$2^{ème}$ année Licence en Electrotechnique - (Semestre 4)

THEORIE DU SIGNAL- SERIE DE TD N°1

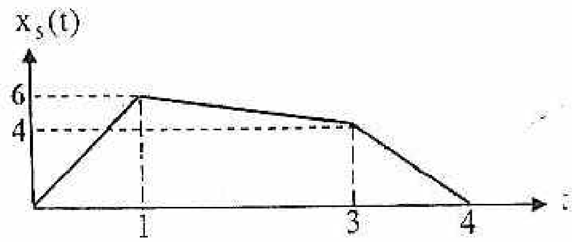
EX1 : Tracer les signaux suivants : Tracer les signaux suivants : $x_1(t) = \varepsilon(t)$; $x_2(t) = \varepsilon(t-1)$;

$x_3(t) = 2\varepsilon(t+1)$; $x_4(t) = \text{rect}(t)$; $x_5(t) = \text{rect}\left(\frac{t-2}{3}\right)$; $x_6(t) = \text{tri}(t)$; $x_7(t) = 2\text{tri}\left(\frac{t-3}{2}\right)$;

$x_8(t) = x_7(t).x_5(t)$

EX2 : Les signaux suivants sont-ils à énergie finie, à puissance moyenne finie ou ni l'un ni l'autre ?

$x_1(t) = e^{-4t}\varepsilon(t)$, $x_2(t) = 2(\cos(3t)+\sin(3t))$, $x_3(t) = \text{tri}\left(\frac{t-1}{2}\right)$ et $x_4(t) = e^{-2\pi t^2}$



On donne :

1-Fonction Echelon-unité $\varepsilon(t)$

$$\varepsilon(t) = \begin{cases} +1 & \text{si} \quad t > 0 \\ 0 & \text{si} \quad t < 0 \end{cases}$$

2- Fonction Rectangulaire rect (t) :

$$\text{rect}(t) = \begin{cases} 1 & \text{si} \quad |t| < \frac{1}{2} \\ 0 & \text{si} \quad |t| > \frac{1}{2} \end{cases}$$

3-Fonction triangulaire tri (t) :

$$\text{tri}(t) = \begin{cases} 1-|t| & \text{si} \quad |t| < 1 \\ 0 & \text{si} \quad |t| > 1 \end{cases}$$

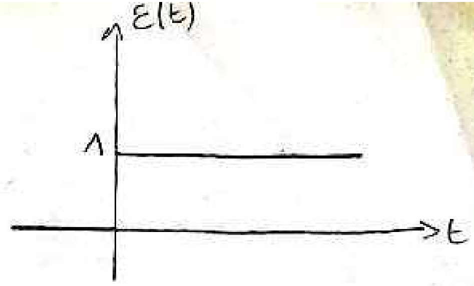
$4 - \int(\alpha t+\beta)^n dt = \frac{1}{\alpha(n+1)}(\alpha t+\beta)^{n+1} + C$

$\alpha \in R^*$ et $n \in N$

$5 - \int_{-\infty}^{+\infty} e^{-\pi t^2} dt = 1$ (Intégrale de Gauss)

Exo1 :

1) $x_1(t) = \varepsilon(t)$

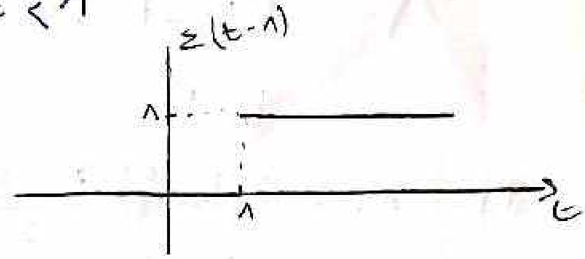


$$\varepsilon(t) = \begin{cases} 0 & \text{si } t < 0 \\ 1 & \text{si } t > 0 \end{cases}$$

2) $x_2(t) = \varepsilon(t-1)$

$$\varepsilon(t) = \begin{cases} 1 \text{ si } & t > 0 \\ 0 \text{ si } & t < 0 \end{cases}$$

$$\varepsilon(t-1) = \begin{cases} 1 \text{ si } & t-1 > 0 \Rightarrow t > 1 \\ 0 \text{ si } & t-1 < 0 \Rightarrow t < 1 \end{cases}$$

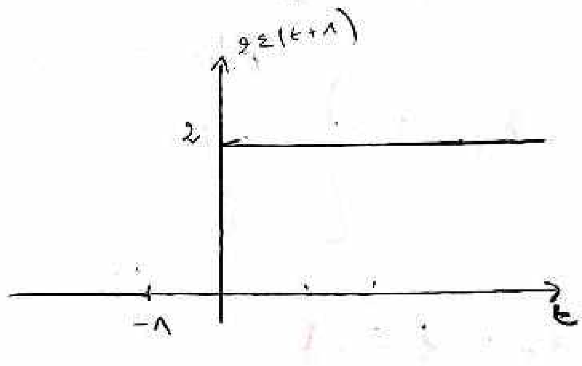


$$\varepsilon(t-1) = \begin{cases} 1 \text{ si } & t > 1 \\ 0 \text{ si } & t < 0 \end{cases}$$

3) $x_3(t) = 2\varepsilon(t+1)$

$$\varepsilon(t) = \begin{cases} 0 & \text{si } t < 0 \\ 1 & \text{si } t > 0 \end{cases}$$

$$2\varepsilon(t+1) = 2\begin{cases} 0 & \text{si } t+1 < 0 \\ 1 & \text{si } t+1 > 0 \end{cases}$$

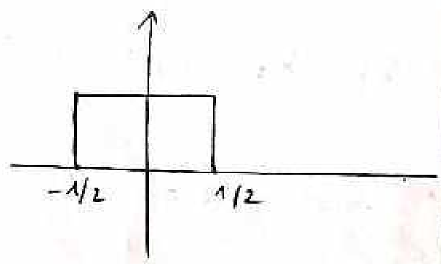


$$2 \cdot \varepsilon(t+1) = 2\begin{cases} 1 \text{ si } & t > -1 \\ 0 \text{ si } & t < -1 \end{cases}$$

$$2 \cdot \varepsilon(t+1) = \begin{cases} 2 \text{ si } & t > -1 \\ 0 \text{ si } & t < -1 \end{cases}$$

4) $x_4(t) = \text{rect}(t)$

$$\text{rect}(t) = \begin{cases} 1 & \text{si } -\frac{1}{2} < t < \frac{1}{2} \\ 0 & \text{ailleurs.} \end{cases}$$



5) $x_5(t) = \text{rect}\left(\frac{t-2}{3}\right)$

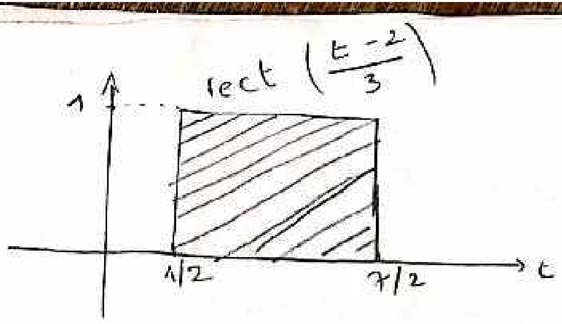
$$\text{rect}\left(\frac{t-2}{3}\right) = \begin{cases} 1 \text{ si } & -\frac{1}{2} < \frac{t-2}{3} < \frac{1}{2} \\ 0 & \text{Ailleurs} \end{cases}$$

$$-\frac{1}{2} < \frac{t-2}{3} < \frac{1}{2}$$

$$-\frac{3}{2} < t-2 < \frac{3}{2}$$

$$-\frac{3}{2}+2 < t < \frac{3}{2}+2$$

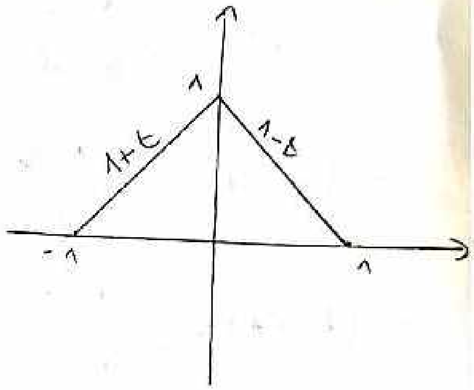
$$\frac{1}{2} < t < \frac{7}{3}$$



$$\text{rect}\left(\frac{t-2}{3}\right) = \begin{cases} 1 & \text{si } \frac{1}{2} < t < \frac{7}{2} \\ 0 & \text{Ailleurs.} \end{cases}$$

6). $x_6 = \text{tri}(t)$

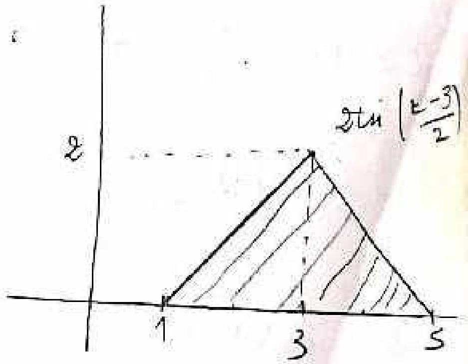
$$\text{tri}(t) = \begin{cases} 1+t & -1 < t < 0 \\ 1-t & 0 < t < 1 \\ 0 & \text{Ailleurs} \end{cases}$$



7)- $x_7 = 2\,\text{tri}\left(\frac{t-3}{2}\right)$ $\quad 1 < t < 3$

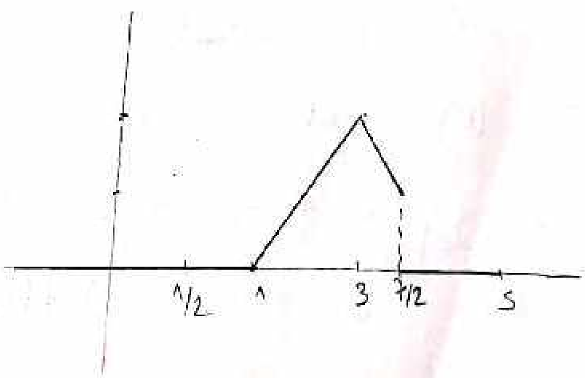
$$x_7(t) = 2\begin{cases} 1+\frac{t-3}{2} \;\left(\frac{t-1/2}{2}\right) & -1 < \frac{t-3}{2} < 0 \\ 1-\frac{t-3}{2} \;\left(\frac{5-t}{2}\right) & 0 < \frac{t-3}{2} < 1 \quad (3 < t < 5) \\ 0 & \text{Ailleurs} \end{cases}$$

$$x_7 = 2\begin{cases} \frac{t-1}{2} & \text{si } 1 < t < 3 \\ \frac{5-t}{2} & \text{si } 3 < t < 5 \\ 0 & \text{Ailleurs} \end{cases}$$



$$x_7 = \begin{cases} t-1 & \text{si } 1 < t < 3 \\ 5-t & \text{si } 3 < t < 5 \\ 0 & \text{Ailleurs} \end{cases}$$

8)- $x_8 = x_7(t) \cdot x_5(t)$



Exo 2 :

$W_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt$ et $P_x = \lim_{\alpha \to \infty} \frac{1}{\alpha} \int_{-\alpha/2}^{\alpha/2} |x(t)|^2 dt$.

