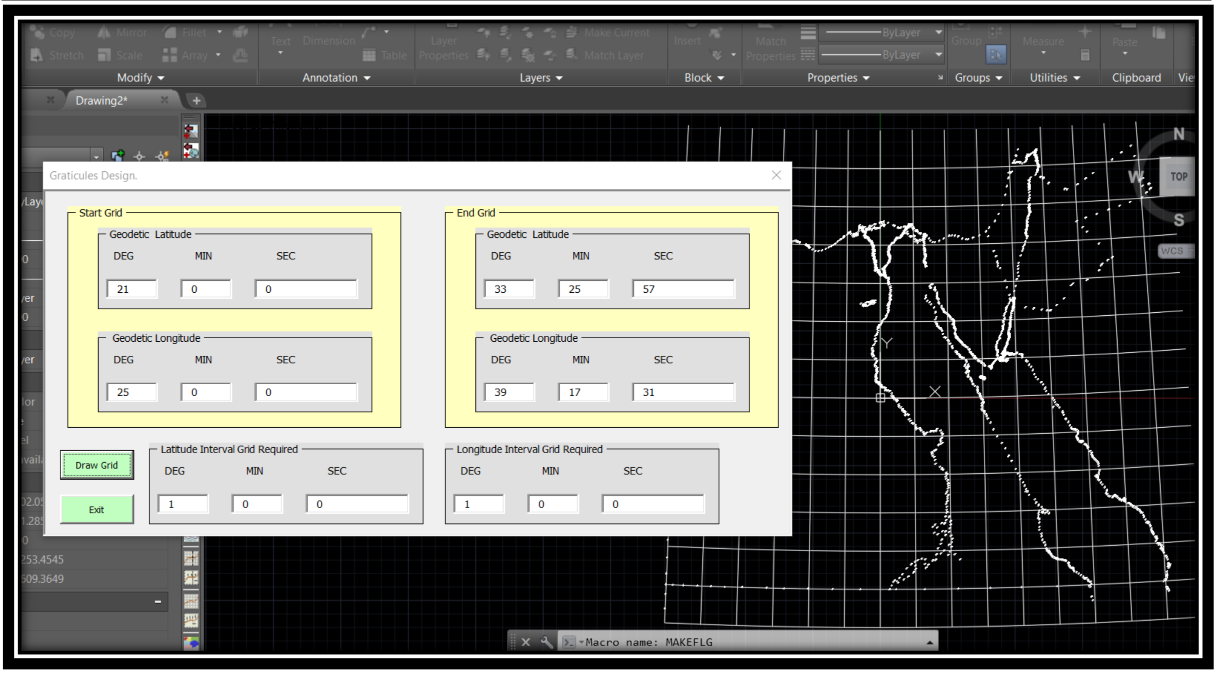

Figure (A-16): Graticules Design in an automatic real map, Egypt map graticules 1º x 1º.

