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أشغال عامة - ٢٠١٣

فونديشن رابعة
أشغال
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Foundations

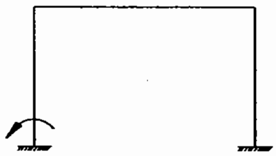
3

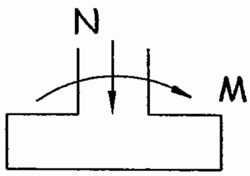
Isolated Footings with M & N

3 Isolated footings subjected to M & N:-

- تتولد العزوم على القواعد نتيجة :-

(1) عزم صريح على العمود (مثل الاعمدة فى ال fixed frames).

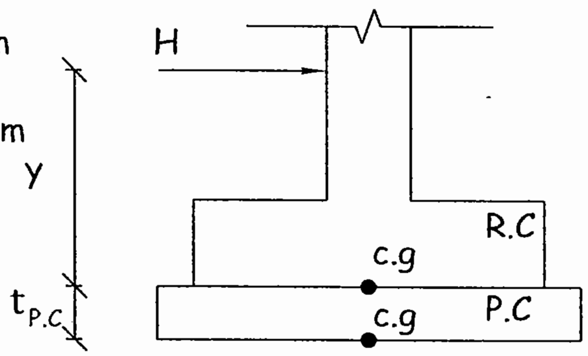




(2) قوة افقية تؤثر على العمود على مسافة من c.g القاعدة.

$M = H * y \longrightarrow$ if $t_{P.C} < 20$ cm

$M = H * (y + t_{P.C}) \longrightarrow$ if $t_{P.C} \geqslant 20$cm

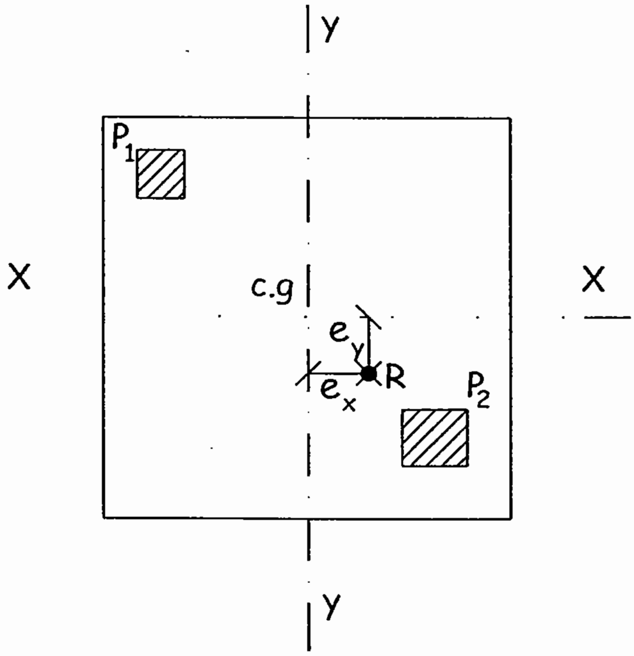


(3) مركز التحميل لا ينطبق على c.g الاساس.

$M_x = R * e_y$

$M_y = R * e_x$

where $R = P_1 + P_2$



❋❋ Types of moments on footing:-

① Permenant Moment:- العزم الدائم

- وهى العزوم الناتجة عن الأحمال الدائمة Gravity loads - Dead loads
وهى عزوم تكون ثابتة المقدار والاتجاه.
- ويكون التعامل معها بأن نلغيها عن طريق أسلوب ترحيل القواعد.

② Variable Moment:- العزم المتغير

- وهى العزوم الناتجة عن الأحمال المتغيرة L.L - Wind loads - earthquake loads
وهى عزوم تكون متغيرة الاتجاه ولكن بقيمة ثابتة.
- ويكون التعامل معها بأن نأخذ تأثيرها معنا عند تصميم القواعد بحيث يكون الاجهاد اسفل الاساس هو

$$f_{act.} = \frac{N}{A} \pm \frac{M \cdot y}{I} \quad \text{but} \quad f_{act.} \neq \frac{N}{A}$$

❋ ملحوظة هامة جدا :-

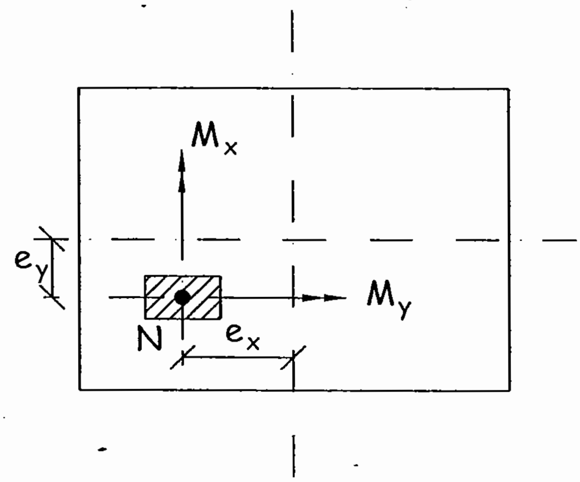
- اذا لم يذكر نوع العزم فى المسألة فاننا نعتبره Variable.
او بمعنى اخر :-

لا تستخدم اسلوب ترحيل القواعد الا اذا ذكر صراحة ان العزم دائم او ناتج عن احمال دائمة.