* Un signal est à Energie finie si $W_x = \ell_1 \geq 0$ constante finie

$\Rightarrow P_x = 0$

* Un signal est à puissance moyenne finie $P_x = \ell_2 > 0$ Valeur finie non nulle.

1) - $\Rightarrow W_x \Rightarrow \infty$

$x_1(t) = e^{-4t} \varepsilon(t)$

$x_1(t) = \begin{cases} e^{-4t} & \text{si } t > 0 \\ 0 & \text{si } t < 0 \end{cases}$ $\rightarrow \lim_{x \to +\infty} x(t) = 0$, $\lim_{x \to -\infty} x(t) = 0$

Il faut commencé par l'energie

$W_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{0} 0^2 dt + \int_{0}^{+\infty} (e^{-4t})^2 dt = \int_0^{\infty} e^{-8t} dt = -\frac{1}{8} \{e^{-8t}\}_0^{\infty}$

$= -\frac{1}{8}[0-1] = \frac{1}{8}$ W.

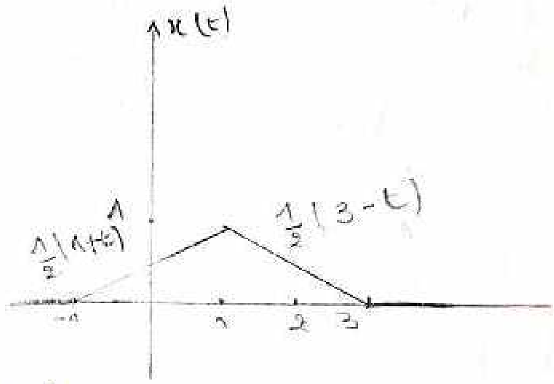
$x_1(t)$ est à Energie finie $\Rightarrow P_x = 0$

3) - $x_3 = \text{tri}\left(\frac{t-1}{2}\right)$

$\text{tri}(t) = \begin{cases} 1+t & -1 < t < 0 \\ 1-t & 0 < t < 1 \\ 0 & \text{Ailleurs} \end{cases}$

$|t| = \begin{cases} t & \text{Si } t > 0 \\ -t & \text{si } t < 0 \end{cases}$

$= \begin{cases} 1 + \frac{t-1}{2} & \text{si } 1 < \frac{t-1}{2} < 0 \\ 1 - \frac{t-1}{2} & \text{Si } 0 < \frac{t-1}{2} < 1 \\ 0 & \text{Ailleurs} \end{cases}$



$x_3(t) = \frac{1}{2} \begin{cases} 1+t & \text{Si } -1 < t < 1 \\ 3-t & \text{si } 1 < t < 3 \\ 0 & \text{Ailleurs} \end{cases}$

$\lim_{t \to +\infty} x_3(t) = 0$ $W_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \underbrace{\int_{-\infty}^{-1} 0^2 dt}_{=0} + \int_{-1}^{1} \frac{1}{4}(1+t)^2 dt + \int_1^3 \frac{1}{4}(3-t)^2 dt + \underbrace{\int_3^{\infty} 0^2 dt}_{=0}$

$$\boxed{\int (\alpha t + \beta)^n dt = \frac{1}{(n+1)\alpha} (\alpha t + \beta)^{n+1} + C} \quad | \alpha \neq 0$$

$W_x = \frac{1}{4} \cdot \frac{1}{1 \times 3} \left[[1+t]^3 \right]_{-1}^{1} + \frac{1}{4} \frac{1}{-1 \times 3} \left[[3-t]^3 \right]_1^3$

$= \frac{1}{12}[2^3 - 0^3] - \frac{1}{12}[0^3 - 2^3] = \frac{8}{12} + \frac{8}{12} = \frac{16}{12} = \frac{4}{3}$ W Signal à energie finie

$P_x = 0$

commence par l'energie si les deux limite tends vers 0 sinon par la puissance

$x_2(t) = 2(\cos(3t) + \sin(3t))$

$\lim_{x \to \pm\infty} x_2(t) \neq 0$

Il faut commencé par P_x

$P_x = \lim \frac{1}{\alpha} \int_{-\alpha/2}^{\alpha/2} |x(t)|^2 dt$, I

$= \lim_{\alpha \to \infty} \frac{1}{\alpha} \int_{-\alpha/2}^{\alpha/2} 4[\cos(3t) + \sin(3t)]^2 dt$

$I = 4 \int_{-\alpha/2}^{\alpha/2} \underbrace{\cos^2(3t) + \sin^2(3t)}_{1} + \underbrace{2\sin(3t)\cos(3t)}\, dt = 4 \int_{-\alpha/2}^{\alpha/2} 1 + \sin(6t)\, dt$

$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$A = B = X \Rightarrow \sin(2x) = 2\sin(x)\cos(x)$

$= 4\left[t - \frac{\cos 6t}{6}\right]_{-\alpha/2}^{\alpha/2}$

$= 4\left(\frac{\alpha}{2} - \frac{\cos(3\alpha)}{6}\right) - \left(-\frac{\alpha}{2} - \frac{\cos(-3\alpha)}{6}\right)$

$= 4\left(\frac{\alpha}{2} - \cancel{\frac{\cos(3\alpha)}{6}} + \frac{\alpha}{2} + \cancel{\frac{\cos(3\alpha)}{6}}\right)$ $= \lim_{\alpha \to \infty} \frac{1}{\alpha} I = \lim_{\alpha \to \infty} \frac{1}{\alpha} 4\alpha = 4$ Jaul

$= 4\alpha.$

$x_5(t) = ?$

Il faut commencé par l'energie.

$W_x = \int_{-\infty}^{+\infty} |x_5(t)| dt = \int_{-\infty}^{0} 0^2 dt + \int_0^1 (6t)^2 dt + \int_1^3 (\quad)^2 dt + \int_3^4 (\quad)^2 dt + \int_4^{+\infty} 0^2 dt.$

$x(1) = 6$
$x(3) = 4$

$x(t) = At + B$

$6 = A + B$ — ①

$4 = 3A + B$ — ②

$1 - 2 \Leftrightarrow 2 = -2A \Rightarrow A = -1$

$B = 6 - A = 6 - 1 = 7$

$\boxed{x(t) = 7 - t}$

$x(3) = 4$
$x(4) = 0$

$x(t) = At + B$

$4 = 3A + B$ — ①

$0 = 4A + B$ — ②

① - ② $\Leftrightarrow 4 = -A \Rightarrow A = -4$

$B = -4A = 16$

$x(t) = 16 - 4t = 4[4 - t]$

$$W_x = \int_{-\infty}^{0} 0^2 dt + 36\int_0^1 t^2 dt + \int_1^3 (7-t)^2 dt + \frac{16}{3}\int_3^4 (4-t)^2 dt + \int_4^{\infty} 0^2 dt$$

$$= \frac{36}{3} t^3 \Big]_0^1 + \frac{1}{3\times(-1)} (7-t)^3 \Big]_1^3 + \frac{16}{3(-1)} \Big[4 - t^3 \Big] \Big]_3^4$$

$$= 12(1^3 - 0^3) - \frac{1}{3}(4^3 - 6^3) - \frac{16}{3}(0^3 - 4^3) = \frac{204}{3} \text{ Joule.}$$

Signal a energie finie $\Rightarrow P_x = 0$

$$x_4 = e^{-2\pi t^2}$$

$$\lim_{t \to \pm\infty} x_4(t) = \lim_{t \to \pm\infty} e^{-2\pi t^2} = 0$$

Donc on va calculer l'energie du signal:

$$W_{x_4} = \int_{-\infty}^{+\infty} |x_4(t)|^2 dt = \int_{-\infty}^{+\infty} x_4^2(t)\, dt = \int_{-\infty}^{+\infty} (e^{-2\pi t^2})^2 dt = \int_{-\infty}^{+\infty} e^{-4\pi t^2} dt$$

On sait que:

Par le changement de variable suivant.

$x = 2t \rightarrow u^2 = 4t^2$

$du = 2dt \rightarrow dt = \frac{1}{2} du.$

$t \to +\infty \qquad x \to +\infty$

$t \to -\infty \qquad x \to -\infty$

$$W_{x_4} = \int_{-\infty}^{+\infty} e^{-\pi x^2} \left(\frac{1}{2} du\right) = \frac{1}{2}\int_{-\infty}^{+\infty} e^{-\pi x^2} du = \frac{1}{2}$$

W_{x_4} = constante $\Rightarrow$ Energie finie.

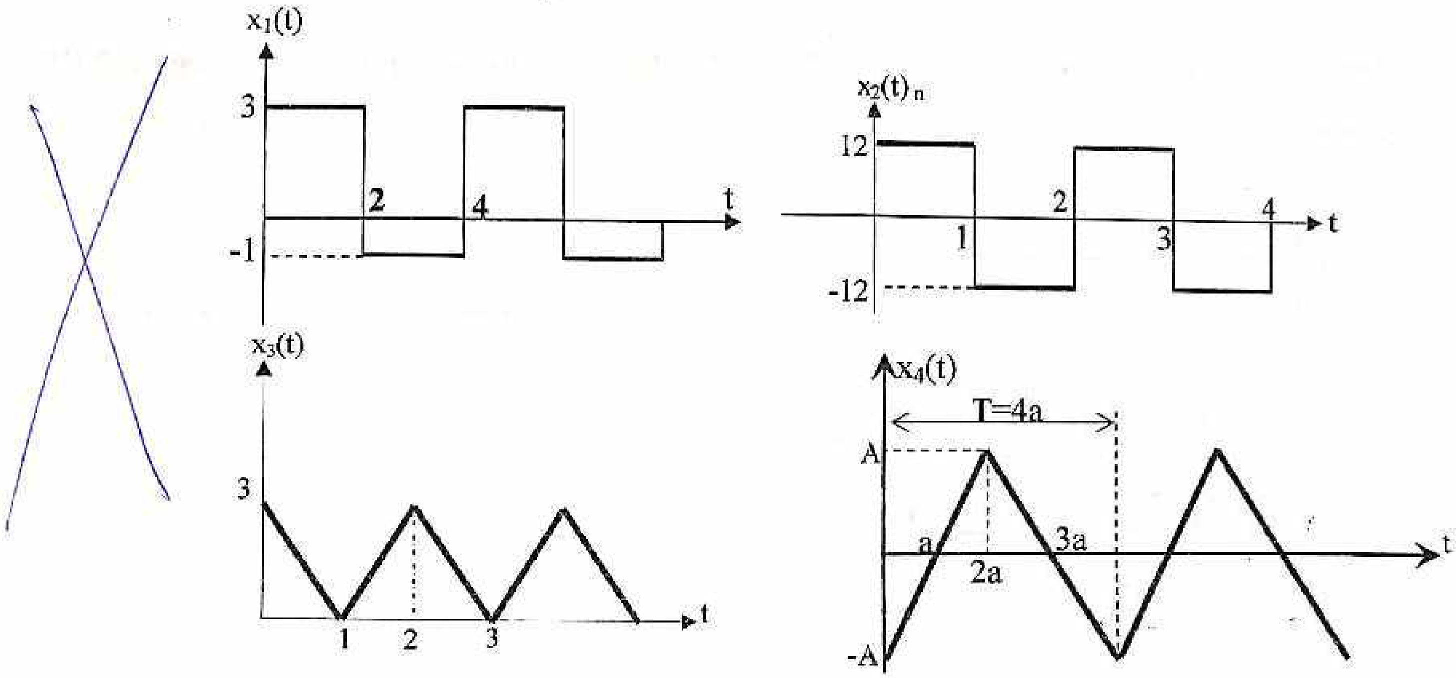
Donc, $x_4(t)$ est a puissance moyenne nulle

$$\boxed{P_{x_4} = 0}$$

Département d'Electronique - 2éme Année Licence en Electronique

Série de TD N°2 de l'unité : Théorie du Signal

Exercice N°1 : Décomposer en série de Fourier les signaux périodiques suivants puis tracer leur spectre d'amplitude pour $(n \leq 4)$