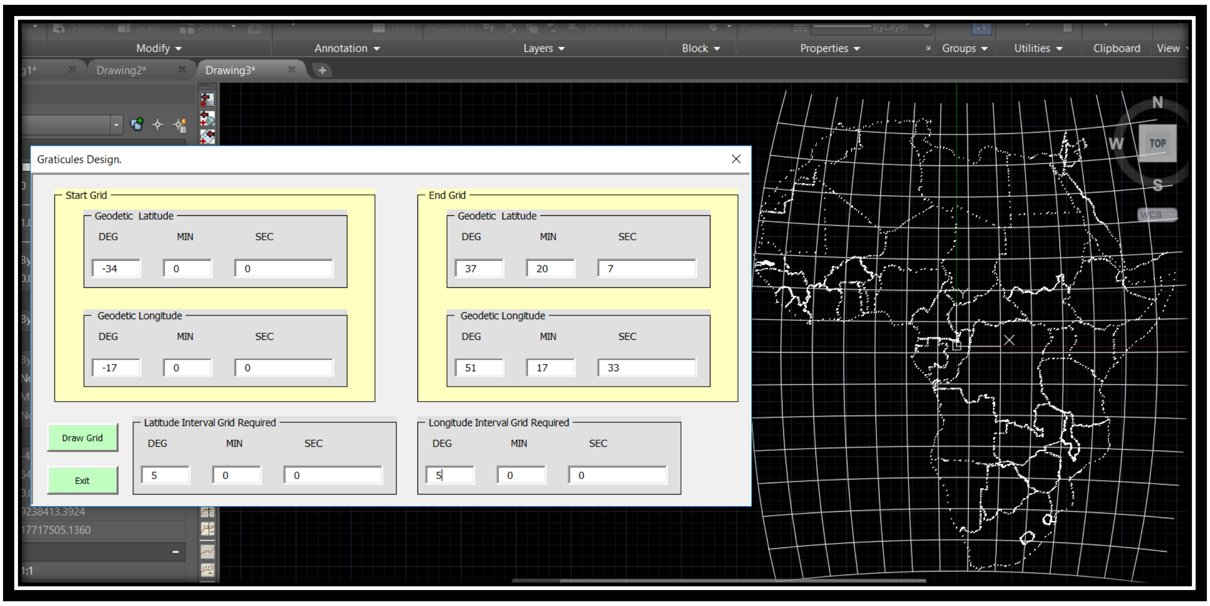

Figure (A-17): Graticules Design in an automatic real map, Africa map graticules 5º x 5º.

- The latitude interval deferent could be different to longitude interval in this option, the polar area could use it , in addition to design the map index of any country, the graticules are designed in north area as 5' (ΔØ) x 5º (Δλ), this dimensions are ≈ 9 km x 9 km at Ø = 89º , figure (A-18).

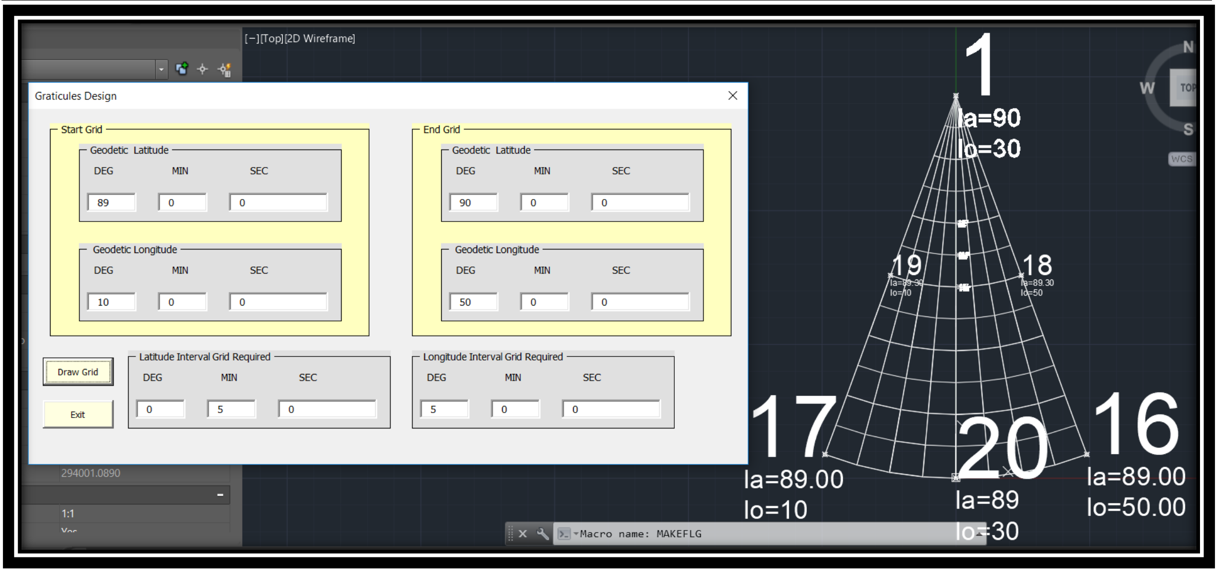

Figure (A-18): The graticules Design in an automatic real map, North area graticules 5' (ΔØ) x 5º (Δλ).

- The map index of any area or country is designed by this option . the latitude interval deferent and longitude interval are chosen , see sec 5.3.1.

In figure (A-1 9), the proposal map index in Egypt of maps are 1:100000 as dim 30'(ΔØ) x 40'(Δλ).

In figure (A-20), the proposal map index in Egypt from Ø = 29 to 31 N & λ = 29 to 31 E for maps are 1:10000 as dim 3'(ΔØ) x 4'(Δλ), each map is 1:100000 divide to 100 maps by scale 1:10000. We can easily divide it into any country by any scale map index.