(A) Design of Footing subjected to normal force and Permenant Moment:-

- اذا كان العمود معرض للعزوم الدائمة M_x , M_y فانه يتم ترحيل القاعدة عن مكانها بمقدار e_x , e_y من محور العمود .

$$e_x = \frac{M_y}{N} \quad , \quad e_y = \frac{M_x}{N}$$



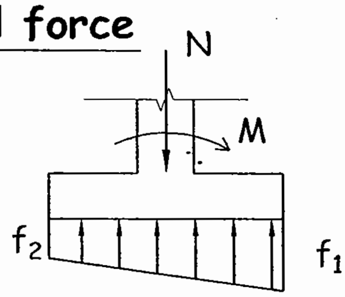
∴ بعد الترحيل

$$\sum M_{x_{c.g}} = M_x - \underbrace{N * e_y}_{=M_x} = \text{Zero}$$

$$\sum M_{y_{c.g}} = M_y - \underbrace{N * e_x}_{=M_y} = \text{Zero}$$

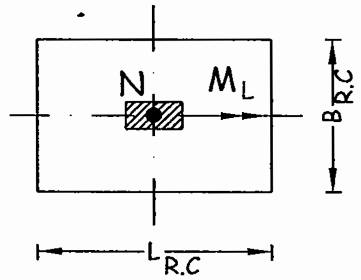
∴ فى النهاية نتيجة الترحيل اصبحت القاعدة عليها N.f فقط .

(B) Design of Footing subjected to normal force and Variable Moment:-



1 Case of Single Moment:-

- فى حالة وجود عزم واحد variable على القاعدة . يوضع فى اى اتجاه ↷ او ↶ بحيث يوازى الطول الكبير للقاعدة (L) .



∴ M_L // L

- وفى هذه الحالة نأخذ معنا تأثير العزم حيث تكون الاجهادات اسفل القاعدة هى :-

$$f_1 = \frac{N}{A} + \frac{M\,y}{I}$$

$$f_2 = \frac{N}{A} - \frac{M\,y}{I}$$

- ويتم اختيار ابعاد الاساس B , L بما يتحقق الشروط الآتية :-

$f_1 \leqslant q_{all.}$

$f_2 > \text{Zero}$ "No tension on soil"

$f_2 \simeq \frac{f_1}{1.5}$ ⟹ to avoid tilting of the footing.

- Steps of Design :-

① Calculate the footing dimensions:

* if $t_{P.C} < 20cm$

assume $L_{R.C} - B_{R.C} = b - a$ → ①

and

$$\frac{N}{L_{R.C} * B_{R.C}} + \frac{6M}{B_{R.C} * (L_{R.C})^2} = q_{all.} \quad ②$$

from 1, 2 get $L_{R.C}$ & $B_{R.C}$

⇒ check

$$\frac{N}{L_{R.C} * B_{R.C}} - \frac{6M}{B_{R.C} * (L_{R.C})^2} > Zero$$

الابعاد بعد التقريب

if UNSAFE

1- Increase L
or
2- Increase B
or
3- Increase L & B

& recheck

then get

$$L_{P.C} = L_{R.C} + 2t_{P.C}$$
$$B_{P.C} = B_{R.C} + 2t_{P.C}$$

* if $t_{P.C} \geq 20cm$

assume $L_{P.C} - B_{P.C} = b - a$ → ①

and

$$\frac{N}{L_{P.C} * B_{P.C}} + \frac{6M}{B_{P.C} * (L_{P.C})^2} = q_{all.} \quad ②$$

from 1, 2 get $L_{P.C}$ & $B_{P.C}$

⇒ check

$$\frac{N}{L_{P.C} * B_{P.C}} - \frac{6M}{B_{P.C} * (L_{P.C})^2} > Zero$$

الابعاد بعد التقريب

if UNSAFE

1- Increase L
or
2- Increase B
or
3- Increase L & B

& recheck

then get

$$L_{R.C} = L_{P.C} - 2t_{P.C}$$
$$B_{R.C} = B_{P.C} - 2t_{P.C}$$

مع محاولة الحفاظ على الشرط $L - B = b - a$

- ولكن هناك اسلوب اخر بديل لحساب ابعاد القاعدة بدلا من حل معادلتين فى مجهولين من الدرجة الثالثة كالاتى :-

❋ to calculate footing dimensions (L,B) in case of N & M_L "variable":-

$$A_{footing} = \left(\frac{N}{q_{all.}}\right) + \text{(some increase)}$$

For effect of N only

Emperical increase for effect of M_L

where : $$A_{footing} = \frac{N}{q_{all.}} + \frac{6\,M}{\sqrt{N * q_{all.}}} = L * B = \checkmark\checkmark \; m^2 \rightarrow (1)$$