Exercice N°2 : Soit les signaux suivants : $x(t) = \text{rect}(t)$, $y(t) = \text{tri}(t)$ et $z(t) = e^{-|t|}$

1- Tracer les signaux puis calculer leurs transformée de Fourier

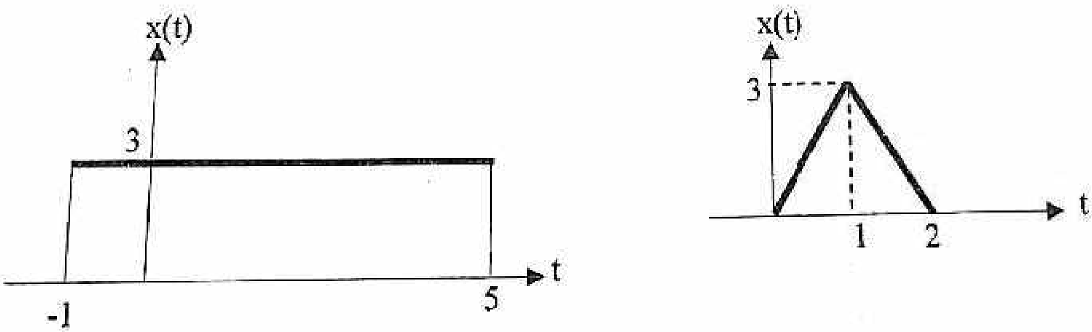
2- Tracer leur spectre d'amplitude

Exercice N°3 :

1- Montrer que : si $TF\left[x\left(\frac{t-\tau}{T}\right)\right] = Te^{-j2\pi f\tau}X(Tf)$; $T > 0$

Avec : $TF[x(t)] = X(f)$

2- Calculer la transformée de Fourier des signaux suivants :

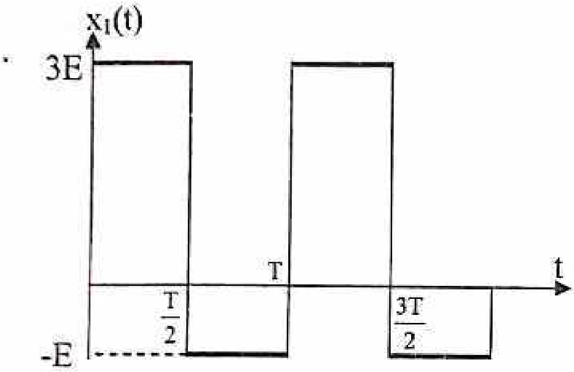


-Département d'Electrotechnique

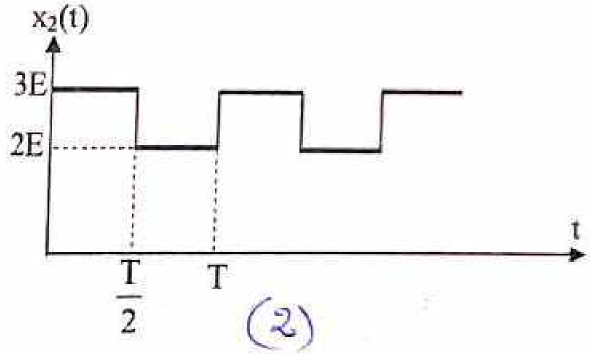
2éme Année Licence en Electrotechnique

Série de TD N°2 de l'unité : Théorie du Signal

Décomposer en série de Fourier les signaux périodiques suivants puis tracer leur spectre d'amplitude pour ($n \leq 4$)



(1)



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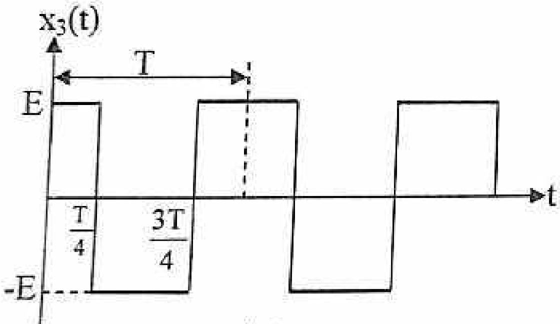
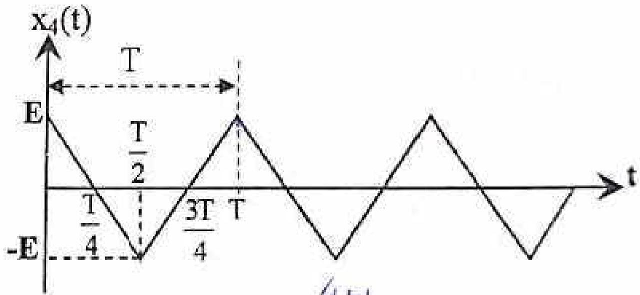


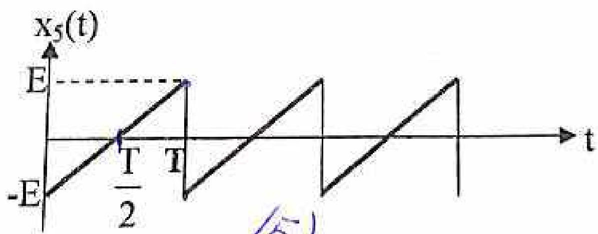
pair

(3)



pair.

(4)



impair (l'axe de temps)

(5)

Exo 1.

Decomposer en serie de Fourier puis tracer leur spectre d'amplitude pour $(n \leq 4)$.

$$a_0 = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\,dt \quad , \quad a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos(n\omega t)\,dt \quad ; \quad b_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin(n\omega t)\,dt.$$

$$a_0 = \frac{1}{T}\int_0^T x_1(t)\,dt = \frac{1}{T}\left\{\int_0^{T/2} 3E\,dt + \int_{T/2}^{T} -E\,dt.\right\}$$

$$= \frac{1}{T}\left\{\left[3Et\right]_0^{T/2} + \left[-Et\right]_{T/2}^{T}\right\}.$$

$$= \frac{1}{T}\left[\left(3E\frac{T}{2} - 0\right) + \left(-E.T - (-E.T/2)\right)\right] = \frac{1}{T}\left(\frac{3E}{2}.T - \frac{ET}{2}\right) = E$$

$$a_n = \frac{2}{T}\int_0^{T/2} 3E\cos(n\omega t) - \frac{2}{T}\int_{T/2}^{T} E\cos(n\omega t)\,dt.$$

$$= \frac{6E}{T}\int_0^{T/2}\cos(n\omega t)\,dt - \frac{2E}{T}\int_{T/2}^{T}\cos(n\omega t)\,dt$$

$$= \frac{6E}{T}\left[\frac{1}{n\omega}\sin(n\omega t)\right]_0^{T/2} - \frac{2E}{T}\left[\frac{1}{n\omega}\sin(n\omega t)\right]_{T/2}^{T}$$

$\omega = 2\pi f = \frac{2\pi}{T}$, $\sin(n\pi) = 0$ $\quad \sin(2\pi n) = 0$

$$a_n = \frac{6E}{T}\frac{1}{n\omega}\left(\sin\left(n\frac{2\pi}{T}.\frac{T}{2}\right) - \sin(0)\right) - \frac{2E}{T}.\frac{1}{n\omega}\left(\sin\left(n\frac{2\pi}{T}.T\right) - \sin\left(n\frac{2\pi}{T}.\frac{T}{2}\right)\right)$$

$$= 0$$

$$b_n = \frac{2}{T}\int_0^{T/2} 3E\sin(n\omega t)\,dt - \frac{2}{T}\int_{T/2}^{T} E\sin(n\omega t)\,dt.$$

$$= \frac{6E}{T}\int_0^{T/2}\sin(n\omega t)\,dt - \frac{2E}{T}\int_{T/2}^{T}\sin(n\omega t)\,dt.$$

$$= \frac{6E}{T}\left[-\frac{1}{n\omega}\cos(n\omega t)\right]_0^{T/2} - \frac{2E}{T}\left[-\frac{1}{n\omega}\cos(n\omega t)\right]_{T/2}^{T}$$

$\omega = 2\pi f = \frac{2\pi}{T}$, $\cos(n\pi) = (-1)^n$ $\quad \cos(2\pi n) = 1.$

$$= \frac{6E}{T}\cdot\frac{-1}{n\omega}\left(\cos\left(n\frac{2\pi}{T}\frac{T}{2}\right) - \cos(0)\right) + \frac{2E}{T}.\frac{1}{n\omega}\left(\cos\left(n.\frac{2\pi}{T}.T\right) - \cos\left(n\frac{2\pi}{T}\frac{T}{2}\right)\right)$$

$$= -\frac{\cancel{6}^{3}E}{\cancel{T}.n\frac{2\pi}{\cancel{T}}}\left((-1)^n - 1\right) + \frac{2E}{\cancel{T}}.\frac{1}{n\frac{2\pi}{\cancel{T}}}.\left(1 - (-1)^n\right)$$

$$= -\frac{3E}{n\pi}.\left((-1)^n - 1\right) + \frac{E}{n\pi}.\left(1 - (-1)^n\right)$$

$$= -\frac{4E}{n\pi}(-1)^n + \frac{4E}{n\pi} = \frac{4E}{n\pi}\left(1 - (-1)^n\right).$$

$$A_n = \sqrt{a_n^2 + b_n^2}$$

$A_0 = a_0 = E$

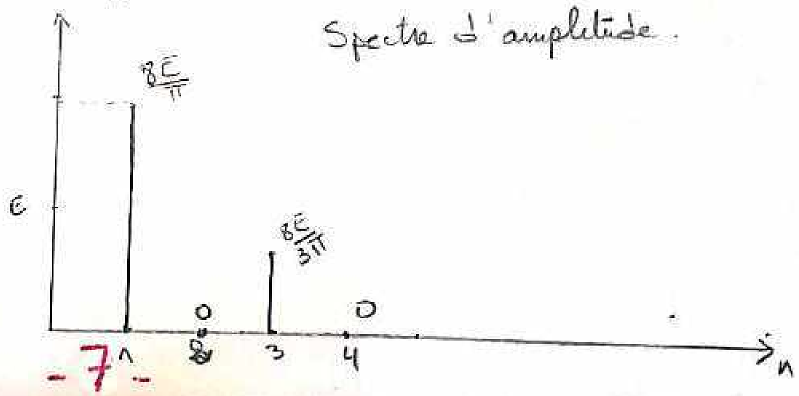
$A_1 = \underline{8E}$

$A_2 = 0$

$A_3 = \frac{8E}{\pi}$

$A_4 = 0$

Spectre d'amplitude.