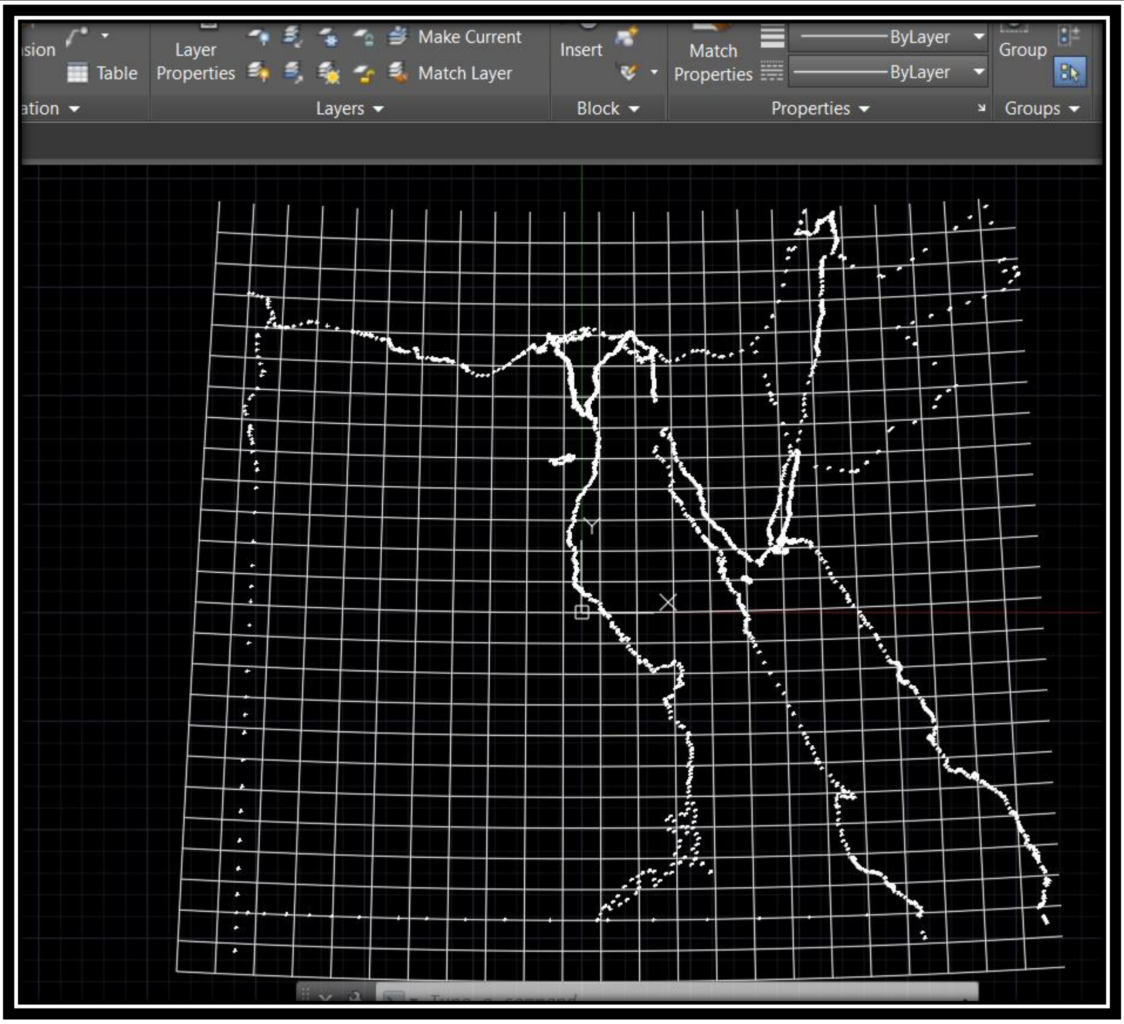

Figure (A-19): Proposal map index in Egypt of maps are 1:100000 by dim 30'(ΔØ) x 40'(Δλ).