and $$L - B = b - a \rightarrow (2)$$

from 1, 2 get L & B

⟹ check

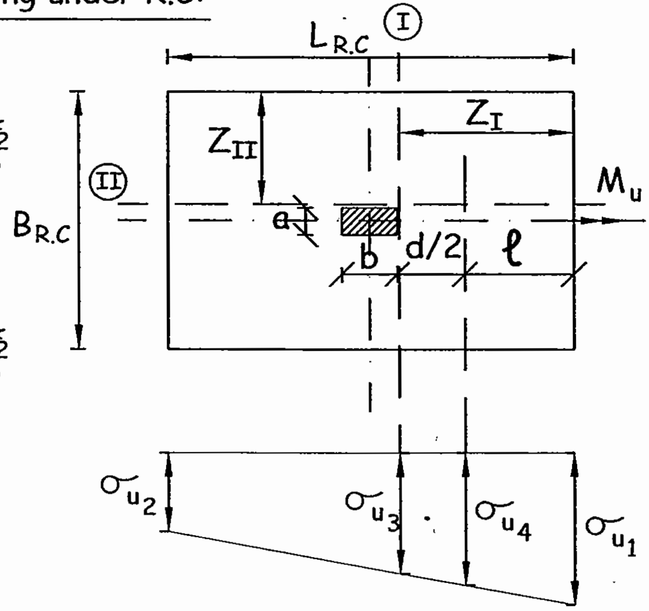
- $$f_1 = \frac{N}{L * B} + \frac{6\,M}{B * (L)^2} \ngtr q_{all.}$$
- $$f_2 = \frac{N}{L * B} - \frac{6\,M}{B * (L)^2} \nless \text{zero}$$

if UNSAFE: increase L & B satisfying the condition $L - B = b - a$

(2) Calculate the ultimate stresses acting under R.C:

$$\sigma_{u_1} = \frac{N * 1.5}{L_{R.C} * B_{R.C}} + \frac{6*1.5\,M_L}{B_{R.C} * (L_{R.C})^2}$$

$$\sigma_{u_2} = \frac{N * 1.5}{L_{R.C} * B_{R.C}} - \frac{6*1.5\,M_L}{B_{R.C} * (L_{R.C})^2}$$



③ Critical sections for moments:-

- على وش العمود مباشرة من الناحيتين (مع ناحية رأس السهم).

For Sec. (I-I)

- get $Z_I = \frac{1}{2}\,[L_{R.C} - b]$
- get $\sigma_{u_3} = \sigma_{u_2} + \frac{L_{R.C} - Z_I}{L_{R.C}}\,[\sigma_{u_1} - \sigma_{u_2}]$
- get $\sigma_{ave_I} = \frac{\sigma_{u_1} + \sigma_{u_3}}{2}$

$\therefore M_{u_I} = \sigma_{ave_I} * \frac{(Z_I)^2}{2} * 1m$

$\therefore d_I = C_1 \sqrt{\frac{M_{u_I\,(KN.m)} * 10^6}{f_{cu\,(N/mm^2)} * 1000}}$ (mm)

For Sec. (II-II)

- get $Z_{II} = \frac{1}{2}\,[B_{R.C} - a]$
- get $\sigma_{ave_{II}} = \frac{\sigma_{u_1} + \sigma_{u_2}}{2} = \frac{N * 1.5}{L_{R.C} * B_{R.C}}$

$\therefore M_{u_{II}} = \sigma_{ave_{II}} * \frac{(Z_{II})^2}{2} * 1m$

$\therefore d_{II} = C_1 \sqrt{\frac{M_{u_I\,(KN.m)} * 10^6}{f_{cu\,(N/mm^2)} * 1000}}$ (mm)

Choose the bigger from d_I , d_{II}

> Critical هو ال M_{u_I} سيكون $L - B = b - a$ طالما التزمنا بالشرط

④ Check Shear:-

- على بعد d/2 من وش العمود من ناحية رأس سهم العزم الموجود اصلا على الاساس.

- Calculate $\ell = Z_I - \frac{d}{2}$
- get $\sigma_{u_4} = \sigma_{u_2} + \frac{L_{R.C} - \ell}{L_{R.C}}\,[\sigma_{u_1} - \sigma_{u_2}]$
- get $\sigma_{ave_{shear}} = \frac{\sigma_{u_1} + \sigma_{u_4}}{2}$

$\therefore Q_{su} = \sigma_{ave_I} * \ell * 1m$ KN/m'

$q_{su} = \frac{Q_{su} * 10^3}{1000 * d} \not> q_{scu}$

if UNSAFE increase d and recheck

⑤ Check Punching:-

$$Q_p = 1.5 * N - \underset{punch}{\sigma_{ave}} * \underset{\text{punching area}}{[(a+d)(b+d)]} \quad (KN)$$

where: $\underset{punch}{\sigma_{ave}} = \dfrac{\sigma_{u_1} + \sigma_{u_2}}{2}$

$$q_p = \frac{Q_p * 10^3}{d_{(mm)} * [(a+d)_{(mm)} (b+d)_{(mm)}] * 2} \quad (N/mm^2)$$

Check $q_p \ngtr q_{pcu}$ ← مقاومة الخرسانة فى القص الثاقب

where: $$q_{pcu} = 0.316 \left(0.5 + \frac{a}{b}\right) \sqrt{\frac{F_{cu}}{\gamma_c}} \quad N/mm^2$$

If UNSAFE ⟹ Increase d & recheck ⟹ نعيد الحسابات من اول

⑥ Calculate final thickness of R.C:

$$t = d + 7\ cm$$

d ← safe for shear & punching

⑦ R.F.T:

- $$A_{s_I} = \frac{M_{u_I} * 10^6}{F_y * J * d_{(mm)}} = ✓✓ \quad mm^2/m'$$
- $$A_{s_{II}} = \frac{M_{u_{II}} * 10^6}{F_y * J * d_{(mm)}} = ✓✓ \quad mm^2/m'$$

and check $\underset{mm/m'}{A_{s_{min.}}} = 1.5 * d_{mm}$ or $5\phi12/m'$ } ايهما اكبر

⑧ Details:

See the Example.