x_2 n'est ni pair ni impair

$$a_0 = \frac{1}{T}\int_0^T x_2(t)\,dt = \frac{1}{T}\left\{\int_0^{T/2} 3E\,dt + \int_{T/2}^{T} 2E\,dt\right\}$$

$$= \frac{1}{T}\left\{[3E.t]_0^{T/2} + [2E.t]_{T/2}^{T}\right\} = \frac{1}{T}\left\{[3E.T/2 - 3E.0] + [2E.T - 2E\tfrac{T}{2}]\right\}$$

$$= \frac{1}{\cancel{T}}\left[\frac{3E\cancel{T}}{2} + E.T\right] = \frac{5E}{2}$$

$$a_n = \frac{2}{T}\int_0^T x_2(t)\cos(n\omega t)\,dt = \frac{2}{T}\left\{\int_0^{T/2} 3E\cos(n\omega t) + \int_{T/2}^{T} 2E\cos(n\omega t)\,dt\right\}$$

$$= \frac{2}{T}\left\{3E\left[\frac{1}{n\omega}\sin(n\omega t)\right]_0^{T/2} + 2E\left[\frac{1}{n\omega}.\sin(n\omega t)\right]_{T/2}^{T}\right.$$

$$\boxed{\sin(n\pi) = 0 \quad , \quad \sin(2n\pi) = 0} \qquad \omega = \frac{2\pi}{T} = 2\pi f.$$

$$= \frac{2}{T}\left\{\frac{3E}{n\omega}\left[\sin\left(n\frac{2\pi}{T}\frac{T}{2}\right) - \sin\left(n\frac{2\pi}{T}.0\right)\right] + \frac{2E}{n\omega}\left[\sin\left(n\frac{2\pi}{T}.T\right) - \sin\left(n\frac{2\pi}{T}\frac{T}{2}\right)\right]\right\} = 0$$

$$b_n = \frac{2}{T}\int_0^{T/2} 3E\sin(n\omega t)\,dt + \frac{2}{T}\int_{T/2}^{T} 2E\sin(n\omega t)\,dt$$

$$= \frac{6E}{T}\left\{\left(-\frac{1}{n\omega}\cos(n\omega t)\right) - \frac{4E}{T}\left(-\frac{1}{n\omega}\cos(n\omega t)\,dt\right.\right.$$

$$\boxed{\cos(n\pi) = 1 \qquad \cos(2n\pi) = 1}$$

$$= \frac{2}{T}\left\{-\frac{3E}{n\omega}\left[\cos\left(n\frac{2\pi}{T}\frac{T}{2}\right) - \cos\left(n\frac{2\pi}{T}.0\right)\right] - \frac{2E}{n\omega}\left[\cos\left(n\frac{2\pi}{T}T\right) - \cos\left(n\frac{2\pi}{T}.\frac{T}{2}\right)\right]\right.$$

$$= \frac{2}{T}\left\{-\frac{3E}{n\frac{2\pi}{T}}\left[(-1)^n - 1\right] - \frac{2E}{n\frac{2\pi}{T}}\left[1 - (-1)^n\right]\right\}$$

$$= -\frac{3E}{n\pi}\left[(-1)^n - 1\right] - \frac{2E}{n\pi}\left[1 - (-1)^n\right]$$

$$= \frac{3E}{n\pi} - \frac{2E}{n\pi} - \frac{3E}{n\pi}(-1)^n + \frac{2E}{n\pi}(-1)^n$$

$$= \frac{E}{n\pi} - \frac{E}{n\pi}(-1)^n = \frac{E}{n\pi}\left(1 - (-1)^n\right).$$

$$x_2(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega t) + \sum_{n=1}^{\infty} b_n\sin(n\omega t)\ldots$$

$$A_n = \sqrt{a_n^2 + b_n^2} \qquad A_0 = a_0$$

$a_n = 0 \Rightarrow A_n = b_n$

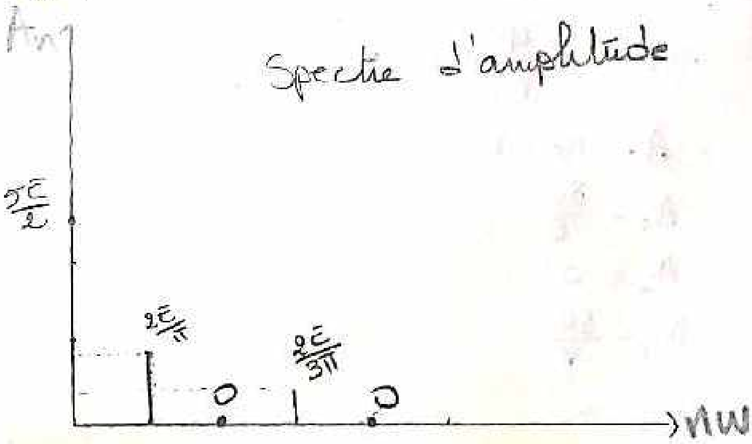
$A_0 = \frac{5E}{2}$

$A_1 = \frac{2E}{\pi}$

$A_2 = 0$

$A_3 = \frac{2E}{3\pi}$

$A_4 = 0$



$x_3(t)$ est pair $\Rightarrow \boxed{b_n = 0}$

$$a_0 = \frac{2}{T}\int_0^{T/2} x_3(t)\,dt = \frac{2}{T}\left\{\int_0^{T/4} E\,dt + \int_{T/4}^{T/2} -E\,dt\right\}$$

$$= \frac{2}{T}\left\{(E.T)_0^{T/4} - (E.T)_{T/4}^{T/2}\right\}$$

$$= \frac{2}{T}\left[E.\frac{T}{4} - \left(E.\frac{T}{2} - E.\frac{T}{4}\right)\right] = 0$$

$$a_n = \frac{4}{T}\int_0^{T/2} x_3(t)\cos(n\omega t)\,dt$$

$$= \frac{4}{T}\left\{\int_{T/2}^{T/4} E.\cos(n\omega t)\,dt - \int_{T/4}^{T/2} E.\cos(n\omega t)\,dt\right\}$$

$$= \frac{4E}{T}\left\{\left[\frac{1}{n\omega}\sin(n\omega t)\right|_0^{T/4} - \left[\frac{1}{n\omega}\sin(n\omega t)\right|_{T/4}^{T/2}\right\}$$

$$= \frac{4E}{Tn\frac{2\pi}{T}}\left\{\left[\sin\left(n\frac{2\pi}{T}.\frac{T}{4}\right) - \left(\sin\left(n\frac{2\pi}{T}\frac{T}{2}\right) - \sin\left(n\frac{2\pi}{T}.\frac{T}{4}\right)\right)\right]\right\}$$

$$a_n = \frac{2E}{n\pi}\left[\sin\left(n\frac{\pi}{2}\right) + \sin\left(n\frac{\pi}{2}\right)\right]$$

$$\boxed{a_n = \frac{4E}{n\pi}.\sin\left(n\frac{\pi}{2}\right)}$$

$$x_3(t) = \sum_{n=1}^{\infty} \frac{4E}{n\pi}\sin\left(n\frac{\pi}{2}\right).\cos(n\omega t).$$

$A_n = \sqrt{a_n^2 + b_n^2}$ $\quad A_0 = a_0$

$b_n = 0$ $\quad A_n = |a_n| = \left|\frac{4E}{n\pi}\sin\left(n\frac{\pi}{2}\right)\right|$

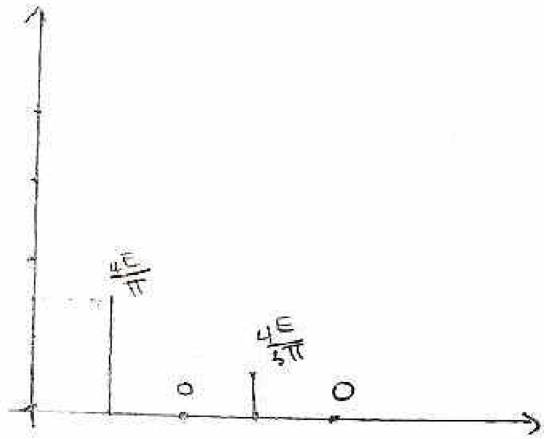
$A_0 = 0$

$A_1 = \frac{4E}{\pi}$

$A_2 = 0$

$A_3 = \frac{4E}{3\pi}$

$A_4 = 0$



$x_4(t)$ est pair $\Rightarrow \boxed{b_n = 0}$

$$x_1(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega t)$$

$$a_0 = \frac{2}{T}\int_0^{T/2} x_4(t)\,dt \qquad x_4(t) = at + b = -\frac{4E}{T}t + E$$

$$a = \frac{-E-E}{T/2 - 0} = \frac{-2E}{T/2} = -\frac{4E}{T}$$

$at = 0 \qquad b = E$

$$a_0 = \frac{2}{T}\int_0^{T/2}\left(-\frac{4E}{T}t + E\right)dt.$$

$$\int (at+b)^n = \frac{1}{a(n+1)} (at+b)^{n+1} + C$$

$$a_0 = \frac{2E}{T} \int_0^{T/2} \left(-\frac{4}{T} t + 1 \right) dt$$

$$= \frac{2E}{T} \left[\frac{1}{-\frac{4}{T}(2)} \left(-\frac{4}{T} t + 1 \right)^2 \right]_0^{T/2}$$

$$a_0 = \frac{2E}{T} \left[-\frac{8}{T} \left(-\frac{4}{T} t + 1 \right)^2 \right]_0^{T/2}$$

$$= \frac{2E}{T} \left[-\frac{8}{T} \left[-2+1 \right]^2 + \frac{8}{T} \right]$$

$$-\frac{8}{T}(1) \quad + \frac{8}{T}$$

$$a_n = \frac{4}{T} \int_0^{T/2} \left(-\frac{4}{T} Et + E \right) \cos(n\omega t)\, dt$$