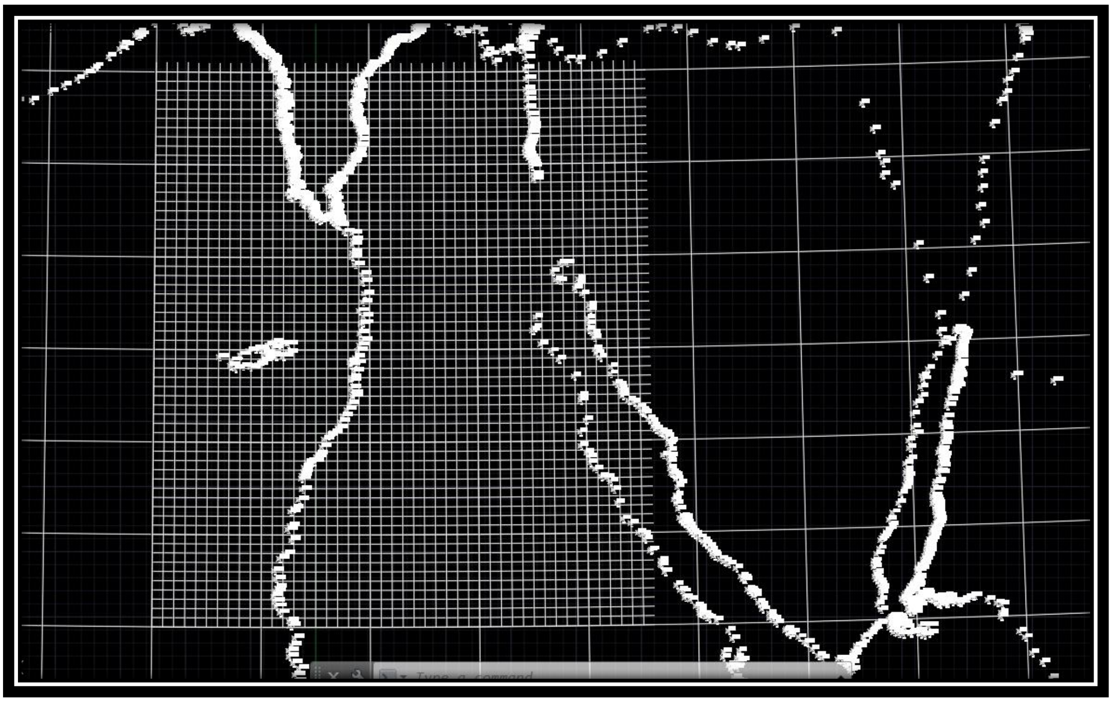

Figure (A-20): Proposal map index in Egypt latitude 28º : 31º N & longitude 30º : 33º E, maps 1:10000 by dim 3'(ΔØ) x 4'(Δλ)

جامعة بنها
كلية الهندسة بشبرا
قسم هندسة المساحة

تطوير خريطة حقيقية في صورة أوتوماتيكية

رسالة دكتوراه الفلسفة
المقدمة إلى كلية الهندسة بشبرا
قسم هندسة المساحة

من المهندس
عبد الرحمن سيد أحمد عرفة

بكالوريوس في هندسة المساحة (1997)
ماجستير في المساحة والجيوديسيا (2005)
كلية الهندسة بشبرا

تحت إشراف

الأستاذ الدكتور / أحمد عبد الستار شاكر أستاذ المساحة والجيوديسيا

الأستاذ الدكتور /عبد الله أحمد سعد أستاذ المساحة والجيوديسيا

كلية الهندسة بشبرا
جامعة بنها

القاهرة في 2019

جامـــــعة بنـــــها
كلية الهندسة بشبرا
قسم هندسة المساحة

القبول النهائي للرسالة

تطوير خريطة حقيقية في صورة أوتوماتيكية

رســــــــــالة مقدمة من

المهندس/ عبدالرحمن سيد أحمد عرفة

بكالوريوس هندسة المساحة (1997)

ماجستير في المساحة والجيودسيا (2005)

كجزء من متطلبات الحصول علي درجة دكتوراه الفلسفة في هندسة المساحة تخصص المساحة والجيوديسيا

وقد تمت مناقشة الرسالة والتوصية بالموافقة علي منح درجة دكتوراه الفلسفة في هندسة المساحة تخصص المساحة والجيوديسيا من لجنة الممتحنين

أعضاء لجنة الحكم والمناقشة

أ.د / أحمــــد عبد الســـــتار شـــــاكر **التوقيع:** ....

أستاذ المساحة والجيوديسيا بكلية الهندسة بشبرا - جامعة بنها

أ.د / ســـعــد زكي محمـد بـلـــــبل **التوقيع:** ....

أستاذ المساحة والجيوديسيا بكلية الهندسة بشبرا - جامعة بنها

أ.د / محمد الحسيني عبد الخالق الطوخي **التوقيع:** ....

أستاذ المساحة والجيوديسيا بكلية الهندسة - جامعة عين شمس

أ.د / عبدالله أحــمــد ســـعـــــــد **التوقيع:** ....