Example:-

It is required to design a rectangular isolated footing to support a column carries a normal load of 1300 KN and a moment due to gravity loads of 200 KN.m.

Given:-

- $t_{P.C} = 30$ cm   Column's dimensions (35X65 cm)
- $q_{all.} = 1.5\ kg/cm^2$
- $f_{cu} = 30\ N/mm^2$, $f_y = 360\ N/mm^2$

Solution:-

∵ The given moment is due to gravity loads

∴ The moment is permenant moment, its direction is parellel to the dimension L of the footing.

$$e = \frac{M}{N} = \frac{200}{1300} = 0.154\ m$$

① Calculate the footing area :-

∵ $t_{P.C} = 30\ cm > 20\ cm$

∴ $A_{P.C} = \frac{1300}{150} = 8.67\ m^2 = B_{P.C} * L_{P.C}$ ------→ ①

$L_{P.C} - B_{P.C} = b - a$

$L_{P.C} - B_{P.C} = 0.65 - 0.35 = 0.3\ m$ ------------→ ②

Solving ① & ② :-

$8.67 = B_{P.C} * (0.3 + B_{P.C}) \Longrightarrow B_{P.C} = 2.79 \approx 2.80\ m$

$L_{P.C} = 0.3 + B_{P.C} = 0.3 + 2.80 = 3.10\ m$

$L_{R.C} = 3.10 - 2 * 0.30 = 2.50\ m$

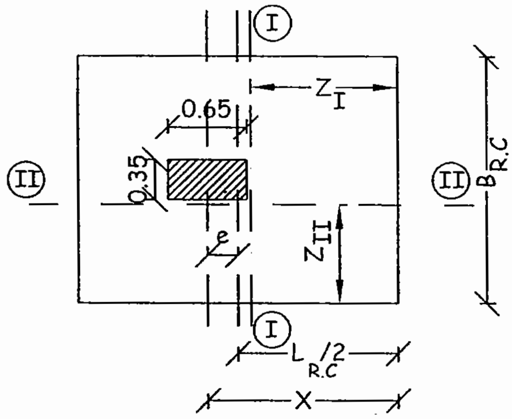
$B_{R.C} = 2.80 - 2 * 0.30 = 2.20\ m$

② Ultimate loads :-

$$q_u = \frac{1.5 * 1300}{2.50 * 2.20} = 355 \ KN/m^2$$

$$X = \frac{L_{R.C}}{2} + e$$

$$= \frac{2.50}{2} + 0.154 = 1.404 \ m$$



③ Critical section for moment (M) :-

** For sec (I-I)

$$Z_I = X - \frac{0.65}{2} = 1.404 - \frac{0.65}{2} = 1.079 \ m$$

$$\therefore M_{u_I} = 355 * \frac{1.079^2}{2} * 1m$$

$$= 206.65 \ KN.m$$

$$d_I = C_1 \sqrt{\frac{M_{(KN.m)} * 10^6}{f_{cu\ (N/mm^2)} * 1000}}$$

$$= 5 \sqrt{\frac{206.65 * 10^6}{30 * 1000}}$$

$$= 414.9 \ mm$$

** For sec (II-II)

$$Z_{II} = \frac{1}{2}(2.20 - 0.35) = 0.925 \ m$$

$$\therefore M_{u_{II}} = 355 * \frac{0.925^2}{2} * 1m$$

$$= 151.87 \ KN.m$$

$$d_{II} = C_1 \sqrt{\frac{M_{(KN.m)} * 10^6}{f_{cu\ (N/mm^2)} * 1000}}$$

$$= 5 \sqrt{\frac{151.87 * 10^6}{30 * 1000}}$$

$$= 355.75 \ mm$$

$$\boxed{\text{take } d = d_I = 430 \ mm}$$

④ Check Shear :-

$$\ell = Z_I - \frac{d}{2} = 1.079 - \frac{0.43}{2} = 0.864 \text{ m}$$

$$Q_{su} = q_u * \ell * 1m = 355 * 0.864 * 1m = 306.7 \quad KN/m'$$

$$q_{su} = \frac{Q_{su} * 10^3}{d * B} = \frac{306.7 * 10^3}{430 * 1000} = 0.713 \quad N/mm^2$$

$$q_{scu} = 0.16 \sqrt{\frac{F_{cu}}{\gamma_c}} = 0.16 \sqrt{\frac{30}{1.5}} = 0.715 \quad N/mm^2$$