$$a_n = \frac{4E}{T} \int_0^{T/2} \left(-\frac{4}{T} t + 1 \right) \cos n\omega t \, dt$$

$$\int u v' = [uv] - \int v\, du.$$

$$u = -\frac{4}{T} t + 1 \longrightarrow u' = -\frac{4}{T}$$

$$v' = \cos n\omega t \longrightarrow v = \frac{1}{n\omega} \sin n\omega t.$$

$$a_n = \frac{4E}{T} \left[\left(-\frac{4}{T} t + 1 \right) \frac{1}{n\omega} \sin n\omega t \Big]_0^{T/2} - \int_0^{T/2} -\frac{4}{Tn\omega} \sin n\omega t \, dt \right]$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

$$a_n = \frac{4E}{T} \left[- \int_0^{T/2} \frac{-4}{Tn\omega} \sin n\omega t \, dt \right]$$

$$a_n = \frac{4E}{T} \left[\frac{4}{Tn\omega} \int_0^{T/2} \sin n\omega t \, dt \right]$$

$$a_n = \frac{16E}{T^2 n^2 \omega^2} \left[\cos n\omega t \right]_0^{\pi/\omega}$$

$$\left((-1)^n - 1 \right)$$

$$a_n = \frac{-16E}{T^2 n^2 \omega^2} \left[(-1)^n - 1 \right]$$

$$a_n = \frac{-16E}{\frac{4\pi^2}{\omega^2} n^2 \omega^2} \left((-1)^n - 1 \right)$$

$$\boxed{a_n = \frac{-4E}{n^2 \pi^2} \left((-1)^n - 1 \right)}$$

$$x_4(t) = \frac{-4E}{n^2\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} \cos n\omega t$$

$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{a_n^2} = |a_n|.$$

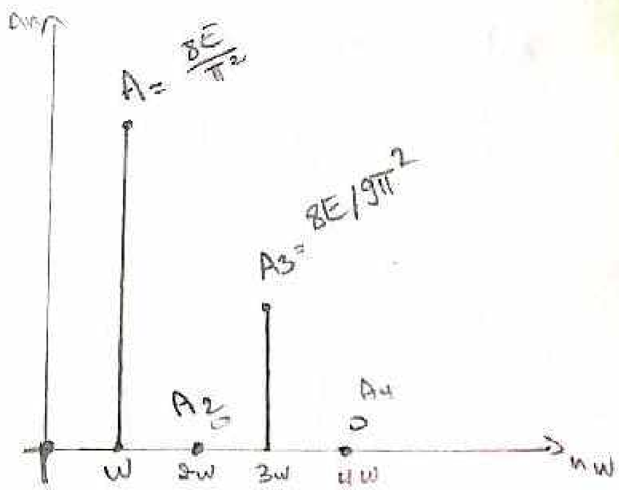
$A_0 = a_0 = 0$

$A_1 = 8E/\pi^2$

$A_2 = 0$

$A_3 = \frac{8E}{9\pi^2}$

$A_4 = 0$



$x_s(t) \to$ impair $\Rightarrow a_n = a_0 = 0$

$$x_s = \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$b_n = \frac{4}{T}\int_0^{T/2} x_s(t) \sin n\omega t \, dt$$

$$x_s(t) = at + b = \frac{2E}{T} - E$$

$$\boxed{x_s(t) = E\left(\frac{2}{T}t - 1\right)}$$

$$b_n = \frac{4}{T}\int_0^{T/2} E\left(\frac{2}{T}t - 1\right) \sin n\omega t \, dt$$

$$b_n = \frac{4E}{T}\int_0^{T/2} \left(\frac{2}{T}t - 1\right) \sin n\omega t \, dt$$

$$\int u v = u.v - \int du.v$$

$$u = \frac{2}{T}t - 1 \longrightarrow u' = \frac{2}{T}$$

$$v' = \sin n\omega t \longrightarrow v = -\frac{1}{n\omega}\cos n\omega t$$

$$b_n = \frac{4E}{T}\left[\left[-\left(\frac{2}{T}t - 1\right)\frac{1}{n\omega}\cos n\omega t\right]_0^{T/2} - \int_0^{T/2} -\frac{2}{Tn\omega}\cos n\omega t \, dt\right]$$

$$b_n = \frac{4E}{T}\left[\left[\ominus\left(\frac{2}{T}t - 1\right)\frac{1}{n\omega}\cos n\omega t\right]_0^{\pi/\omega} + \frac{2}{Tn\omega}\left[\underbrace{\frac{1}{n\omega}\sin n\omega t}_{\sin n\pi = 0}\right]_0^{\pi/\omega}\right]$$

$$b_n = -\frac{4E}{Tn\omega}\left[\left[\frac{2}{T}t - 1\right] \cdot \cos n\omega t\right]_0^{t/\omega}$$

$$b_n = -\frac{4E}{Tn\omega} = -\frac{2E}{n\pi}$$

$$\boxed{x_s(t) = \sum_{n=1}^{\infty} \frac{-2E}{n\pi}\sin n\omega t}$$

$A_n = |b_n|$

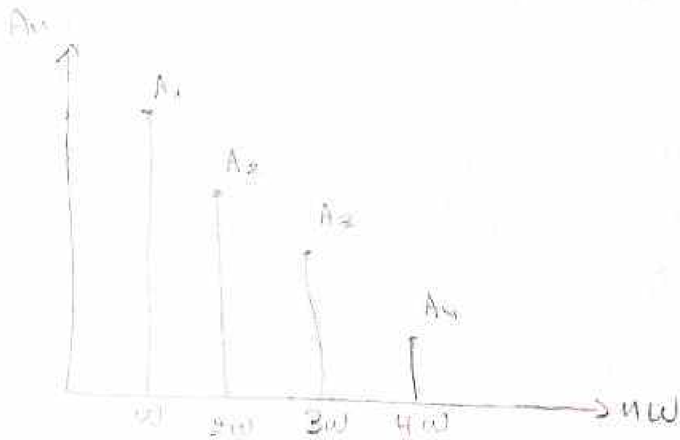
$A_0 = a_0 = 0$

$A_1 = |b_1| = 2$

$A_2 = |b_2| = \frac{E}{\pi}$

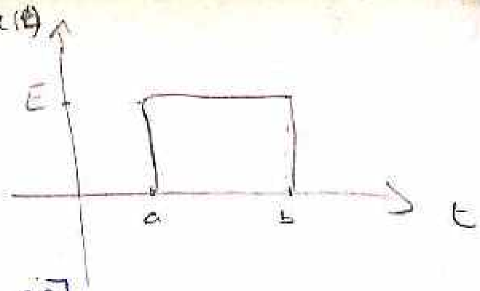
$A_3 = |b_3| = 2E/3\pi$

$A_4 = |b_4| = \frac{E}{2\pi}$



Exemple : 1ère méthode :

$$\mathcal{L}[x(t)] = X(P) = \int_0^{\infty} x(t)\, e^{-Pt}\, dt$$



$$\mathcal{L}[x(t)] = X(P) = \int_a^b E\, e^{-Pt}\, dt$$

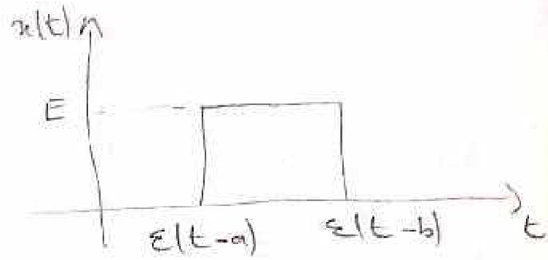
$$X(P) = E \int_a^b e^{-Pt}\, dt.$$

$$X(P) = E\left[-\frac{1}{P} e^{-Pt}\right]_a^b = E\left[-\frac{1}{P} e^{-bP} + \frac{1}{P} e^{-aP}\right]$$

$$\boxed{X(P) = \frac{E}{P}\left(e^{-aP} - e^{-bP}\right)}$$

$$\mathcal{L}[\varepsilon(t)] = \frac{1}{P} \quad ; \quad \mathcal{L}[\varepsilon(t-a)] = \frac{1}{P} e^{-aP}$$

2ème méthode :



$$x(t) = E\left(\varepsilon(t-a) - \varepsilon(t-b)\right)$$

$$[x(t)] = X(P) = E\left(\frac{1}{P} e^{-aP} - \frac{1}{P} e^{-bP}\right)$$

$$\boxed{X(P) = \frac{E}{P}\left(e^{-aP} - e^{-bP}\right)}.$$

Série TD N°3

Exo 1 :

$$x_1(t) = a E_0\left(\varepsilon(t) - \varepsilon(t-T)\right) - E_0\left(\varepsilon(t-T) - \varepsilon(t-2T)\right)$$

$$x_1(t) = E_0\left[a\varepsilon(t) - a\varepsilon(t-T) - \varepsilon(t-T) + \varepsilon(t-2T)\right]$$

$$x_1(t) = E_0\left[a\varepsilon(t) - \varepsilon(t-T)(a+1) + \varepsilon(t-2T)\right]$$

$$\mathcal{L}[x_1(t)] = E_0\left[\frac{a}{P} - \frac{a+1}{P} e^{-TP} + \frac{1}{P} e^{-2TP}\right]$$

$$X_1(P) = \frac{E_0}{P}\left(a - (a+1) e^{-TP} + e^{-2TP}\right)$$

2ème méthode :

$$X_1(P) = \int_0^{\infty} x_1(t)\, e^{-Pt}\, dt$$

$$X_1(P) = \int_0^T a E_0\, e^{-Pt}\, dt + \int_T^{2T} -E_0\, e^{-Pt}\, dt.$$

$$X_1(P) = a E_0 \int_0^T e^{-Pt}\, dt - E_0 \int_T^{2T} e^{-Pt}\, dt$$

$$X_1(P) = a E_0\left[-\frac{1}{P} e^{-Pt}\right]_0^T - E_0\left[-\frac{1}{P} e^{-Pt}\right]_T^{2T}$$

$$X_1(P) = a E_0\left[-\frac{1}{P} e^{-PT} + \frac{1}{P}\right] - E_0\left[-\frac{1}{P} e^{-2TP} + \frac{1}{P} e^{-PT}\right]$$

$$X_1(P) = E_0\left[-\frac{a}{P} e^{-PT} + \frac{a}{P} + \frac{1}{P} e^{-2TP} - \frac{1}{P} e^{-PT}\right]$$

$$X_1(P) = E_0\left[\frac{a}{P} - (a+1)\frac{1}{P} e^{-PT} + \frac{1}{P} e^{-2TP}\right]$$

$$X_1(P) = \frac{E_0}{P}\left(a - (a+1) e^{-TP} + e^{-2TP}\right)$$

$$x_2(t) = \begin{cases} \frac{E}{a} t & 0<t<a \\ E & a<t<2a \\ -\frac{E}{2a} t + 2E & 2a<t<4a \end{cases}$$