أستاذ المساحة والجيوديسيا بكلية الهندسة بشبرا - جامعة بنها

تاريخ المناقشة: 19 / 5 / 2019

تعريف مقدم البحث

الاسم	:	عبد الرحمن سيد أحمد عرفة عرفة
تاريخ الميلاد	:	11 / 7 / 1971
الميلاد والإقامة	:	منيل شيحة – محافظة الجيزة
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المؤهلات الدراسية	:	بكالوريوس الهندسة المساحية يوليو 1997 كليـة الهندسة بشبرا جامعة الزقازيق / فرع بنها(جامعة بنها حاليا)
	:	ماجستير في المساحة والجيوديسيا مارس 2005 بعنوان ضبط الارصاد التقليدية باستخدام المتجهات الخطية. كليـة الهندسة بشبرا جامعة الزقازيق/ فرع بنها(جامعة بنها حاليا)
سابق الخبرة	:	منذ التخرج في كلية الهندسة وحتي 2012 مهندس مساحة ومدير إدارة بشركة المقاولات المصرية – مختار إبراهيم .
	:	من 2012 وحتي 2014 مدير عام المساحة بشركة هورس للأعمال الهندسية.
	:	من 2014 الي الان مدير عام المساحة بشركة سيركون للمقاولات العامة.

ملخص البحث

يعتمد إنتاج الخرائط في إتمامه على علم إسقاط الخرائط و هو تمثيل السطح الكري على مستوى أو سطح قابل للإفراد وهو ما يعبر عنه رياضياً بتحويل الإحداثيات الجغرافية (خطوط الطول و العرض) إلي إحداثيات شماليات و شرقيات (إحداثيات مسقطة)؛ و يكون هذا السطح إما مستوى أو مخروط أو أسطوانة ولكلٍ أنواع عديدة ومعادلات خاصة بكل نوع مثل أن يكون سطح الإسقاط معتدلاً أو مستعرضاً أو مائلاً؛ وأحياناً يتماس السطحين وأحياناً يتقاطعان وأحياناً يكون الإسقاط لهدف معين مثل أن يكون الهدف إنتاج خريطة تحافظ على المساحة وأخرى تحافظ على الشكل وثالثة تحافظ على الإتجاهات أو المسافات وأحيانا يتم الإسقاط على سطح واحد كما في غالب الأنواع ومنها إسقاط مركيتور وأحياناً يتم الإسقاط على أسطح متعددة مثل إسقاط مريكتور المستعرض الدولي والإسقاط المخروطي المتعدد. وهناك الإسقاط الهندسي وهناك الإسقاط الرياضي والأول يكون الإسقاط من نقطة حقيقية يمكن تخيل أشعة تخرج منها(إسقاط مركزي) أو تكون الأشعة متوازية (إسقاط عمودي) و الثاني يعتمد على معادلات رياضية تحقق شرطاً محدداً يطلب سلفاً وليس له نقطة إسقاط حقيقة يمكن تخيل الإسقاط منها.

ويتم إختيار نوع الإسقاط بما يحقق الهدف من الخريطة فإذا كان الهدف المساحات نختار إسقاط متساوي المساحات فهو يزيل تشوه المساحات ويظهر تشوهات أخري؛ ويفضل إسقاط على آخر عند تحقيق أقل تشوه فنجد هناك إسقاط للخرائط يناسب القطبين ولا يناسب منطقة الإستواء وهكذا. ويمكن القول بأن هذه الأنواع العديدة والتصنيفات الكثيرة كانت لهدف تقليل التشوه في الخريطة والتي لايمكن التخلص منها تماماً عند الإسقاط.

وتشوه الخريط ينتج عن إسقاط السطح الكري للأرض إلى مستوى أو إلى سطح قابل للإفراد حيث تنتج إختلافات فى العلاقات الرياضية بين النقاط وبعضها؛ أى أن الأطوال والإنحرافات والمساحات والأشكال المأخوذة من إحداثيات الخريطة تختلف عن نظائرها فى الحقيقة مما يسبب مشاكل للعارفين وغير العارفين بعلم إسقاط الخرائط وهذا التشوه يتغير من مكان لآخر ومن نقطة لأخرى وليس ثابتاً في الخريطة مما يشكل صعوبة للمتخصصين في الجوديسيا حال التعامل مع غير المتخصصين عند تسليم الأعمال المساحية. ونجد خريطة تحافظ على الشكل تحدث تشوها في المساحات والأطوال ونجد خريطة تحافظ على المساحة تحدث تشوها في الأشكال والإتجاهات والمسافات؛ وهكذا لا نستطيع أن نحصل على خريطة مسقطة خالية من كل أنواع التشوهات.