$$\because q_{su} < q_{scu} \Rightarrow \therefore \text{SAFE}$$

⑤ Check Punching Shear :-

على بعد $\frac{d}{2}$ من وش العمود من كل ناحية

$$Q_{p_u} = 1.5 * 1300 - q_u (X * Y)$$

$$= 1.5 * 1300 - 355 * (1.08 * 0.78)$$

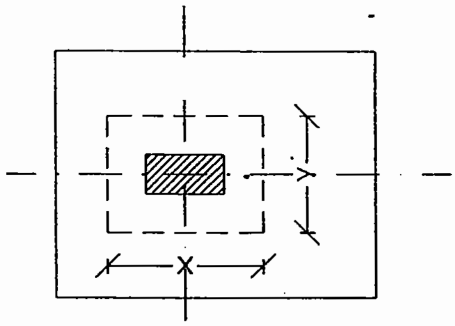
$$= 1650.95 \quad KN$$

$$q_{p_u} = \frac{1650.95 * 10^3}{430 * [1080 + 780] * 2}$$

$$= 1.03 \quad N/mm^2$$

$$q_{p_{cu}} = 0.316 \left(0.5 + \frac{0.35}{0.65}\right) \sqrt{\frac{30}{1.5}}$$

$$= 1.413 \quad N/mm^2$$



$X = 0.65 + 0.43 = 1.08$ m

$Y = 0.35 + 0.43 = 0.78$ m

$$\because q_{p_u} < q_{p_{cu}} \Rightarrow \therefore \text{SAFE}$$

⑥ Final Thickness :-

$$d_{final} = 430 \text{ mm}$$

$$t_{final} = 430 + 70\,(\text{cover}) = 500 \text{ mm}$$

⑦ R.F.T :-

$$A_{s_{min.}} = \begin{cases} 1.5 * d_{mm} = 1.5 * 430 = 645 \text{ mm}^2/\text{m}' \\ 5 \phi 12/\text{m}' = 565 \text{ mm}^2/\text{m}' \end{cases}$$

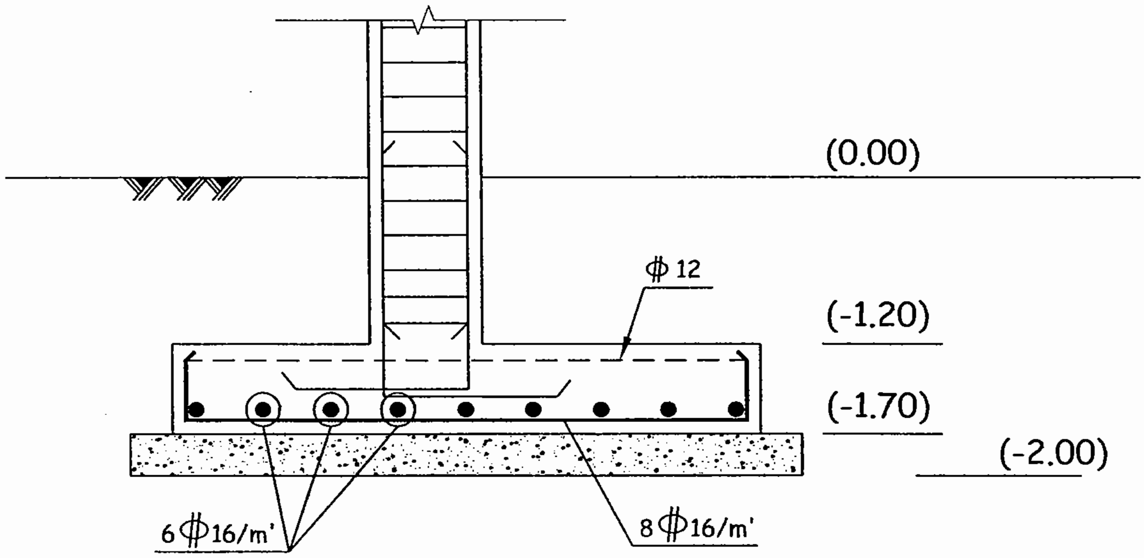
$$A_{s_I} = \frac{M_{u_I} * 10^6}{F_y * 0.826 * d} = \frac{206.65 * 10^6}{360 * 0.826 * 430} = 1616.16 \text{ mm}^2/\text{m}'$$

8 ϕ 16/m'