$t = 2a$ $x(t) = -\frac{E}{2a} t + b$

$x(2a) = E = -\frac{E}{2a} 2a + b_1$

$\boxed{b_1 = 2E}$

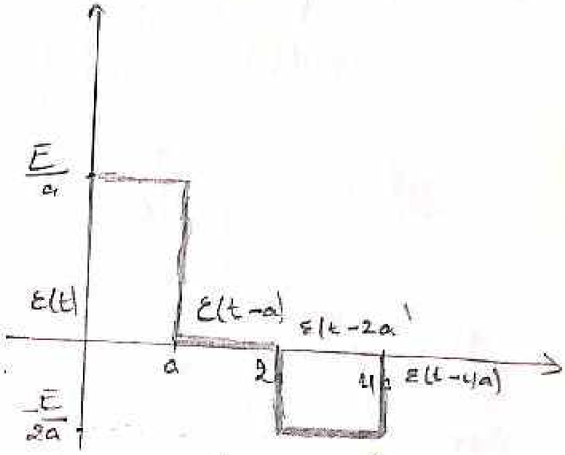
$t = 4a$

$x(4a) = 0 = -\frac{E}{2a} 4a + b_1$

$\boxed{b_1 = 2E}$

$$\begin{cases} \mathcal{L}\left[\frac{dx}{dt}\right] = X(P)\,p - x_0 \\ X(P) = \dfrac{\mathcal{L}\left[\frac{dx}{dt}\right] + \cancel{x_0}^{=0}}{p} \end{cases}$$

$$X_2(P) = \frac{1}{P} \mathcal{L}\left[\frac{dx(t)}{dt}\right]$$



$$\frac{dx_2(t)}{dt} = \begin{cases} \frac{E}{a} & 0<t<a \\ 0 & a<t<2a \\ -\frac{E}{2a} & 2a<t<4a \end{cases}$$

$$\frac{dx_2(t)}{dt} = \frac{E}{a}\left(\varepsilon(t) - \varepsilon(t-a)\right) - \frac{E}{2a}\left(\varepsilon(t-2a) - \varepsilon(t-4a)\right)$$

$$\frac{dx_2(t)}{dt} = \frac{E}{a}\left(\varepsilon(t) - \varepsilon(t-a) - \frac{1}{2}\varepsilon(t-2a) + \frac{1}{2}\varepsilon(t-4a)\right)$$

$$\mathcal{L}\left[\frac{dx_2(t)}{dt}\right] = \frac{E}{a}\left[\frac{1}{P} - \frac{1}{P}e^{-aP} - \frac{1}{2P}e^{-2aP} + \frac{1}{2P}e^{-4aP}\right)$$

$$\mathcal{L}\left[\frac{dx_2(t)}{dt}\right] = \frac{E}{aP}\left(1 - e^{-aP} - \frac{1}{2}e^{-2aP} + \frac{1}{2}e^{-4aP}\right)$$

$$X_2(P) = \frac{1}{P}\mathcal{L}\left[\frac{dx_2(t)}{dt}\right]$$

$$X_2(P) = \frac{E}{aP^2}\left(1 - e^{-aP} - \frac{1}{2}e^{-2aP} + \frac{1}{2}e^{-4aP}\right.$$

$$X_2(P) = \int_0^\infty (x_2(t))\, e^{-Pt}\, dt$$

$$X_2(P) = \int_0^a \frac{E}{a} t\, e^{-Pt}\, dt + \int_a^{2a} E\, e^{-Pt}\, dt + \int_{2a}^{4a}\left(\frac{-E}{2a} + 2E\right) e^{-Pt}\, dt$$

Ex02:

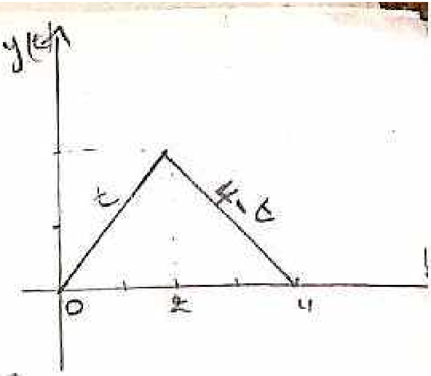
$$y(t) = 2\,\mathrm{tri}\left(\frac{t-2}{2}\right)$$

$$\mathrm{tri}(t) = \begin{cases} 1-t & 0<t<1 \\ 1+t & 1<t<0 \\ 0 & \text{Ailleurs} \end{cases}$$

$$\mathrm{tri}\left(\frac{t-2}{2}\right) = 2\begin{cases} 1-\left(\frac{t-2}{2}\right) & 0<\frac{t-2}{2}<1 \\ 1+\left(\frac{t-2}{2}\right) & -1<\frac{t-2}{2}<0 \\ 0 & \text{Ailleurs} \end{cases}$$

$$2\,\text{tri}\left(\frac{t-2}{2}\right) = 2\begin{cases} \frac{4-t}{2} & 2<t<4 \\ \frac{t}{2} & 0<t<2 \\ 0 & \text{Ailleurs} \end{cases}$$

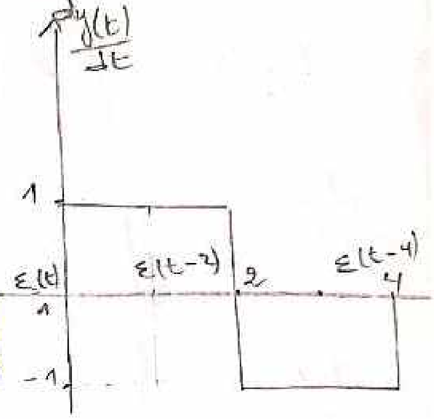
$$2\,\text{tri}\left(\frac{t-2}{2}\right) = \begin{cases} 4-t & 2<t<4 \\ t & 0<t<2 \\ 0 & \text{Ailleurs} \end{cases}$$



2)- $\mathcal{L}\left[\frac{dx(t)}{dt}\right] = X(P)p - \cancel{x_0}$ (0,0)

$X(P) = \frac{1}{P}\,\mathcal{L}\left[\frac{dx(t)}{dt}\right]$ } propriété de la dérivée

$$\frac{dy(t)}{dt} = \begin{cases} 1 & 0<t<2 \\ -1 & 2<t<4 \\ 0 & \text{Ailleurs} \end{cases}$$



$\frac{dy(t)}{dt} = 1\big(\varepsilon(t) - \varepsilon(t-2)\big) - 1\big(\varepsilon(t-2) - \varepsilon(t-4)\big)$

$\frac{dy(t)}{dt} = \left[\varepsilon(t) - \varepsilon(t-2) - \varepsilon(t-2) + \varepsilon(t-4)\right]$

$\frac{dy(t)}{dt} = \left[\varepsilon(t) - 2\varepsilon(t-2) + \varepsilon(t-4)\right]$

$\mathcal{L}\left[\frac{dy(t)}{dt}\right] = \left(\frac{1}{P} - \frac{2}{P}e^{-2P} + \frac{1}{P}e^{-4P}\right)$

$\mathcal{L}\left[\frac{y(t)}{dt}\right] = \frac{1}{P}\left(1 - 2e^{-2P} + e^{-4P}\right)$

$$\boxed{Y(P) = \frac{1}{P}\,\mathcal{L}\left[\frac{dy(t)}{dt}\right] = \frac{1}{P^2}\left(1 - 2e^{-2P} + e^{-4P}\right)}$$

Exo 3 :

$e(t) = R\,i(t) + \frac{1}{C}\int_0^t i_1(t)\,dt$ ... ①

$\frac{1}{C}\int_0^t i_1(t)\,dt = R_1 i_2(t) + S(t)$ ... ②

$S(t) = R_2\, i_2(t)$ .... ③

$i(t) = i_1(t) + i_2(t)$ ——— ④

$E(P) = R\,I(P) + \frac{1}{CP} I_1(P)$ ... ①

$\frac{1}{CP} I_1(P) = R_1 I_2(P) + S(P)$ .... ②

$S(P) = R_2 I_2(P)$ —— ③

$I(P) = I_1(P) + I_2(P)$

③ $\Leftrightarrow \boxed{I_2(P) = \frac{1}{R_2} S(P)}$

$$\frac{1}{CP} I_1(P) = R_1 \left(\frac{1}{R_2}\right) S(P) + S(P)$$

$$\frac{1}{CP} I_1(P) = \left(\frac{R_1}{R_2} + 1\right) S(P) = \left(\frac{R_1 + R_2}{R_2}\right) S(P).$$

$$\boxed{I_1(P) = \frac{(R_1+R_2)\,CP}{R_2} \cdot S(P)}$$

$$I(P) = \frac{(R_1+R_2)CP}{R_2} S(P) + \frac{1}{R_2} S(P)$$

$$\boxed{I(P) = \frac{(R_1+R_2)CP + 1}{R_2}}$$

$$E_P = \frac{R(R_1+R_2)CP + R}{R_2} S(P) + \frac{R_1 + R_2}{R_2} S(P)$$

$$E_P = \frac{RC(R_1+R_2)P + (R + R_1 + R_2)}{R_2} S(P).$$

$$\boxed{H(P) = \frac{S(P)}{E(P)} = \frac{R_2}{RC(R_1+R_2)P + (R_1+R_2+R)}}$$

$$\boxed{H(P) = \frac{K}{\tau P + 1}}$$

$$H(P) = \frac{R_2 / R_1 + R_2 + R}{\frac{RC(R_1+R_2)}{R_1+R_2+R} P + 1}$$

$$K = \frac{R_2}{R_1 + R_2 + R} \quad , \quad \tau = \frac{R(R_1+R_2)}{R_1 + R_2 + R}$$