على ما نعتقد لجأ العلماء لعلم إسقاط الخرائط هروبا من المعادلات الجيودسية الكبيرة والصعبة في التعامل وقتها حيث لا يوجد آلات حاسبة ولا حاسب آلي وظلت المشكلة قديماً في الإسقاط نفسه حيث معادلات إسقاط الخرائط أحياناً ليست بسيطة

على أنه بعد الإسقاط لا يهتم المختصين إلا بإحداثيات الخريطة والمعادلات الرياضية البسيطة الخاصة بها. وهنا قفزة عن الحقيقة حيث حسابات أي خريطة تفرض أننا علي مستوى وحيد أما الواقع فالراصد يتحرك علي مستويات كثيره بعدد وقفات الرصد التي رسمت منها الخريطة فكل نقطة محتلة بالتيودوليت لها مستوي أفقي خاص بها ويتم تجاهل علاقة هذه المستويات ببعضها البعض. فالارصاد علي مستويات متعددة والحسابات للخريطة المسقطة علي مستوي وحيد. وسبب آخر في ظني أن النظرة الأولي في علم إسقاط الخرائط بدأت من النظرة الكلية الشمولية لمحاولة تمثيل الأرض ككل على خريطة واحدة؛ فالكرة والمجسم الإهليجي (الإلبسويد) مجسمين لا يمكن الوصول لحد نهاية لهما فكلما حاولنا الوصول لحد نهاية إستمر البسط بلا انقطاع ولذا تم التفكير في الإسقاط لهذا المجسم على سطح له نهايات مثل المستوى أو المخروط أو الإسطوانة.

قد يتسائل مساح بسيط لماذا نقوم بإسقاط الخريطة لرسم مدينة أو قرية لا يظهر فيها كروية الأرض بعد؛ كما أنه يمكن رسمها بدون هذة الإشكالية؛ إن عمل خريطة للأرض ككل يبدو بمشكلتين الأولى هي حدوث التشوهات المختلفة بسبب الإسقاط والثانية أن الخريطة لها حدود على الورق والأرض ليس لها حدود بسبب كرويتها وهي غير متناهية البسط فهي في بسط متواصل بلا انقطاع.

لو نظرنا لمنطقة محددة بين خطي طول وخطي عرض مثلاً نجد أنها منطقة لها حدود حقيقية يمكن تحديدها في الطبيعة ويمكن تمثيلها على الخريطة بهذه الحدود ويمكن تكرار هذا الأمر على منطقة مجاورة وهكذا تكرر. أي أن نجعل نظرتنا تبعيضية غير شمولية لتمثل خرائطنا التفصيلية والطبوغرافية. لو تمت النظرة المبعضة غير الشاملة لأمكن تخيل خريطة بدون الإسقاط على هذه المساحة المحدودة والتي لا يظهر فيها تأثير كروية الأرض واضحاً; **كما أننا لا نتصور تمثيل الأرض ككل بدون علم إسقاط الخرائط وتبقي هذه الخرائط جغرافية عالمية أو قارية وهي خلاف ما نصبوا إليه من إنتاج خرائط طبوغرافية وتفصيلية بدون إسقاط الخرائط بل ويمكن تمثيل الدول والقارات كذلك وتنفيذ المشاريع العابرة للقارات بدون مشاكل.** على أنه يجب التفريق والفصل بين الهدف وهو رسم الخريطة وإنتاجها بأي طريقة وبين الحسابات عليها ولو تم هذا الفصل لأمكن لنا أن نعود بالحسابات علي سطح الارض مباشرة وانتفاء معضلة التشوهات في الخريطة.

و عليه فإن فكرة هذا البحث هى التخلص من هذه التشوهات وذلك بتمثيل الأرض كما هى ونتعامل مع الأرض على حالها – بدون خطوة الإسقاط من المجسم الإهليجي إلي سطح إسقاط – في صورة خريطة حقيقية أوتوماتيكية نقوم بتمثيل العمليات المطلوبة من مسافات وإنحرافات ومساحات خالية تماما من التشوهات. لنتجاوز المشاكل التي تنتج من جراء الإسقاط خاصة ونحن فى زمن أجهزة الحاسب والخرائط الرقمية والإحتياج الشديد لها فى مجال نظم المعلومات الجغرافية.