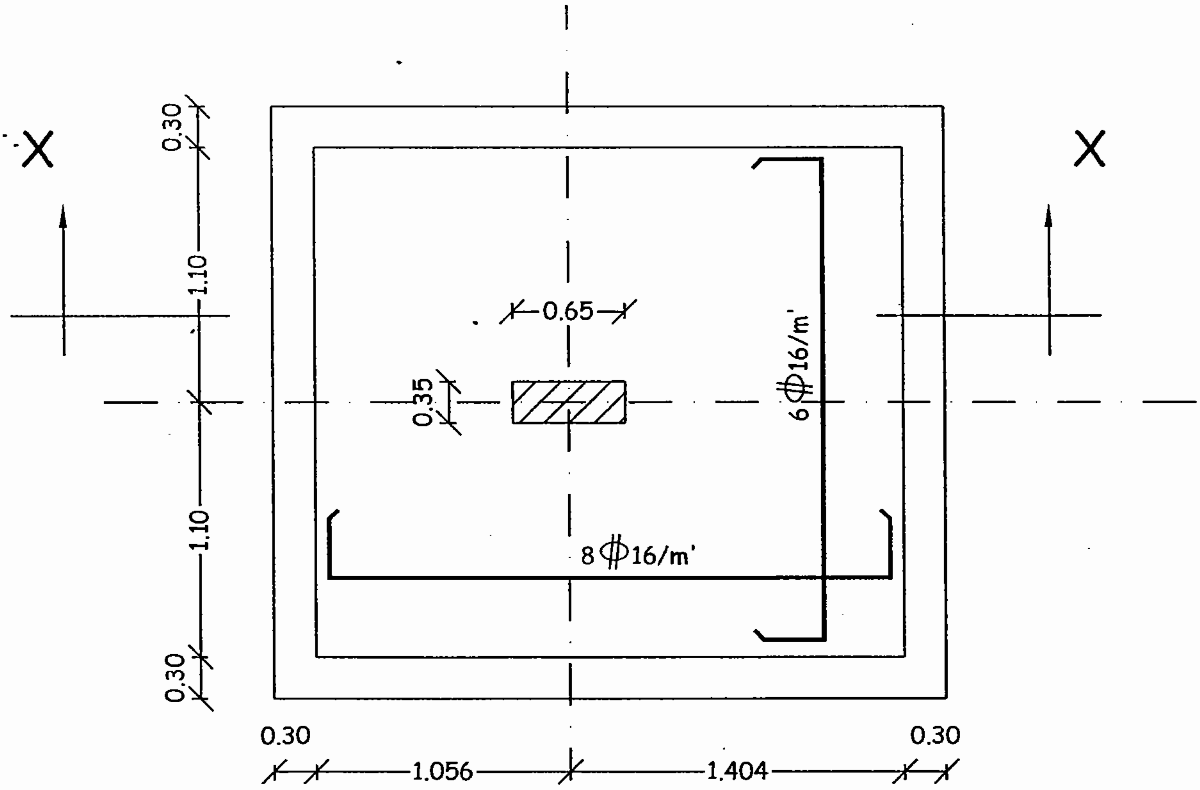
$$A_{s_{II}} = \frac{M_{u_{II}} * 10^6}{F_y * 0.826 * d} = \frac{151.87 * 10^6}{360 * 0.826 * 430} = 1187.7 \text{ mm}^2/\text{m}'$$

6 ϕ 16/m'

⑧ Details:



SEC. (X-X)



PLAN

Example:-

It is required to design a rectangular isolated footing to support a column of (300 * 650 mm) subjected to a normal load of 1200 KN and a moment of 150 KN.m.

Given:-

- $t_{P.C} = 40$ cm
- $q_{all.} = 1.3$ kg/cm^2
- $f_{cu} = 30$ N/mm^2 , $f_y = 360$ N/mm^2

Solution:-

(1) Calculate the footing area :-

$\because \quad t_{P.C} = 40 \text{ cm} > 20 \text{ cm}$

$$\therefore \quad A_{P.C} = \frac{N}{q_{all.}} + \frac{6\,M}{\sqrt{N * q_{all.}}} = \frac{1200}{130} + \frac{6 * 150}{\sqrt{1200 * 130}}$$

$$= 11.5 \text{ m}^2 = B_{P.C} * L_{P.C} \quad \text{------------} \rightarrow (1)$$

$L_{P.C} - B_{P.C} = b - a$

$$L_{P.C} - B_{P.C} = 0.65 - 0.30 = 0.35 \text{ m} \quad \text{------------} \rightarrow (2)$$

Solving (1) & (2) :-

$$11.5 = B_{P.C} * (0.35 + B_{P.C}) \Rightarrow B_{P.C} = 3.22 \approx 3.25 \text{ m}$$

$$L_{P.C} = 0.35 + B_{P.C} = 0.35 + 3.25 = 3.60 \text{ m}$$

Checks:-

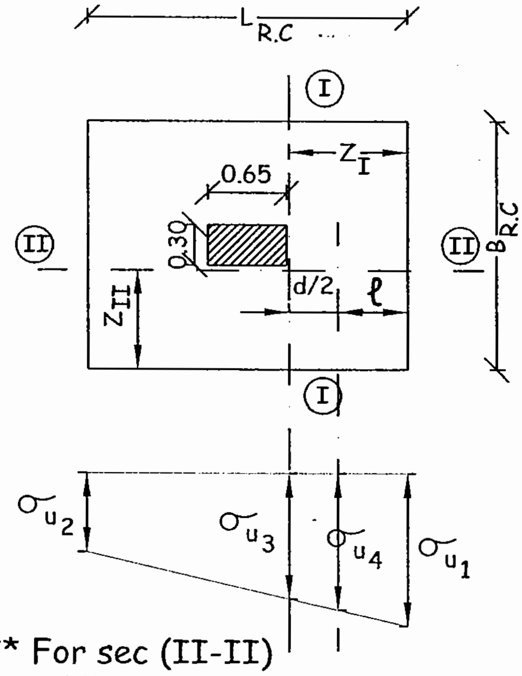
$$f_1 = \frac{1200}{3.55 * 3.25} + \frac{6 * 150}{3.25 * (3.55)^2} = 125.98 \text{ KN/m}^2 < q_{all.}$$

$$f_2 = \frac{1200}{3.55 * 3.25} - \frac{6 * 150}{3.25 * (3.55)^2} = 82 \text{ KN/m}^2 > \text{zero}$$

$$L_{R.C} = 3.55 - 2 * 0.40 = 2.75 \text{ m}$$

$$B_{R.C} = 3.25 - 2 * 0.40 = 2.45 \text{ m}$$

② Ultimate loads :-



$$\sigma_{u_1} = \frac{1.5 * 1200}{2.75 * 2.40} + \frac{1.5 * 6 * 150}{2.40 * (2.75)^2}$$