2). $R_1 = R_2 = R$ $\qquad\qquad H(P) = \dfrac{\alpha}{P + 3\alpha}$

$$H(P) = \frac{R}{2R^2CP + 3R} = \frac{1}{2RCP + 3}$$

$$\boxed{H(P) = \frac{\frac{1}{2RC}}{P + \frac{3}{2RC}}} \Longrightarrow \boxed{\alpha = \frac{1}{2RC}}$$

$$S(P) = H(P) \cdot E(P)$$

$$S(P) = \frac{\alpha}{P+3\alpha} \cdot E(P)$$

$$e(t) = 2E_0(\varepsilon(t) - \varepsilon(t-T)) + E_0(\varepsilon(t-T) - \varepsilon(t-2T))$$

$$e(t) = E_0(2\varepsilon(t) - 2\varepsilon(t-T) + \varepsilon(t-T) - \varepsilon(t-2T))$$

$$e(t) = E_0(2\varepsilon(t) - \varepsilon(t-T) - \varepsilon(t-2T))$$

$$\mathcal{L}[e(t)] = E(P) = E_0\left(\frac{2}{P} - \frac{1}{P}e^{-TP} - \frac{1}{P}e^{-2TP}\right)$$

$$\boxed{E(P) = \frac{E_0}{P}\left(2 - e^{-TP} - e^{-2TP}\right)}$$

$$S(P) = \frac{\alpha}{P+3\alpha} \cdot \frac{E_0}{P}\left(2 - e^{-TP} - e^{-2TP}\right)$$

$$S(P) = \frac{E_0\,\alpha}{P(P+3\alpha)}\left(2 - e^{-TP} - e^{-2TP}\right) \qquad \frac{1}{(P+a)(P+b)} = \frac{1}{b-a}\left(\frac{1}{P+a} - \frac{1}{P+b}\right)$$

$$\frac{1}{P(P+3\alpha)} = \frac{1}{3\alpha}\left(\frac{1}{P} - \frac{1}{P+3\alpha}\right)$$

$$S(P) = \frac{E_0}{3}\left(\frac{1}{P} - \frac{1}{P+3\alpha}\right)\left(2 - e^{-TP} - e^{-2TP}\right)$$

$$S(P) = \frac{E_0}{3}\left[\frac{2}{P} - \frac{1}{P}e^{-TP} - \frac{1}{P}e^{-2TP} - \frac{2}{P+3\alpha} + \frac{e^{-TP}}{(P+3\alpha)} + \frac{e^{-2TP}}{P+3\alpha}\right]$$

$$\mathcal{L}^{-1}[S(P)] = s(t) = \frac{E_0}{3}\left[2 - \varepsilon(t-T) - \varepsilon(t-2T) - 2e^{-3\alpha t} + \varepsilon(t-T)e^{-3\alpha(t-T)}\right.$$

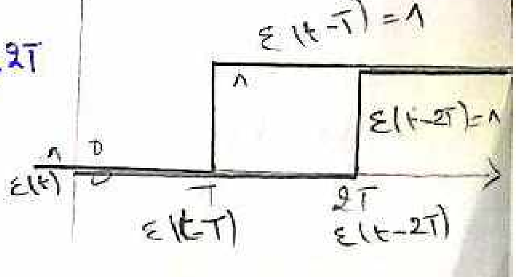
$$\left. + \varepsilon(t-2T)e^{-3\alpha(t-2T)}\right]$$

$$2 - 1 - 2e^{-3\alpha} + e^{-3\alpha(t-T)}$$

$$2 - 1 - 1 - 2e^{-3\alpha t} + e^{-3\alpha(t-T)} + e^{-3\alpha(t-2T)}$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{e^{-b}}{P+a}\right] = \varepsilon(t-b)\,e^{-a(t-b)}}$$

$$S(t) = \frac{E_0}{3}\begin{cases} 2 - 2e^{-3\alpha t} & 0 < t < T \\ 1 - 2e^{-3\alpha t} + e^{-3\alpha(t-T)} & T < t < 2T \\ -2e^{-3\alpha t} + e^{-3\alpha(t-T)} + e^{-3\alpha(T-2T)} & t > 2T \end{cases}$$

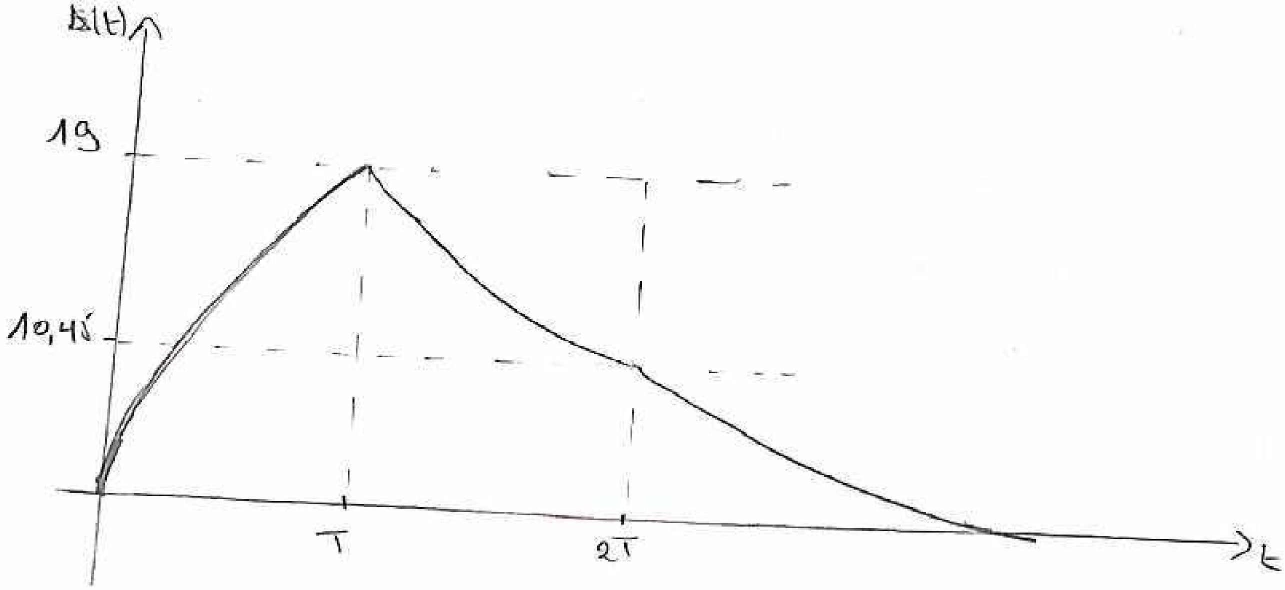


$S(0) = 0$

$S(\infty) = 0$

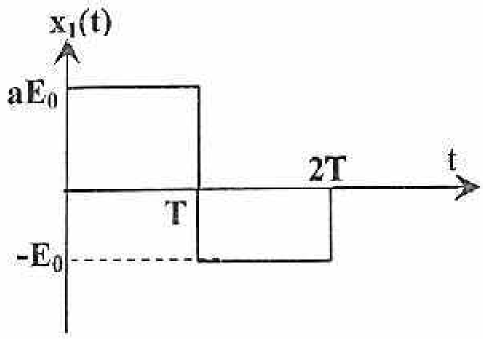
$S(T) = 10(2 - e^{-3}) = 19V$

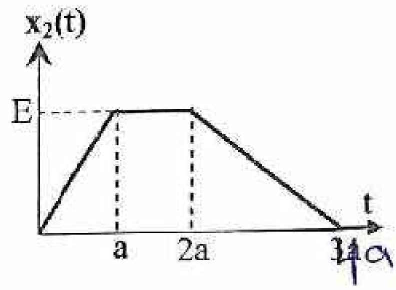
$S(2T) = 10(1 - 2e^{-6} + e^{-3}) = 10{,}45\,V$

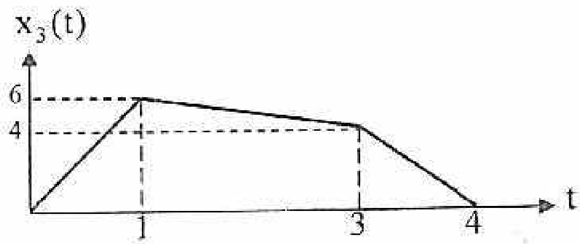


2éme Année Licence en Electrotechnique
Série de TD N°3 de « Théorie du Signal »

Exercice N°1 : Calculer la transformée de Laplace des signaux suivants :







Exercice N°3 : le signal $y(t) = 2\text{tri}\left(\frac{t-2}{2}\right)$

1- Tracer le signal y(t)

2- En utilisant la propriété de la dérivée par rapport au temps, calculer sa transformée de Laplace

Exercice N°4 : Soit le système du premier ordre représenté en **Figure 1** (le condensateur est initialement déchargé).

1- Trouver sa fonction de transfert. Déduire sa constante de temps τ et son gain en régime continu k.

2- On suppose que $R_1 = R_2 = R$. Ecrire la fonction de transfert sous forme : $H(p) = \frac{\alpha}{p+3\alpha}$

Déterminer la constante α en fonction de R et C.

1-1- Trouver en fonction de t et α, l'expression de sa réponse au signal e(t) représenté en **Figure 2.**

1-2- On suppose que E_0 =30V et $T = \frac{1}{\alpha}$. Calculer s(0) , s(T), s(2T) et s(∞). Tracer l'allure de s(t).

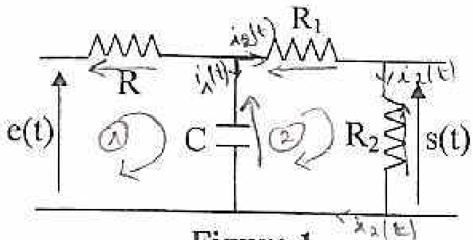


Figure 1

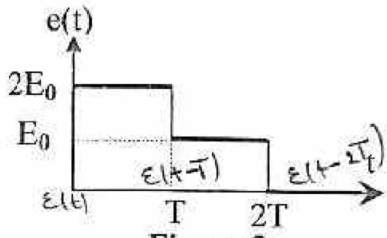


Figure 2

Exercice N°4 : Soit le système du premier ordre représenté en **figure 3.** (La bobine est initialement déchargée)

1-Trouver sa fonction de transfert. Déduire sa constante de temps τ et son gain en régime continu k.

2-Ecrire sa fonction de transfert sous forme : $H(p) = \frac{\lambda}{p + 2\lambda}$. Déterminer la constante λ en fonction de R et L

3-Trouver en fonction de E_0, λ, T et t, l'expression de sa réponse au signal e(t) représenté en **figure 4.**

4- On suppose que E_0 = 20 Volts et $T = \frac{1}{\lambda}$. Calculer s(0), s(T), s(2T) et s(∞).Tracer l'allure de s(t).