أهداف البحث:

- التخلص من التشوه الناتج من الخرائط بسبب عملية إسقاط الخرائط.
- انتاج خرائط رقمية ممثلة بخطوط الطول و العرض دون الحاجة للإسقاط.
- التعامل مع النسخ الرقمية من الخرائط على هيئة خطوط الطول والعرض وحساب المسافات والإنحرافات والمساحات منها بمعادلاتها الجيوديسية خاليه من التشوهات.
- وضع مقترحات لترقيم وترتيب الخرائط يبسط عملية الترقيم والإستدلال على الخريطة بمقاييس الرسم المختلفة بما لا يحمل تداخلا بين الخرائط عند مناطق التجاور – مثل التداخل والغموض بين الخرائط في منطقة التجاور بين أحزمة الإسقاط المصرية في نظام مركيتور المصري المستعرض.وكذلك نظام مريكتور الدولي المستعرض

تم تقديم الرسالة في ستة أبواب :

الباب الأول: مقدمة و فيه تم ذكر أهمية إنتاج الخرائط في تقدم الدول واعتماد إنتاج الخرائط على علم إسقاط الخرائط والمعني الرياضي له وهو تحويل الإحداثيات الجيوديسية إلي إحداثيات مسقطة شماليات وشرقيات والتي تنتج معها تشوهات الخريطة. بعدها تم ذكر الهدف من الرسالة وأخيراً تم عمل تلخيص لأبواب الرسالة المختلفة.

الباب الثاني: وتم فيه ذكر فكرة مبسطة عن علم إسقاط الخرائط . وذكرت بعض التعريفات الشهيرة في البحث وتاريخ علم إسقاط الخرائط وتطوراته ومقترحات العلماء بنظرة سريعة وتم استعراض التصنيفات المختلفة في علم إسقاط الخرائط وهي تصنيفات تربط علاقة سطح الإسقاط وسطح الجسم الناقص .

- فمن حيث سطح الإسقاط فأحياناً يتم الإسقاط على مستوى أو أسطوانة أو مخروط .
- ومن حيث الوضع النسبي للسطحين معتدل ومائل ومستعرض.
- ومن حيث تقارب السطحين يكون السطحين متقاطعين أو متماسين.
- ومن حيث خصائص الخريطة فخريطة تحافظ على الشكل وأخرى تحافظ على المساحة وثالثة تحافظ على المسافات.
- ومن حيث كيفية إنتاج الخريطة فأحيانا تتم بإسقاط هندسي تخرج الأشعة من مركز تخيلي أو تكون متوازية وعمودية على سطح الإسقاط وأحياناً يكون الإسقاط رياضياً نقوم بحساب الشماليات والشرقيات من معادلات رياضية دون تخيل هندسي لها من نقطة إسقاط حقيقية .
- ومن حيث عدد سطوح الإسقاط فأحياناً يتم الإسقاط على سطح وحيد؛ وأحياناً أخري تتعدد سطوح الإسقاط لنفس المنظومة بغية تقليل التشوه والذي يزاداد كلما إبتعدنا عن مناطق التماس والتقاطع.

وأخيرا تم إستعراض أنواع التشوهات وكيفية تضخم التشوهات لتصل في بعض الأحيان إلي أضعاف كثيرة. و نجد تأثر إنتاج خريطة تحافظ على الشكل في تشوه المساحات والمسافات؛ كذلك تأثر إنتاج خريطة تحافظ على المساحة في تشوه الأشكال والمسافات وهكذا.

الباب الثالث : وتم فيه إستعراض المعادلات الرياضة المستخدمة لحساب المساحات والمسافات والإنحرافات من إحداثيات الخريطة (الإحداثيات المسقطة). ثم الحسابات المناظرة علي المرجع الجيوديسي في الخريطة المقترحة من معادلات المسألة المباشرة والعكسية الجوديسية في الخطوط القصيرة والطويلة والتحويلات من الإحداثيات الجيوديسية الي الإحداثيات الكارتيزية المتعامدة ومناقشة المساحة علي سطح الإهليج واستنباط قانون لحساب مساحة مثلث كري أو أهليجي بمعلومية أطوال أضلاعة الجيودسية ثم إضافة تصحيح كري له.

الباب الـــــرابع: تطرقنا الي حساب بسيط على سطح كرة والمسافة الفراغية بين النقطتين والتي تمثل علاقة الوتر بالمنحني بين نقطتين وقيمة الفرق بما يظهر كروية الارض وأن كروية الارض تنتفي عملياً في المسافات القصيرة. تم إختيار مجموعة خرائط للدراسة في نظام الإسقاط المصري والذي تتم فيه الحسابات على مجسم القطع الناقص هلمرت 1906 وقمنا بحساب الفرق بين المسافات من الخريطة وعلى سطح الإهليج ومدي تأثير هذه الفروق على طباعة الخريطة والفروق بينهم في مقاييس الرسم المختلفة وتم إختيار مجموعة خرائط في منتصف الحزام الأوسط لمصر عند خط طول 31 درجة شرقا في منطقة التشوه الصغري ومجموعة أخري عند طرف الحزام في منطقة التشوه العظمى. ثم تم هذا العمل في حالة عالمية حيث تم إختيار حزام رقم 31 في نظام مركيتور المستعرض الدولي مسقطاً من المرجع الجيوديسي الدولي WGS84 وتم اختيار مجموعة في منتصف الحزام وأخري في عند الحد الشرقي للحزام وتمت الدراسة في منطقة الإستواء وأخري عند دائرة عرض 30° وثالثة عند دائرة عرض 60° وأخيراً عند 70 ° و 80 ° وأعيدت حسابات الفروق ومدي تاثيرها على رسم الخريطة وأخيراً رسم خريطة بدون الإسقاط وخطوات تنفيذ ذلك .