$$= 347.1 \text{ KN/m}^2$$

$$\sigma_{u_2} = \frac{1.5 * 1200}{2.75 * 2.40} - \frac{1.5 * 6 * 150}{2.40 * (2.75)^2}$$

$$= 198.4 \text{ KN/m}^2$$

③ Critical section for moment (M) :-

** For sec (I-I)

$$Z_I = \frac{1}{2}(2.75 - 0.65) = 1.05 \text{ m}$$

$$\sigma_{u_3} = \sigma_{u_2} + \frac{L_{R.C} - Z_I}{L_{R.C}} [\sigma_{u_1} - \sigma_{u_2}]$$

$$= 198.4 + \frac{2.75 - 1.05}{2.75} [347.1 - 198.4]$$

$$= 290 \text{ KN/m}^2$$

$$\sigma_{ave._I} = \frac{347.1 + 290}{2} = 319 \text{ KN/m}^2$$

$$\therefore M_{u_I} = 319 * \frac{1.05^2}{2} * 1\text{m}$$

$$= 175.84 \text{ KN.m}$$

$$d_I = 5 \sqrt{\frac{175.84 * 10^6}{30 * 1000}}$$

$$= 382.79 \text{ mm}$$

** For sec (II-II)

$$Z_{II} = \frac{1}{2}(2.40 - 0.30) = 1.05 \text{ m}$$

$$\sigma_{ave._{II}} = \frac{347.1 + 198.4}{2} = 273 \text{ KN/m}^2$$

$$\therefore M_{u_{II}} = 273 * \frac{1.05^2}{2} * 1\text{m}$$

$$= 150.49 \text{ KN.m}$$

$$d_{II} = 5 \sqrt{\frac{150.49 * 10^6}{30 * 1000}}$$

$$= 354.13 \text{ mm}$$

take $d = d_I = 430$ mm

④ Check Shear :-

$$\ell = Z_I - \frac{d}{2} = 1.05 - \frac{0.43}{2} = 0.835 \text{ m}$$

$$\sigma_{u_4} = \sigma_{u_2} + \frac{L_{R.C} - \ell}{L_{R.C}} [\sigma_{u_1} - \sigma_{u_2}]$$

$$= 198.4 + \frac{2.75 - 0.835}{2.75} [347.1 - 198.4] = 301.95 \text{ KN/m}^2$$

$$\sigma_{ave._{shear}} = \frac{347.1 + 301.95}{2} = 324.5 \text{ KN/m}^2$$

$$Q_{su} = \sigma_{ave._{shear}} * \ell * 1m = 324.5 * 0.835 * 1m = 270.95 \text{ KN/m'}$$

$$q_{su} = \frac{Q_{su} * 10^3}{d * B} = \frac{270.95 * 10^3}{430 * 1000} = 0.63 \text{ N/mm}^2$$

$$q_{scu} = 0.16 \sqrt{\frac{F_{cu}}{\gamma_c}} = 0.16 \sqrt{\frac{30}{1.5}} = 0.715 \text{ N/mm}^2$$

$$\because q_{su} < q_{scu} \Rightarrow \therefore \text{SAFE}$$

⑤ Check Punching Shear :-

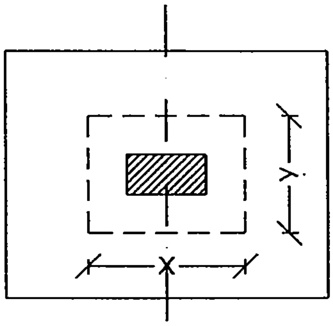
$$\sigma_{ave._{punching}} = \frac{347.1 + 198.4}{2} = 273 \text{ KN/m}^2$$

$$Q_{p_u} = 1.5 * 1200 - 273 * (1.08 * 0.73)$$

$$= 1584.77 \text{ KN}$$

$$q_{p_u} = \frac{1584.77 * 10^3}{430 * [1080 + 730] * 2}$$

$$= 1.018 \text{ N/mm}^2$$



$$X = 0.65 + 0.43 = 1.08 \text{ m}$$

$$Y = 0.30 + 0.43 = 0.73 \text{ m}$$

$$q_{p_{cu}} = 0.316 \left(0.5 + \frac{0.30}{0.65}\right) \sqrt{\frac{30}{1.5}}$$

$$= 1.36 \text{ N/mm}^2$$

$$\because q_{p_u} < q_{p_{cu}} \Rightarrow \therefore \text{SAFE}$$

⑥ Final Thickness :-

$$d_{final} = 430 \quad mm$$

$$t_{final} = 430 + 70\,(cover) = 500 \quad mm$$

⑦ R.F.T :-

$$A_{s_{min.}} = \left\{ \begin{array}{ll} 1.5 * d_{mm} = 1.5 * 430 = 645 & mm^2/m' \\ 5 \phi 12/m' = 565 & mm^2/m' \end{array} \right\} \therefore O.K$$