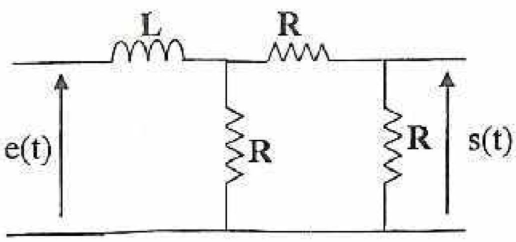


Figure 3

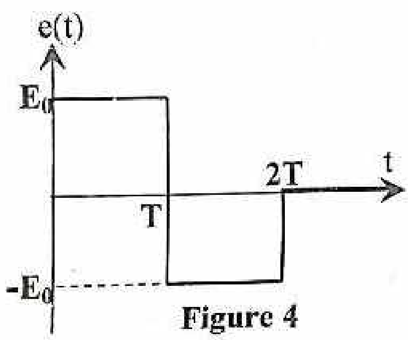
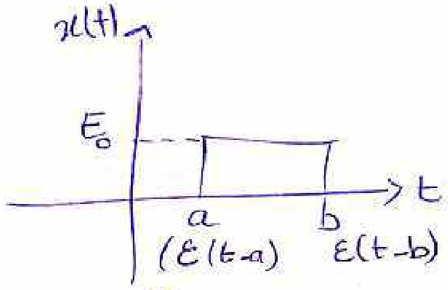


Figure 4

Série TD (03) :

Introduction :



1°/ $X(P) = \int_0^{\infty} x(t) \cdot e^{-Pt}\, dt$

$$X(P) = \int_a^b E_0 \cdot e^{-Pt}\, dt$$

$$= E_0 \int_a^b e^{-Pt}\, dt.$$

$$= E_0 \left[\frac{-1}{P} e^{-Pt} \right]_a^b$$

$$= E_0 \left[-\frac{1}{P} e^{-Pb} + \frac{1}{P} e^{-Pa} \right]$$

$$\boxed{X(P) = \frac{E_0}{P}\left(e^{-Pa} - e^{-Pb} \right)}$$

2°/ $x(t) = E_0 \left(\varepsilon(t-a) - \varepsilon(t-b) \right).$

$$\mathcal{L}[\varepsilon(t)] = \frac{1}{P} . \quad \mathcal{L}[\varepsilon(t-a)] = \frac{1}{P} e^{-aP}$$

$$\mathcal{L}[x(t)] = X(P) = E_0 \left(\frac{1}{P} e^{-aP} - \frac{1}{P} - e^{-bP} \right)$$

$$X(P) = \frac{E_0}{P}\left(e^{-aP} - e^{-bP} \right).$$

Exercice (01) :

. Signal (01) :

$$x_1(t) = a E_0 \left(\varepsilon(t) - \varepsilon(t-T) \right) - E_0 \left(\varepsilon(t-T) - \varepsilon(t-2T) \right).$$

$$x_1(t) = E_0 \left(a\varepsilon(t) - a\varepsilon(t-T) - \varepsilon(t-T) + \varepsilon(t-2T) \right).$$

$$x_1(t) = E_0 \left(a\varepsilon(t) - (a+1)\varepsilon(t-T) + \varepsilon(t-2T) \right)$$

$$\mathcal{L}[x_1(t)] = X_1(P) = E_0 \left(\frac{a}{P} - \frac{(a+1)}{P} e^{-TP} + \frac{1}{P} e^{-2TP} \right)$$

$$\boxed{X_1(P) = \frac{E_0}{P}\left(a - (a+1) e^{-TP} + e^{-2TP} \right)}$$

$$X_1(P) = \int_0^{\infty} x(t) \cdot e^{-tP}\, dt.$$

$$X_1(P) = \int_0^T a E_0 \cdot e^{-tP}\, dt + \int_T^{2T} -E_0 \cdot e^{-tP}\, dt.$$

$$X_1(P) = a E_0 \int_0^T e^{-tP}\, dt - E_0 \int_T^{2T} e^{-TP}\, dt.$$

$$X_1(P) = a E_0 \left[-\frac{1}{P} e^{-tP} \right]_0^T - E_0 \left[-\frac{1}{P} e^{-tr} \right]_T^{2T}$$

$$X_1(P) = a E_0 \left[-\frac{1}{P} e^{-TP} + \frac{1}{P} \right] - E_0 \left[-\frac{1}{P} e^{-2TP} + \frac{1}{P} e^{-TP} \right]$$

$$X_1(P) = \frac{E_0}{P}\left(-a e^{-TP} + a + e^{-2TP} + e^{-TP} \right).$$

$$X_1(P) = \frac{E_0}{P}\left(a - (a+1) e^{-TP} + e^{-2TP} \right).$$

. Signal (02) :

$$\mathcal{L}\left[\frac{dx(t)}{dt}\right] = X(P) . P - \cancel{x_0} \ 0$$

$$X(P) = \frac{1}{P} \mathcal{L}\left[\frac{dx(t)}{dt}\right].$$

$$x_2(t) = \begin{cases} \frac{E}{a} t & 0 < t < a. \\ E & a < t < 2a \\ -\frac{E}{2a} t + 2E & 2a < t < 4a \end{cases}$$

$\frac{E-0}{a-0}$ $\quad \frac{0-E}{4a-2a}$

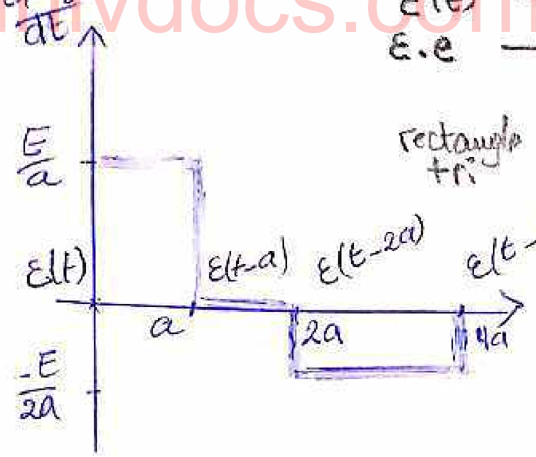
$t = 4a$

sin/cos $\longrightarrow$ P

$\varepsilon(t) \longrightarrow$ P

$\varepsilon . e \longrightarrow$ lim $0 \to W$, $\infty \to P$

rectangle, tri $\longrightarrow$ W



$$\frac{dx_2(t)}{dt} = \frac{E}{a}\left(\varepsilon(t)-\varepsilon(t-a)\right)-\frac{E}{2a}\left(\varepsilon(t-2a)-\varepsilon(t-4a)\right).$$

$$\frac{dx_2(t)}{dt} = \frac{E}{a}\left[\varepsilon(t)-\varepsilon(t-a)-\frac{1}{2}\varepsilon(t-2a)+\frac{1}{2}\varepsilon(t-4a)\right].$$

$$\mathcal{L}\left[\frac{dx_2(t)}{dt}\right] = \frac{E}{a}\left[\frac{1}{p}-\frac{1}{p}e^{-ap}+\frac{1}{2p}e^{-4ap}\right]$$

$$\mathcal{L}\left[\frac{dx_2(t)}{dt}\right] = \frac{E}{ap}\left[1-e^{-ap}-\frac{1}{2}e^{-2ap}+\frac{1}{2}e^{-4ap}\right]$$

$$\boxed{X_1(p) = \frac{E}{ap^2}\left(1-e^{-ap}-\frac{1}{2}e^{-2ap}+\frac{1}{2}e^{-4ap}\right)}.$$

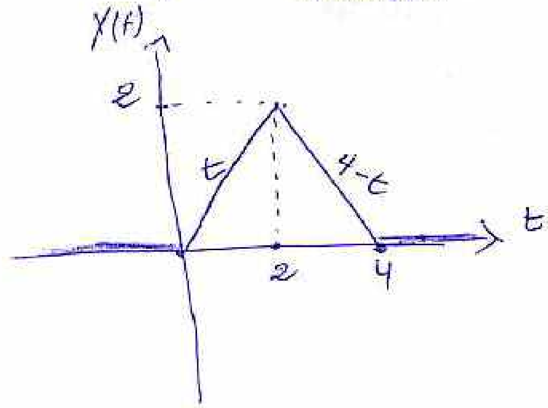
Exercice (02) : $y(t) = 2 . \mathrm{tri}\left(\frac{t-2}{2}\right)$.

$$\mathrm{tri}(t) = \begin{cases} 1+t & -1<t<0. \\ 1-t & 0<t<1. \\ 0 & \text{ailleurs}. \end{cases}$$

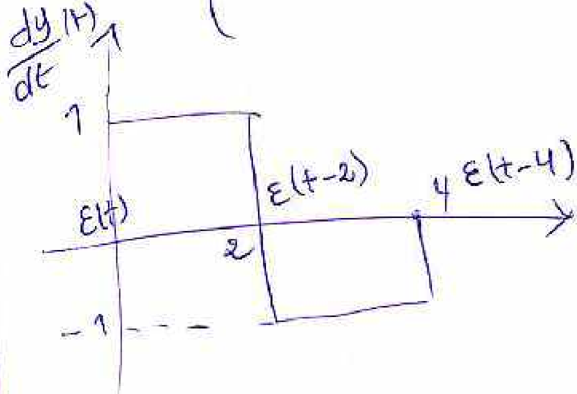
$$y(t) = 2 . \begin{cases} 1+\frac{t-2}{2} & -1<\frac{t-2}{2}<0. \\ 1-\left(\frac{t-2}{2}\right) & 0<\frac{t-2}{2}<1. \\ 0 & \text{ailleurs}. \end{cases}$$

$$y(t) = 2 \begin{cases} \frac{t}{2} & 0<t<2 \\ \frac{4-t}{2} & 2<t<4 \\ 0 & \text{ailleurs}. \end{cases}$$

$$y(t) = \begin{cases} t & 0<t<2 \\ 4-t & 2<t<4 \\ 0 & \text{ailleurs} \end{cases}$$



$$\frac{dy(t)}{dt} = \begin{cases} 1 & 0<t<2 \\ -1 & 2<t<4. \end{cases}$$



$$\frac{dy(t)}{dt} = 1\left(\varepsilon(t)-\varepsilon(t-2)\right)-1\left(\varepsilon(t-2)-\varepsilon(t-4)\right).$$

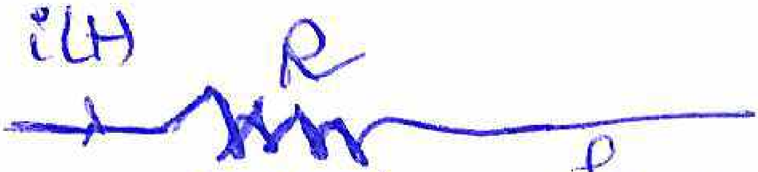
$$\frac{dy(t)}{dt} = \left(\varepsilon(t)-2\varepsilon(t-2)+\varepsilon(t-4)\right).$$

$$\mathcal{L}\left[\frac{dy(t)}{dt}\right] = \left(\frac{1}{p}-\frac{2}{p}e^{-2p}+\frac{1}{p}e^{-4p}\right)$$

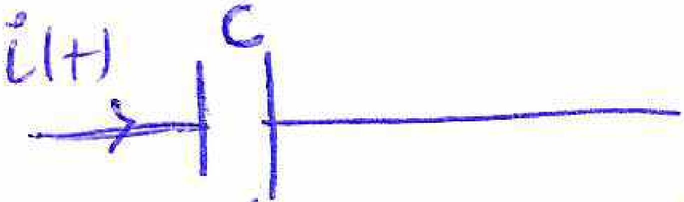
$$\mathcal{L}\left[\frac{dy(t)}{dt}\right] = \frac{1}{p}\left(1-2e^{-2p}+e^{-4p}\right).$$

$$\boxed{Y(p) = \frac{1}{p^2}\left(1-2e^{-2p}+e^{-4p}\right).}$$

Exercice (01):



$$R\,i(t) \xrightarrow{\mathcal{L}} R\,I(p)$$



$$\frac{1}{C}\int_0^t i(t)\,dt \xrightarrow{\mathcal{L}} \frac{1}{Cp}\,I(p).$$

$$L\cdot\frac{di(t)}{dt} \xrightarrow{\mathcal{L}} L\,I(p)\,p$$