الباب الـخامس : وفيه تم إستعراض ترقيم وترتيب الخرائط بنظمها المختلفة في مصر والخريطة المليونية الجغرافية ثم تم وضع مقترح لمنظومة خرائط للعالم تعتمد على أبعاد جغرافية ثابتــة لكل مقياس رسـم وهي تعتمــد على خريطــة 1:100000 بأبعاد مقترحة $(\Delta\lambda)40' \times (\Delta\emptyset)30'$ ثم إنطلاق التقسيم بناءً عليها فتصل الأبعاد في خريطة 1:10000 إلي أبعاد $(\Delta\lambda)4' \times (\Delta\emptyset)3'$ ويتواصل التقسيم فتصل الأبعاد على خريطة 1:1000 إلي أو بابعاد $(\Delta\lambda)\ 24'' \times (\Delta\emptyset)\ 18''$ وحتي خريطة 1:500 تكون بابعاد نصف أبعاد خريطة 1:1000 الأخيرة.

الباب السادس : في هذا الباب تم سرد ملخص للرسالة والذي يتمثل فيما وصلنا إليه من نتائج ومميزات إنتاج خريطة بدون إسقاط حيث لاحاجة لنا بتقسيم أحزمة كما في إسقاط مركيتور المستعرض المصري والدولي وتنتفي مشكلة ترتيب الخرائط في مناطق الجوار بين الأحزمة ونجد أن كل نقطة لها إحداثي جغرافي وحيد وفريد في المرجع الجيوديسي الواحد وهي سهلة جداً لعمل المشاريع الدولية والتي تتصل بدون إنقطاع كالطرق الدولية وخطوط البترول حيث تظل في نظام إحداثي واحد. وحتي المشاريع المحلية حال وقوعها في منطقة التجاور بين أحزمة الإسقاط يحدث إرباكاً شديداً للعارفين فضلاً عن غيرهم ثم التوصيات .

إحتوت الرسالة بعد ذلك على قائمة بالمراجع التى تم الاستعانة بها فى البحث مرتبة ترتيباً أبجدياً للسهولة على القارىء وأخيرا ملحق به شرح مبسط لوظائف برنامج خريطة حقيقية اوتوماتيكية هو يعمل باضافة الوظائف المطلوبة في الاوتوكاد وبرنامج Civil 3D.

أهم التوصيات

- تزامنا مع التطور الكبير جدا في علوم البرمجيات والحاسبات وكذلك القفزات السريعه في رسائل الاقمار الصناعية للجاذبية والاستشعار عن بعد ونظم الملاحة فإننا نوصي بشدة باستخدام وانتاج الخريطة المقترحة (خريطة حقيقة أوتوماتيكية بدون اسقاط).
- استخدام نظام ترقيم الخرائط العالمي المقترح حيث أن الترقيم يدل على المكان والمقياس والابعاد ومناسب جدا لمناطق التجاور بين الدول ويمكن رسم خرائط البلد الواحد في منظومة واحدة.
- استخدام المعادلة المقترحة لحساب المساحة علي سطح الالبسويد خالية من النتشوهات.
- زيادة عرض الخريطة في المناطق الشمالية عن العرض القياسي في نظام ترقيم الخرائط المقترح بسبب تقارب خطوط الطول عند الاقطاب.
- تطوير وإنتاج جهاز TOTAL STATION جيوديسي يعمل بإحداثيات جيوديسية وإستخدام المعادلات الجيودسية المباشرة والعكسية واستخدام بيانات الجيويد عند توافرها لانهاء مشاكل التشوهات في المسافات والانحرافات.
- التوصية لشركات برامج الرسم الكبري مثل Autodesk و Esri و Microsoft بإضافة الخريطة المقترحة الي برامجهم بعد التنسيق مع المؤلفين.