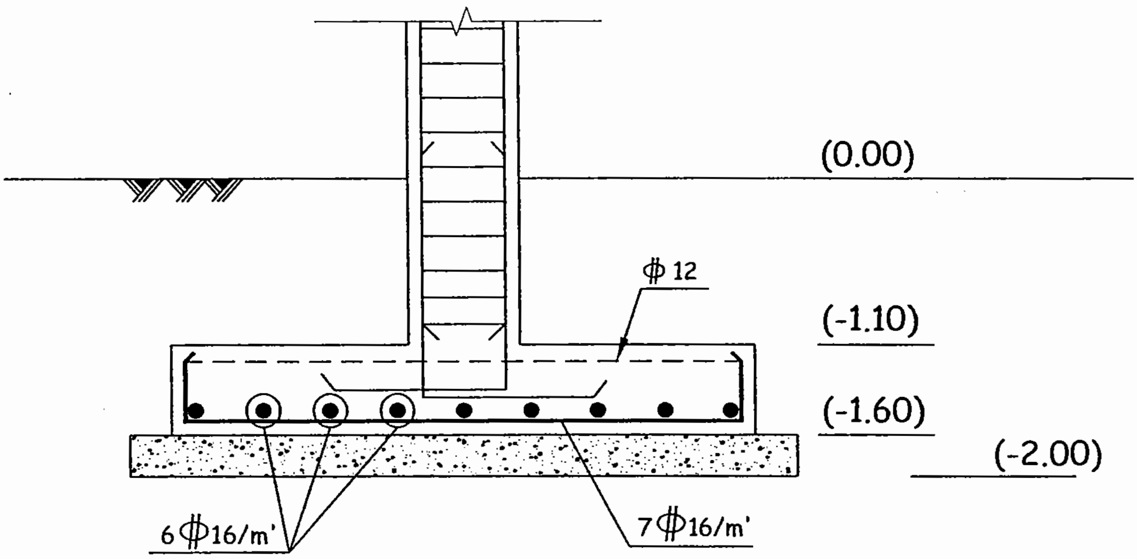
$$A_{s_I} = \frac{M_{u_I} * 10^6}{F_y * 0.826 * d} = \frac{175.84 * 10^6}{360 * 0.826 * 430} = 1375.2 \quad mm^2/m'$$

7 ϕ 16/m'

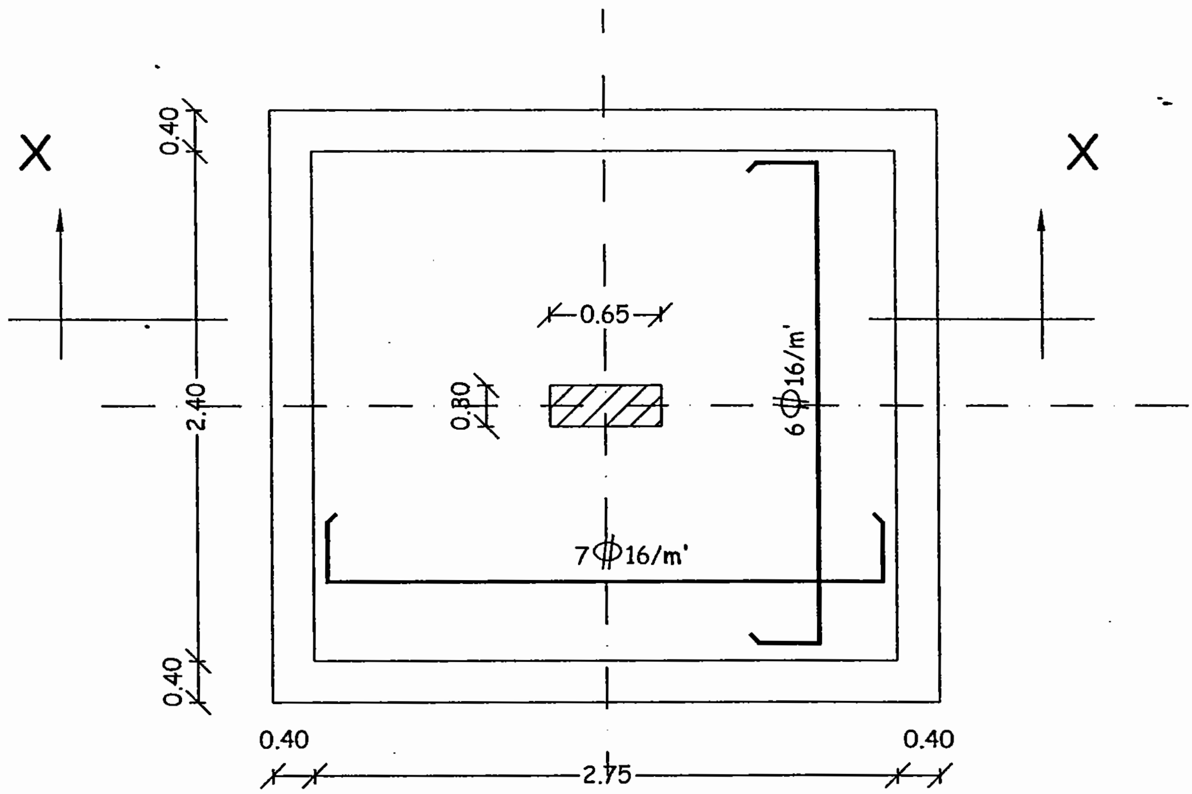
$$A_{s_{II}} = \frac{M_{u_{II}} * 10^6}{F_y * 0.826 * d} = \frac{150.49 * 10^6}{360 * 0.826 * 430} = 1176.95 \quad mm^2/m'$$

6 ϕ 16/m'

⑧ Details:



SEC. (X-X)



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